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PLASTIC ZONE EXTENT IN A CCT SPECIMEN DETERMINED FROM THE CRACK-TIP STRESS FIELD APPROXIMATED BY MEANS OF THE WILLIAMS EXPANSION

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Abstract: *A plate with a central crack under uniaxial tension is studied in this work in order to discuss the influence of the number of terms considered in the Williams expansion (derived for approximation of the crack tip stress field) on the plastic zone size. Two criteria are introduced: Rankine criterion and von Mises criterion. Comparison between FEM solution and results obtained by means of the Williams expansion shows that whereas for short cracks the classical one-parameter fracture mechanics can be used, the plastic zone extent estimated via the Williams expansion for longer cracks agrees better to the numerical solution if more than only the first term of the series is considered.*

Keywords: **Williams power expansion, Plastic zone extent, Crack-tip field, Over-deterministic method, Multi-parameter fracture mechanics.**

1. Introduction

There exists a large group of materials that fail in a brittle manner and whose fracture behavior can be described satisfactorily by means of the well-known stress intensity factor (SIF). On the other hand, a lot of engineering structures is made of materials whose fracture response is more complicated and the SIF as the single-controlling parameter is not sufficient. Some energetic parameters are very often searched in this case in order to assess the fracture behavior properly. Extent of the nonlinear zone around the crack tip, where the fracture processes occur, seems to be one of the important parameters. Furthermore, if a relation between the nonlinear zone extent and boundary condition and/or specimen size and shape is found, the typical phenomenon what is referred to as the size/geometry/boundary effect (Berto & Lazzarin, 2010; Karihaloo et al., 2006) could be minimized/eliminated. In this paper, a multi-parameter approach of approximation of the crack-tip stress field is presented. The so-called Williams expansion (Williams, 1957) is used in order to describe the stress state in a plate with a central crack under uniaxial tension and it is utilized for better estimation of the plastic zone extent.

2. Methodology

Several basic terms and ideas should be presented before the results are introduced.

2.1. Williams power expansion

Williams (1957) derived a power series that can express the stress/displacement crack-tip field in a linear-elastic material with a plane crack with traction-free faces subjected to arbitrary remote loading. If only mode I of loading is considered the expansion for stress tensor components can be written in a form:

$$\sigma_{ij} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} A_n f_{ij}^{\sigma}(\theta, n) \ . \tag{1}$$

For further explanations the expression for the displacement vector components shall be known as well:

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$$u_i = \sum_{n=0}^{\infty} r^{n/2} A_n f_i^u(\theta, n, E, \nu) \,. \tag{2}$$

Symbols with the following meanings are used in Eq. (1) and (2):

σ_{ij} stress tensor components; $i,j \in \{x,y\}$;

u_i displacement vector components; $i \in \{x,y\}$;

r,θ polar coordinates centered at the crack tip;

A_n unknown coefficients of the Williams expansion (WE) terms (depend on the specimen geometry, relative crack length α and loading conditions); they have to be determined numerically;

E, ν material properties (Young's modulus and Poisson's ratio);

f_{ij}^{σ} known functions corresponding to the stress distribution;

f_i^u known functions corresponding to the displacement distribution.

Note that for the presented study only up to ten initial terms of the infinite series are considered, i.e. the expansion has a finite number of terms N.

2.2. Over-deterministic method

The over-deterministic method, ODM, (described in more detail for instance in Ayatollahi & Nejati, 2011) was chosen as a tool for calculation of the coefficients of the Williams expansion terms, A_n. This method is based on the Eq. (2) and it requires knowledge of the displacements field determined by means of the conventional FE analysis in a set of nodes around the crack tip. Efficiency and accuracy of the method have been tested and several recommendations can be found in the paper Šestáková (2013).

2.3. Estimation of the plastic zone size

Two various conditions/criteria for estimation of the plastic zone extent were applied: the Rankine criterion and the von Mises criterion.

2.3.1. Rankine criterion

The Rankine criterion is also known as the normal stress, Coulomb, or maximum stress criterion and it is often used for brittle/quasi-brittle materials. The plastic zone size is estimated by means of the comparison of the principal stress, σ_1, with a critical stress value, σ_c, that is a material property (tensile strength, yield strength, etc.):

$$\sigma_1 \geq \sigma_c \,. \tag{3}$$

2.3.2. Von Mises criterion

The von Mises criterion is also referred to as the HMH criterion and it applies best to ductile materials, such as metals. Based on the criterion, the plastic zone size is defined again through the equation between the equivalent (von Mises) stress, σ_{HMH}, and a critical stress value, σ_c:

$$\sigma_{HMH} \geq \sigma_c \,. \tag{4}$$

The equivalent (von Mises) stress can be defined for instance as:

$$\sigma_{HMH} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{xx} - \sigma_{zz})^2 + 6(\sigma_{yz}^2 + \sigma_{zx}^2 + \sigma_{xy}^2)} \,. \tag{5}$$

3. Numerical Model of the CCT Configuration

Dimensions of the specimen, see Fig. 1, as well as the material properties (of an aluminium alloy) were taken from the paper written by Tay (1995) in order to verify the analysis: W = 50 mm, h = 100 mm, a = 15.6 mm, σ = 82 MPa; E = 73 GPa, ν = 0.3, σ_c = 370 MPa. Because of the small specimen thickness (t = 2.5 mm) the plane stress conditions were applied in the finite element model. Due to the symmetry, only one quarter of the specimen could be simulated. The commercial FE software

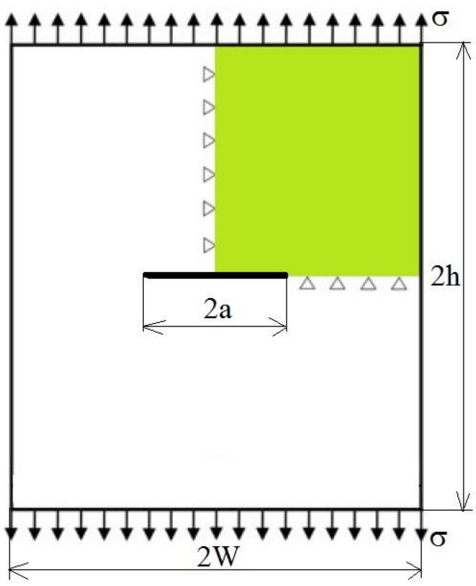

Fig. 1: Schema of the CCT specimen; only the colored quarter of the specimen was modeled.

Ansys was used for the numerical calculations; particularly, PLANE183 elements were applied in order to model the cracked specimen. The crack-tip singularity was simulated via the shifted mid-side nodes in the first row of the triangular elements around the crack tip. The distance of nodes from the crack tip used for application of the ODM was chosen to be 7.8 mm.

4. Results

First, the verification of the method was performed. In Fig. 2a, the comparison of the plastic zone sizes published in Tay (1995) and results calculated by means of the WE when only the first term (as it is usual) is considered. It can be seen that the mutual agreement is very good for both criteria used (note that in the assumed configuration the Rankine criterion is identical to the Tresca criterion). The difference of the experimental data is caused by the nature of the criteria applied, because no redistribution of the stress at the crack tip is considered (it is to be one of the next research areas of the author's collective). Nevertheless, another kind of results shall be shown in this work.

Fig. 2b and 2c (for Rankine and von Mises criterion, respectively) show the plastic zone calculated numerically via FEM together with results calculated by means of the WE under consideration of various numbers of terms used for the reconstruction of the stress field and for determination of the plastic zone.

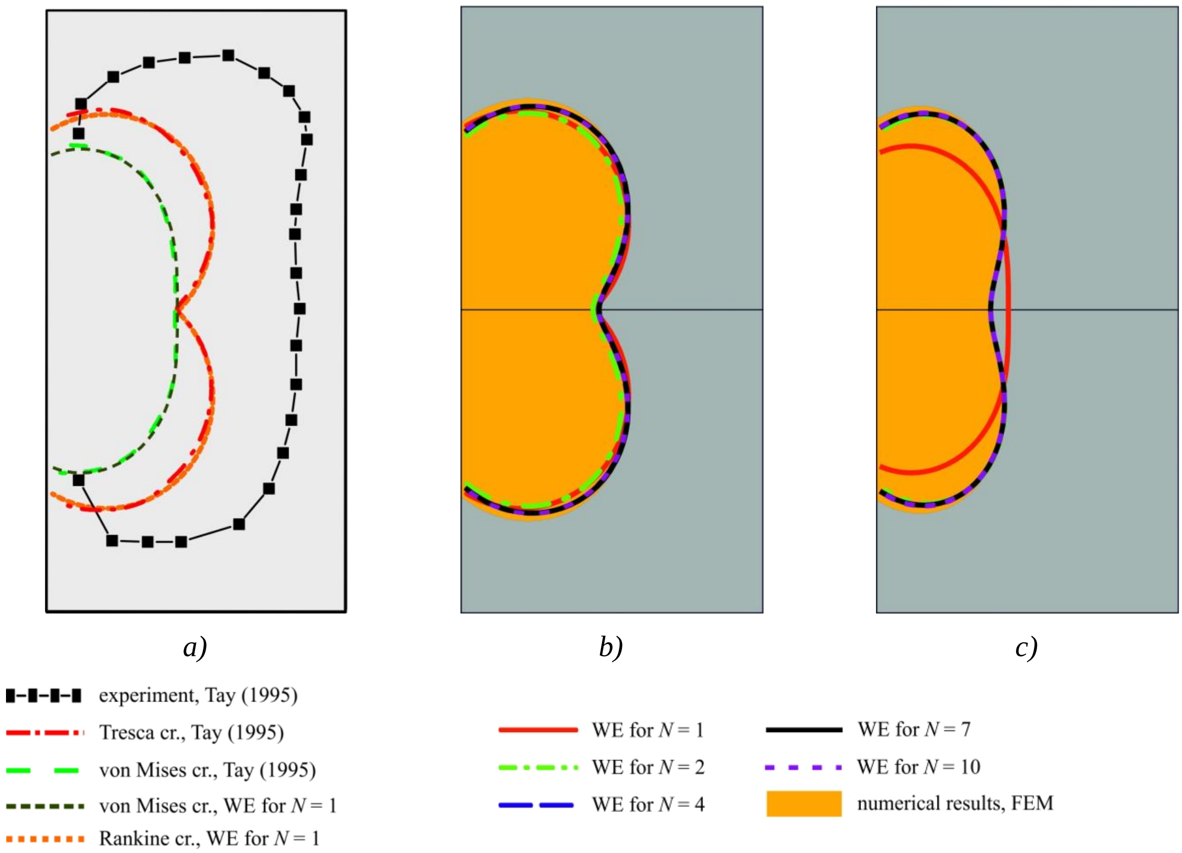

Fig. 2: Comparison of the plastic zone extents: a) Verification of the methodology, i.e. comparison of the data published in Tay (1995) with data calculated by means of the Williams expansion considering only the first term (N = 1); b) Plastic zone determined through FEM vs. plastic zone size calculated by means of the Williams expansion from the Rankine criterion; c) ditto b) from the von Mises criterion.

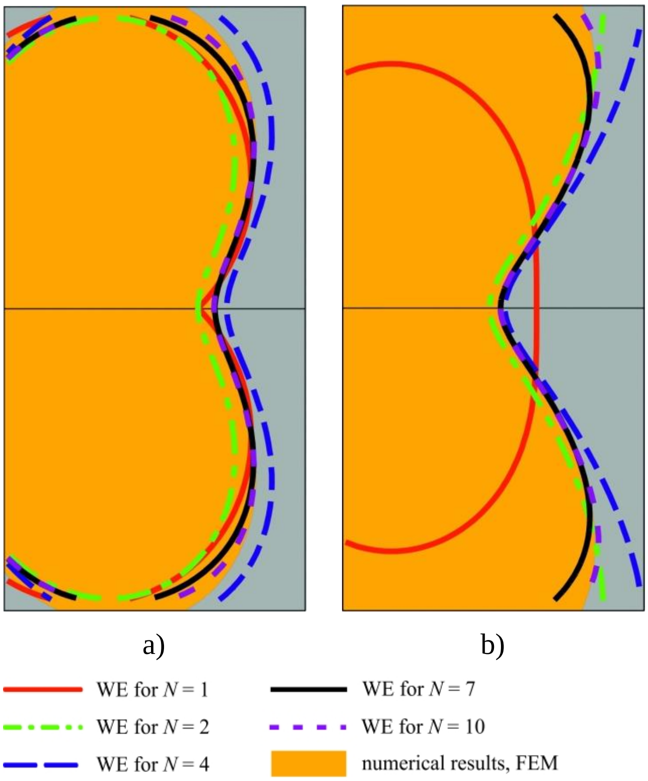

Fig. 3: Comparison of the plastic zone extent for a long crack (a = 40 mm) in CCT specimen calculated via FEM with results determined by means of the Williams expansion with various numbers of terms considered (N = 1, 2, 4, 7 and 10): a) Rankine criterion; b) Von Mises criterion.

As can be seen in Fig. 2a and 2c, the numerical results agree well with the approximation by means of the WE. Furthermore, there are almost no differences regardless of the number of the WE terms considered. Most likely, this is a consequence of the specimen configuration, because the stress field is not affected by the free surface or other boundary conditions. In order to validate this hypothesis, a CCT configuration with a very long crack (a = 40 mm) was modeled, see results in Fig. 3.

Fig. 3 shows that the plastic zone extent for a very long crack calculated by means of the WE differs in dependence on the number of the terms considered. The biggest difference from the FEM solution is clearly apparent if only the first term of the WE is taken into account and the von Mises criterion is applied.

5. Conclusions

In the paper, the plastic zone extent is investigated in a CCT specimen, i.e. in a configuration loaded in mode I. It is shown that for short cracks the size of the plastic zone calculated via FEM is identical to that determined by means of the Williams power expansion considering an arbitrary number of terms of the series. This conclusion is in contradiction to authors experience and it is explained by the configuration of the specimen tested, i.e. the stress distribution around the short cracks is not affected by any boundary conditions (e.g. free surface). This is proved on a configuration with a very long crack, where the plastic zone extent calculated from the WE depends on the number of terms considered and the biggest differences are observed between the FEM solution and the conventional one-parameter fracture mechanics approach when the von Mises criterion is applied.

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