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NUMERICAL SIMULATION OF THE TURBULENT COMPRESSIBLE GAS FLOW IN THE VANELESS MACHINES

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Abstract: *We work with the numerical solution of the turbulent compressible gas flow. We show the numerical simulation of the effect observed in the special vaneless stator-rotor system. The considered axis-symmetrical problem is solved numerically. The system of the Reynolds-Averaged Navier-Stokes equations with the k-omega turbulent model is reformulated into the cylindrical coordinates. The finite volume method is used for the solution of the resulting system of equation in meridian planes. At the boundary, the classical Riemann problem is modified to yield physically relevant boundary conditions, with the aim to keep the conservation laws. Suggested procedures were programmed into the own software, and used on examples. The described method can be used for flow simulation in symmetrical channels of arbitrary apparatuses.*

Keywords: **Turbulent Gas Flow, 3D RANS, Finite Volume Method, Axis-symmetrical Problems, Boundary Conditions.**

1. Introduction

The aim of this contribution is to simulate the viscous compressible gas flow in the slot which is formed by the gap between two coaxial cones. The considered geometry is shown in Fig. 1. The inner cone here acts as a rotor, and outter cone is the stator. The gas enters this shaped channel with the essentially small tangential velocity, and exits with relatively high tangential velocity component. The resulting viscous stresses turn the rotor. In order to simulate this 3D nonstationary process we numerically solve the system of the Navier-Stokes equations for the turbulent flow. We assume the problem to be axis-symmetrical. Therefore we simplify the governing equations and compute 3D solution in the two-dimensional meridian plane. Numerical example of this approach is shown. In our simulation we consider the Reynolds-Averaged Navier-Stokes equations with the k-ω model of turbulence, rewritten into the axis-symmetrical form.

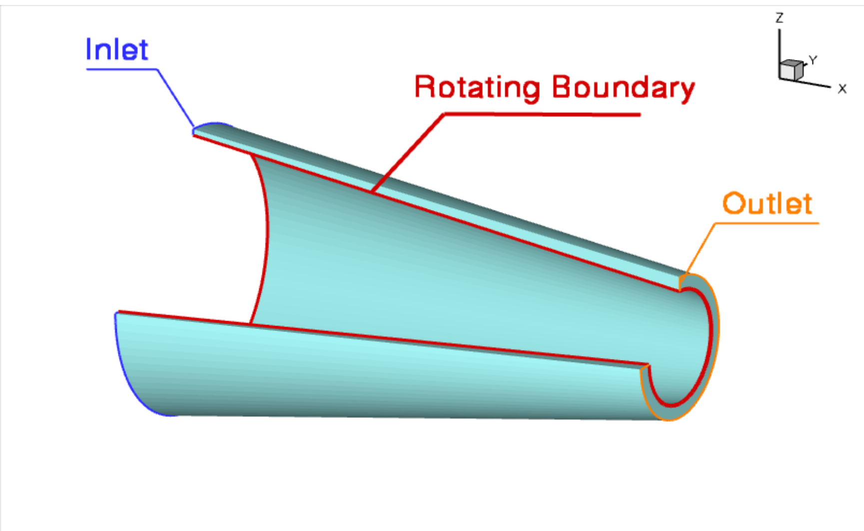

Fig. 1: 3D geometry shape, inner boundary rotates along the x- axis.

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2. The Formulation of the Equations

For the symmetrical three dimensional flow we use the following system of the equations

$$\frac{\partial}{\partial t} q + \frac{\partial}{\partial x} f(q) + \frac{\partial}{\partial y} g(q) - \left(\frac{\partial}{\partial x} r(q) + \frac{\partial}{\partial y} s(q) \right) = -\frac{1}{y} F(q) + \frac{1}{y} G(q), \tag{1}$$

where $q = (\rho, \rho u, \rho v, \rho w, E)$ denotes the state vector, and

$$f(q) = \left[\rho u, \rho u^2 + p, \rho uv, \rho uw, (E+p)u\right],$$

$$g(q) = \left[\rho v, \rho vu, \rho v^2 + p, \rho vw, (E+p)v\right],$$

$$r(q) = \left[0, \tau_{xx}, \tau_{xy}, \tau_{xz}, u\tau_{xx} + v\tau_{xy} + w\tau_{xz} + \gamma\left(\frac{\mu}{Pr} + \frac{\mu_T}{Pr_T}\right)\frac{\partial \varepsilon}{\partial x}\right],$$

$$s(q) = \left[0, \tau_{xy}, \tau_{yy}, \tau_{zy}, u\tau_{xy} + v\tau_{yy} + w\tau_{zy} + \gamma\left(\frac{\mu}{Pr} + \frac{\mu_T}{Pr_T}\right)\frac{\partial \varepsilon}{\partial y}\right],$$

$$F(q) = \left[\rho v, \rho uv, \rho(v^2 - w^2), 2\rho vw, (E+p)v\right],$$

$$G(q) = \left[0, \mu_X\left(\frac{1}{3}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \mu_X \frac{4}{3}\left(\frac{\partial v}{\partial y} - \frac{v}{y}\right), \mu_X\left(\frac{\partial w}{\partial y} - \frac{w}{y}\right),\right.$$

$$\left.\mu_X\left(-\frac{4}{3} v\frac{\partial u}{\partial x} + \frac{1}{3} u\frac{\partial v}{\partial x} + u\frac{\partial u}{\partial y} - w\frac{\partial w}{\partial y}\right) + \gamma\left(\frac{\mu}{Pr} + \frac{\mu_T}{Pr_T}\right)\frac{\partial \varepsilon}{\partial y} - \frac{2\rho k}{3} v\right],$$

with $\mu_X = \mu + \mu_T$, and

$$\tau_{xx} = \left(+\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)\mu_X - \frac{2\rho k}{3},$$

$$\tau_{yy} = \left(-\frac{2}{3}\frac{\partial u}{\partial x} + \frac{4}{3}\frac{\partial v}{\partial y}\right)\mu_X - \frac{2\rho k}{3},$$

$$\tau_{xy} = \tau_{yx} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\mu_X,$$

$$\tau_{xz} = \left(\frac{\partial w}{\partial x}\right)\mu_X, \tau_{yz} = \left(\frac{\partial w}{\partial y}\right)\mu_X.$$

Here p is the pressure, ρ the density, (u,v,w) is the average value vector of velocity: u is the velocity in the direction x, the components v, w are radial and circle velocities, x,y,z denote the cylindrical coordinates: y denote the radius, z the angle of rotation, and t the time. Further, k is the turbulent kinetic energy of flux components of the velocity, ω is the specific turbulent dissipation, Pr is laminar and Pr_T is turbulent Prandtl constant number, μ is the dynamic viscosity coefficient dependent on temperature, $\mu_T = \rho k/\omega$ is the eddy-viscosity coefficient. In the energy equation, E denotes the total energy $E = \rho\varepsilon + \rho k + \frac{1}{2}\rho(u^2 + v^2 + w^2)$, where $\varepsilon = p/\rho(\gamma - 1)$ is the internal energy of a unit mass of the fluid where the constant $\gamma > 1$. Using the integral form of the system (1) we can study a flow with shock waves, too. In this work we assume the system (1) equipped with the k - ω two-equation turbulent model described in (Kok, 2000), and rewritten into the cylindrical coordinates in (Kyncl & Pelant, 2013).

3. Numerical Time Step Method

For the discretization of the system (1) we used the finite volume method. The time interval of interest is devided using the set of partial time-steps (each of them restricted by the so-called CFL condition). The finite volume mesh is constructed as a polygonal approximation of the area studied. In this work we used the structured quadrilateral mesh. The system (1) is integrated over each element and time interval, the Green's theorem is used to compose the finite volume formulation. In order to compute the piecewise constant approximation of the solution at each time instant (and each quadrilateral element) we must evaluate the state on the edges of each quadrilateral. We use the so-called Riemann solver for the inner edges. At the boundary edges we use boundary modifications of this initial-value problem. We applied analysis from (Kyncl & Pelant, 2006), Chapters 4.-8. for the inlet and outlet edges. For the edges corresponding to the wall surface we applied (Kyncl & Pelant, 2008), Chapter 2. The dual mesh was used for the approximation of the higher order terms. In order to achieve higher accuracy in space we used the higher-order scheme with the VAN ALBADA limiter.

4. Examples

The computation was made for geometry in Fig. 1, the aparatus is 0.17 m long. The gas flows from the left towards the right side. We simulated gas rotating at inlet around axis at 960 Rad. s^{-1}. Inner boundary (refering to geometry shown in Fig. 1), representing the rotor, had the velocity fixed at 960 Rad. s^{-1}. We chose T_0 = 293 K, p_0=105000 Pa at inlet (left in Figs. 1 and 2), average pressure p = 100000 Pa at the outlet (the right-hand side in Figs. 1 and 2). Mesh consisted of 88x44 quadrilaterals, 1.000.000 iterations were computed.

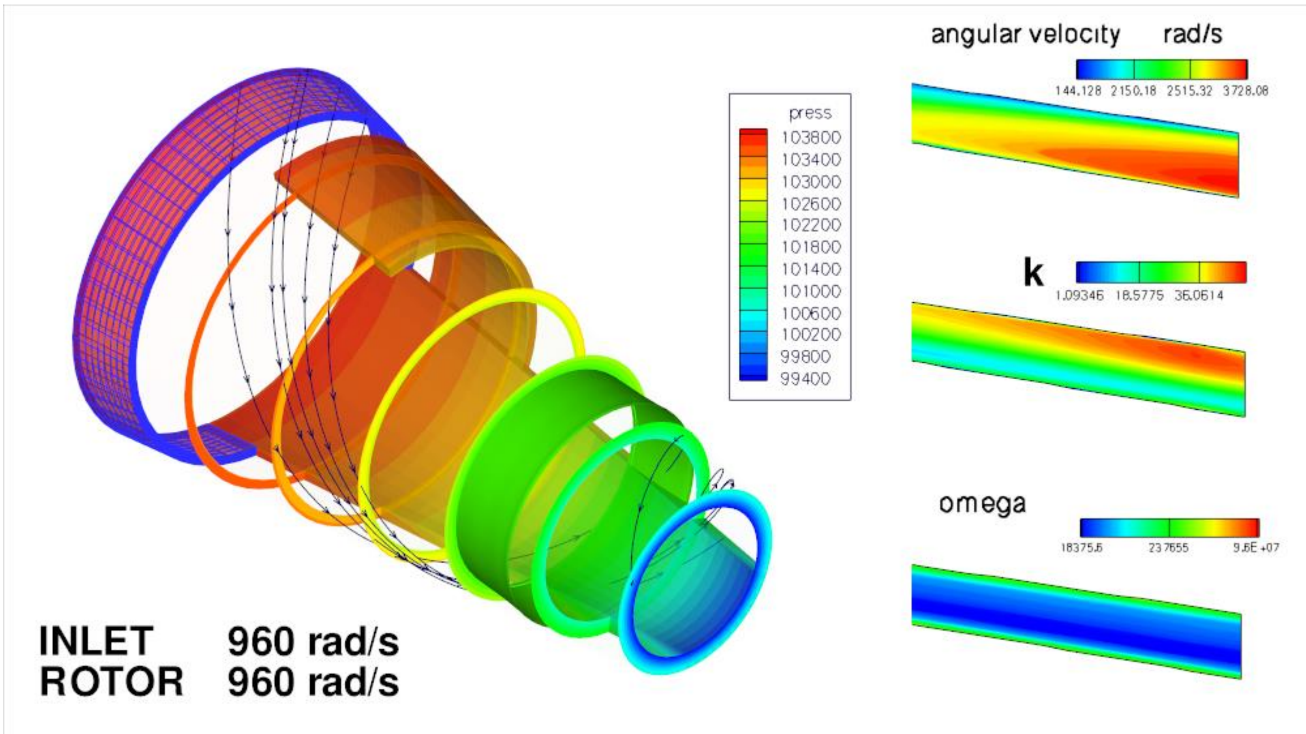

Fig. 2: Turbulent flow, angular velocity, density, and entropy isolines shown at the outlet part, 2D cut, pressure isolines and velocity streamlines shown in 3D.

Further test cases involve the increased rotor speed to 960 Rad. s^{-1}, 1440 Rad. s^{-1}, 1920 Rad. s^{-1}, and the rotor with the slip (inviscid) surface. The aim of these computations was to compare the kinetic energy of the surface velocity component of the gas in the vicinity of the rotor. The Fig. 3 shows the comparison of this restricted part of the kinetic energy $Ek = 0.5 \rho w^2$, where w is the surface (rotational) component of the velocity.

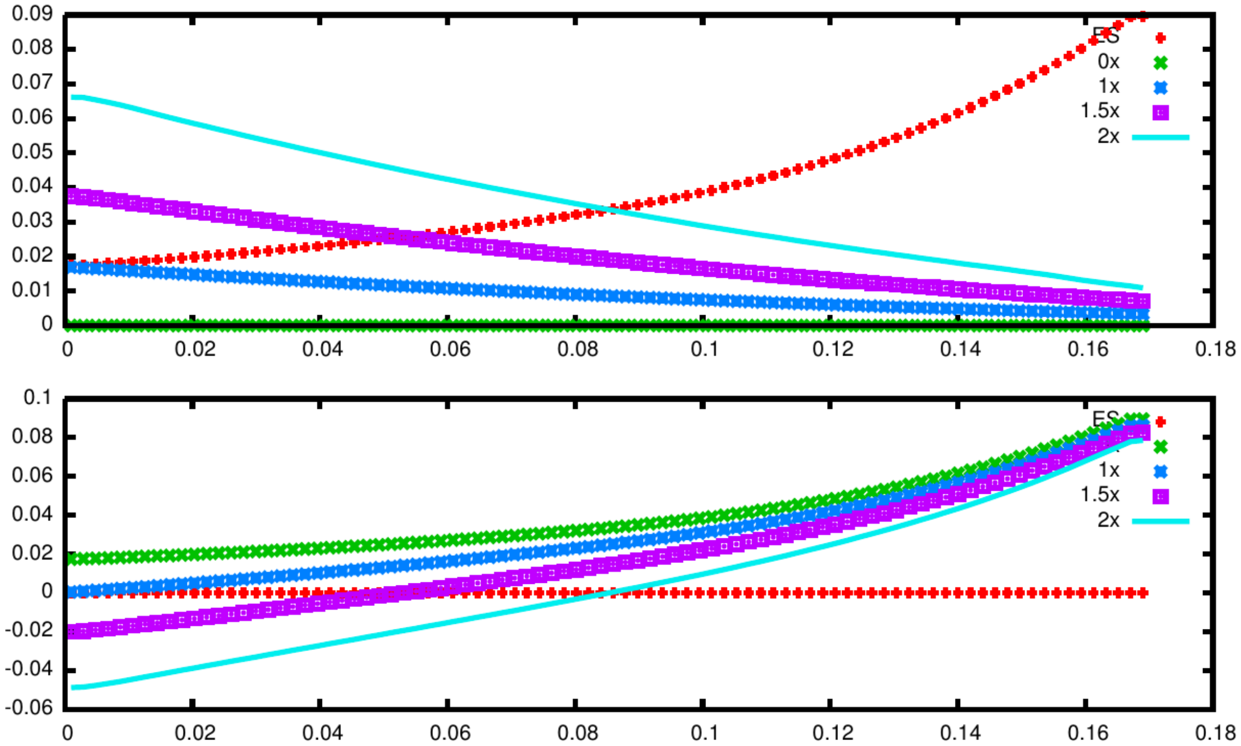

Fig. 3: Computational results for the selected cases with the slip rotor (ES), and the rotor with angular velocities 0 Rad. s^{-1} (0x), 960 Rad.s^{-1} (1x) 1440Rad. s^{-1} (1.5x), 1920 Rad. s^{-1} (2x). Horizontal axis is the x- axis, the vertical axis shows the rotational component of the kinetic energy $Ek = 0.5\rho w^2$ *. The bottom picture shows the difference of this energy Ek and the energy Ek in the case of the slip rotor.*

5. Conclusions

We presented the numerical method suitable for solving the 3D Navier-Stokes equations with $k-\omega$ turbulence model for axis-symmetrical flow in two-dimensional meridian plane. The turbulent closure equations were transformed into the cylindrical coordinates. The originality of this result lies also in the use of the modified Riemann problem for the construction of the boundary conditions. Described method can be used for the flow simulation in symmetrical channels of arbitrary apparatuses. The presented example shows the flow through the real gas turbine with the simple design (without any blades). The shown method can be used for the further optimization of such aparatuses.

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References

Kyncl, M., Pelant, J. (2006) Applications of the Navier-Stokes Equations for 3d Viscous Laminar Flow for Symmetric Inlet and Outlet Parts of Turbine Engines with the Use of Various Boundary Conditions, Report VZLÚ R3998, Prague.

Kyncl, M., Pelant, J. (2008) Applications of the Navier-Stokes Equations for 2d Viscous, Compressible Turbulent Flow on Steady Grids with the (EARSM) Turbulent Model, Report VZLÚ R4300, Prague).

Kok, J. C. (2000) Resolving the Dependence on Free-stream Values for $k-\omega$ Turbulence Model. In AIAA Journal, Vol. 38, No. 7, pp. 1292-1295.

Kyncl, M., Pelant, J. (2013) Simulation of the Propeller Disk Inside the Symmetrical Channel, in Proc. EFM 2013, Kutná Hora, pp. 404-410.

Feistauer, M., Felcman, J., Straškraba, I. (2003) Mathematical and Computational Methods for Compressible Flow. Oxford University Press, Oxford

Pelant, J. (1996-2000) ARTI Reports VZLÚ, Z-65, Z-67 to Z-73. Prague.