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Approximate Calculation of Eigen-Values of Linear Viscously Damped System with Passive Damping Element

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Abstract: The paper presents an approximative method for eigen-solution of non-classically damped linear system representing classically damped structure equipped with passive damping element (viscous damper). The proposed procedure avoids using a numerically demanding state-space approach. It operates in the original dimension of the problem and utilizes the real eigen-modes for its further size reduction. The method is based on the dividing of the damping matrix to classical and non-classical part and the application of the perturbation strategy. The accuracy of the procedure is demonstrated by considering numerical examples.

Theoretical background

The proposed method is based on the division of the damping matrix of the linear viscously damped system, which represents the structure with the damper, in the form:

$$\mathbf{C} = \mathbf{C_p} + \mathbf{d} \cdot \mathbf{d^T}. \tag{1}$$

Matrix $\mathbf{C_p}$ is the proportional part of the damping matrix, while the product consisting of vector $\mathbf{d}$ stands for the non-proportional part arising from the presence of the damper. The eigen-values λ of the complete system are given, see [1], by solving the equation:

$$\frac{1}{\lambda} + \sum_{i=1}^{n} \frac{\tilde{d}_i^{\,2}}{\lambda^2 + \lambda \widetilde{C}_{p_{i,i}} + {\omega_i}^2} = 0, \tag{2}$$

where n is a number of the eigen-modes of the complete system, $\tilde{d}_i$ is given by product of transposed i-th eigen-mode and vector $\mathbf{d}$. Element $\widetilde{C}_{p_{i,i}}$ is i-th diagonal element of modal damping matrix $\widetilde{\mathbf{C}}_p$ calculated from matrix $\mathbf{C}_p$ and ω_i is i-th undamped angular eigen-frequency of the system. The solution of the j-th eigen-value λ_j can be sought in a form:

$$\lambda_j = \lambda_{0,j} + \Delta\lambda_j. \tag{3}$$

Symbol $\lambda_{0,j}$ stands for the known or easily obtained eigen-solution of the same system with e.g. different damping ratio of the damper, while $\Delta\lambda_j$ represents its deviation from the exact solution λ_j. Inserting solution (3) into Eq. (2) and performing few mathematical operations lead to:

$$\begin{aligned} &\prod_{i=1}^{n} \left(\alpha_{i,j} + \beta_{i,j}\Delta\lambda_j + \Delta\lambda_j^2\right) + (\lambda_{0,j} + \Delta\lambda_j) \sum_{l=1}^{n} \tilde{d}_l^{\,2} \prod_{\substack{i=1 \\ i\neq l}}^{n} (\alpha_{i,j} + \beta_{i,j}\Delta\lambda_j + \Delta\lambda_j^2) = 0. \\ &\alpha_{i,j} = \lambda_{0,j}^2 + \lambda_{0,j}\widetilde{C}_{p_{i,i}} + {\omega_i}^2, \qquad \beta_{i,j} = 2\lambda_{0,j} + \widetilde{C}_{p_{i,i}}. \end{aligned} \tag{4}$$

After expansion of Eq. 4 the numerically favorable simplification consisting in keeping only the first or the first two powers of $\Delta\lambda_j$ can be made. Then $\Delta\lambda_j$ is easily calculated and subsequently the eigen-value λ_j can be determined.

Numerical experiments

The accuracy of the suggested approach was analysed for the numerical model of the existing tower with the vibration absorber. The exactness of its calculated eigen-solution was investigated parametrically for a set of various damping ratios of the absorber ($\zeta_d = 0 \div 0{,}9$), while the proportional damping matrix of the tower remained constant (structural damping ratio $\zeta = 0{,}005$). Two approximate eigen-solutions which differed in the selection of the reference values λ_0 in Eq. (3) were calculated. The first selection λ_0 was represented by the eigen-values λ_p corresponding to the classically damped system. In the second case the eigen-values λ_n of the realistically damped system, which were calculated under the assumption of the neglecting of the off-diagonal elements of the modal damping matrix, were chosen. The eigen-values λ_n are often used as the approximate eigen-solution of non-classically damped systems and thus served together with the exact eigen-solution for the comparison and the assessment of the suitability of the proposed method. The accuracy was assessed in terms of the relative errors that were calculated separately for the imaginary and the real part of the eigen-values. The most significant errors for the solutions using the first two powers of $\Delta\lambda$ are depicted in Fig. 1.

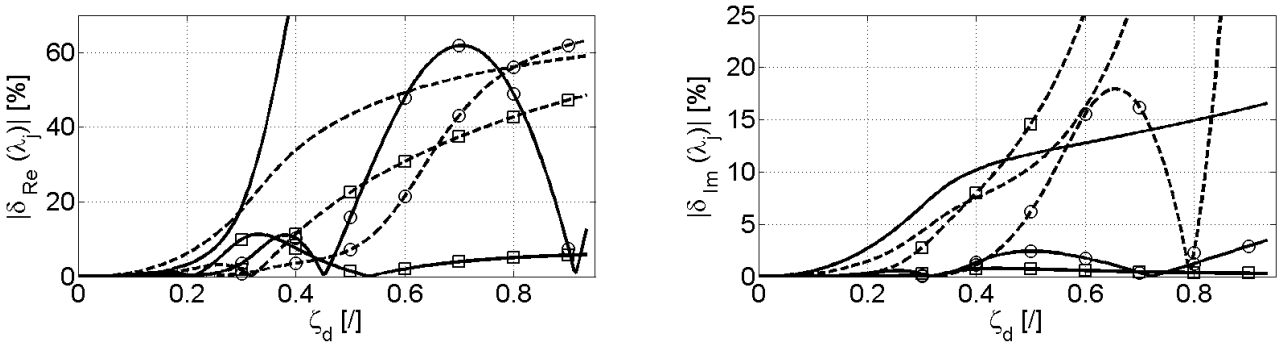

Fig. 1: The percentage relative error of the real (left) and imaginary (right) part of calculated eigen-values λ_j as a function of damping ratio ζ_d (solid line - 2^{nd} eigen-value; dashed line - 3^{rd} eigen-value) (no markers - traditional approximate solution λ_n; ○ - proposed solution based on λ_n; □ - proposed solution based on λ_p)

The comparison of the proposed and the exact eigen-solutions showed higher errors in the real (damping) than in the imaginary (frequency) part of the eigen-values. The method provided acceptable and higher accuracy than the traditional approximate approach in the whole investigated interval of ζ_d for the eigen-values, that were not directly connected with the frequency tuning of the absorber. The significant errors came up for eigen-values which were close to eigen-value of the absorber, i.e. for the second and the third eigen-value. In these cases the method provided sufficient accuracy (below 10%) only in the limited interval in the vicinity of the classical damping ratio. Nevertheless in comparison with the classical approximate solution λ_n the range of the applicability is wider. The best result was reached for the solution using two powers of $\Delta\lambda$ and based on initial values λ_n. In this case the limiting value of the damping ratio of the absorber for which the errors were reasonable was 35%, while for traditional approximative approach it was only 25%. Beyond this value the errors were significant and more sophisticated approach should be employed.

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References

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