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AEROELASTIC DIVERGENCE MODELED BY MEANS OF THE STOCHASTIC RESONANCE

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Abstract: *The divergence is one of the most important and dangerous phenomenon of aeroelastic post-critical states occurring at a prismatic slender beam in a cross-flow. This phenomenon manifests by stable periodic hopping between two nearly constant limits perturbed by random noises. Experimental observation and numerical simulation motivates an idea to model this process as the effect of the stochastic resonance. Being observed and practically used in a number of disciplines in physics (optics, plasma physics, atd.) its mathematical basis follows in the most simple case from properties of the Duffing equation with negative linear part of the stiffness. The occurrence of this phenomenon depends on certain combinations of input parameters, which can be determined theoretically and verified experimentally in the wind tunnel. Parameter combinations leading to the stochastic resonance (or divergence) should be avoided in practice in order to eliminate any danger of the bridge deck collapse due to aeroelastic effects.*

Keywords: Stochastic resonance, Interwell hopping, Non-linear vibration, Aeroelastic divergence, Post-critical states.

1. Phenomenon of the Stochastic Resonance

Stochastic resonance is a phenomenon, which has been surmised in physical chemistry in early forties, see e.g. Kramers (1940). Many years later several branches in theoretical and experimental physics identified this phenomenon and applied this one in optics and plasma physics, see e.g. Inchiosa & Bulsara (1996), or review paper Gammaitoni et al. (1998). Hundreds papers more have been published until now, including also a couple of monographs, for instance McDonnell et al. (2008). Authors outlined some basic properties of the stochastic resonance quite recently, Náprstek (2014). Stochastic resonance represents in principle a very stable interwell hopping of the non-linear oscillator of the Duffing type under suitable combination of the deterministic (harmonic) and white noise related random excitation. Many physical effects can be modeled using this approach.

Let us assume the nonlinear mass-unity oscillator with one degree of freedom under additive excitation, which consists of harmonic and random components:

$$\begin{aligned} \dot{u} &= v; \\ \dot{v} &= -2\omega_b \cdot v - V'(u) + P(t) + \xi(t). \end{aligned} \tag{1}$$

$V(u)$ - potential energy being introduced in a form corresponding with the Duffing equation:

$$V(u) = -\frac{\omega_0^2}{2}u^2 + \frac{\gamma^4}{4}u^4 \qquad \Rightarrow \qquad V'(u) = dV(u)/du = -\omega_0^2 \cdot u + \gamma^4 \cdot u^3 \tag{2}$$

$\xi(t)$ - Gaussian white noise of intensity $2\sigma^2$ respecting conditions:

$$E\{\xi(t)\} = 0 \; ; \qquad E\{\xi(t)\xi(t')\} = 2\sigma^2 \cdot \delta(t - t') \tag{3}$$

$P(t) = P_o \exp(i\Omega t)$ - external harmonic force with frequency Ω. Amplitude P_o should be understood per unit mass. Symbols ω_0 and ω_b have a usual meaning of the circular eigen-frequency and circular damping frequency of the associated linear system. The linear part of the $V'(u)$ is negative making the system metastable in the origin, while the cubic part acts as stabilizing factor beyond a certain interval of displacement u.

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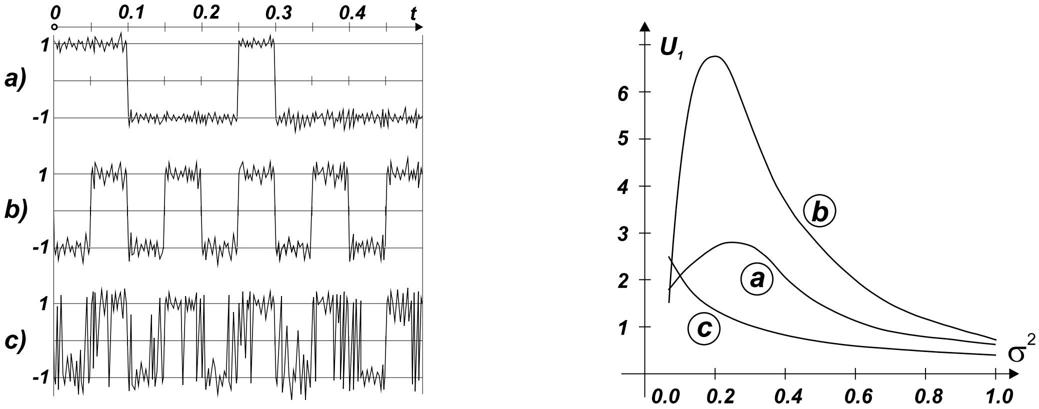

Fig. 1: Response of the system with combined excitation - white noise and harmonic (left). The spectral amplification of the system response due to stochastic resonance effect (right).

Taking into account that random noise in Eq. (1) has an additive character, the appropriate Fokker - Planck (FP), e.g. Pugachev & Sinitsyn (1987), equation can be easily written out:

$$\kappa_u = v\ ; \quad \kappa_v = -2\omega_b \cdot v - V'(u) + P(t)\ ; \kappa_{vv} = 2\sigma^2 \tag{4}$$

$$\frac{\partial p(u,v,t)}{\partial t} = -v\frac{\partial p(u,v,t)}{\partial u} + \frac{\partial}{\partial v}[2\omega_b \cdot v + V'(u) - P(t)]p(u,v,t) + \sigma^2\frac{\partial^2 p(u,v,t)}{\partial v^2} \tag{5}$$

together with boundary conditions:

$$\lim_{u,v\rightarrow\pm\infty} p(u,v,t) = 0\ ; \quad p(u,v,0) = \delta(u,v). \quad \text{(Dirac function)} \tag{6}$$

Some illustrative numerical results have been outlined in Fig. 1. It presents numerical simulations using the basic system Eq. (1). In individual parts the influence of rising white noise intensity σ^2, which acts together with a harmonic force onto the system, can be seen. For very low level of the noise the harmonic component is hardly able to overcome the inter-well barrier and therefore only seldom irregular jumps between stable points occur. In local regimes the system response is relatively small and nearly linear, see Fig. 1 - left (a). Optimal ratio of the noise intensity and the amplitude of the harmonic force results for its certain frequency in the system response containing a visible periodic part corresponding with the frequency of the external harmonic excitation component. The response is not harmonic and contains many higher harmonics. However the basic frequency of the interwell hopping is stable making possible to reconstruct the original harmonic component hidden in the background, see Fig. 1 - left (b). Fig. 1 - left (c) demonstrates the state of a large superiority of the noise. Increasing the noise level can counteract the aforementioned process and thereby the stochastic resonance effect vanishes. However, the useful harmonic component could be still detected when stiffness parameters (ω_0^2, γ^4) and damping (ω_b) are adjusted appropriately with respect to the deterministic excitation component frequency Ω.

General characteristics of the response can be obtained investigating the FP equation (5) together with conditions Eqs (6). Examining the right side of Eq. (5), it is obvious that stationary solution cannot exist due to time dependent coefficient κ_v. This factor is given by deterministic excitation component. Otherwise the stationary solution exists and can be carried out for Hamiltonian systems using the Boltzmans formula, e.g. Cai & Lin (1988) or Pugachev & Sinitsyn (1987). A number of methods can be used to solve Eq. (1), however an evolution character should be kept at this case. Fig. 1 shows the sensitivity of the system.

2. Example - Harmonically Excited Beam Under Influence of Turbulence Noise

The response of a beam, loaded by the wind with turbulent component, known as the aeroelastic divergence, see e.g. Náprstek & Pospíšil (2012) and many others, initiated the idea to use the theory of stochastic resonance for the explanation of hopping of the beam in between two meta-stable positions, see e.g. Gammaitoni et al. (1998), McDonnell et al. (2008) or Náprstek (2014). This kind of the response has been observed during the wind tunnel measurement focused on the self-induced vibration with the large amplitudes in the non-linear range using the special experimental stand. It represents the working

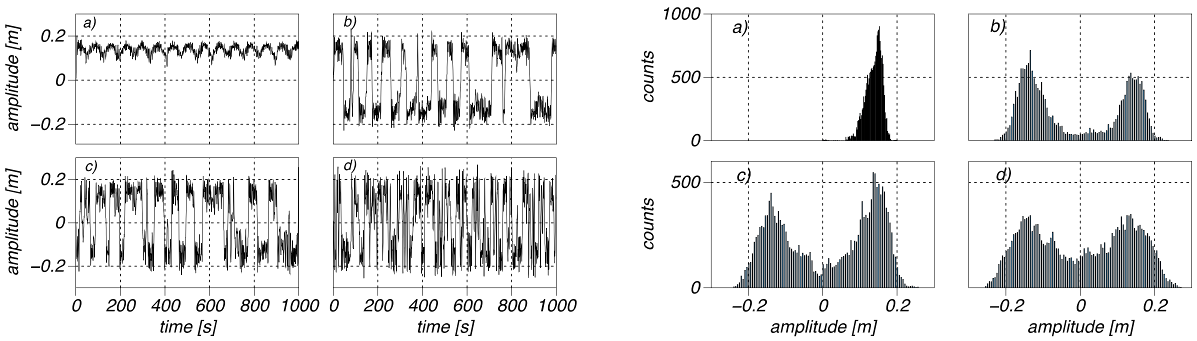

Fig. 2: Time history and probability density functions (histograms) of the response due to change of the noise intensity.

mechanism sensitive to the excitation by the wind. The stand and the experiments are described more in detail in the paper Král et al. (2013).

The effect of the stochastic resonance has been demonstrated by means of numerical simulation on the special experimental stand mentioned above. It accommodates a sectional model of a slender prismatic beam tested in a cross-flow in a wind tunnel. As it can be seen in Fig. 2 - left, time histories for various levels of the noise component are adequate with those demonstrated in Fig. 1 - left. Influence of the noise intensity increase manifests itself from a local nearly linear effect running around one of two semi-stable system positions as far as complex nonlinear process passing through the domain of both semi-stable positions. This result is even more obvious regarding Fig. 2 - right presenting the response probability density for input noise intensities corresponding with the left picture. In particular the probability density shape changes from the local concentration rather of the Gaussian like curve until bimodal probability density being symmetrical and typical for Duffing oscillator with a high level white noise excitation and almost without any harmonic excitation component.

3. Conclusions

The phenomenon of the stochastic resonance has been introduced as a theoretical tool of aeroelastic divergence description. This way reveals to be adequate observing carefully experimental results obtained by comprehensive measurements in a wind channel. With respect to those, the divergence manifests phenomenologically as a periodical hopping between two quasi-static positions with weak random perturbation. This effect can emerge under a relevant combination of periodic (nearly harmonic vortex shading) and random (white noise type) additive excitation as it has been observed experimentally. Conditions of the theoretical stochastic resonance occurrence at the Duffing equation are qualitatively identical and therefore it has been adopted as an adequate theoretical model of the divergence phenomenon. The relevance of this model has been verified analytically by means of the Fokker-Plack equation as well as by numerical solution of corresponding Ito stochastic system. The paper describes also the numerical simulation of the experimental stand used for the aeroelastic testing of profiles, before it has been tested in the turbulent flow. It shows, that under certain "optimal" value of the parameters, the signal-to-noise ratio of the response increases and the resonant-like peak occurs in the amplitude spectra. This makes an optimistic perspective for the experimental analysis, which together with the analytical and numerical ones should continue to obtain better insight into the general tendencies when individual parameters of the system and the input signal are changed.

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