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INTERACTION BETWEEN BENDING MOMENT AND SHEAR FORCE IN ALUMINIUM PLATED STRUCTURAL ELEMENTS

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Abstract: *Resistance of aluminium plated structural elements under interaction between bending moment and shear force according to EN 1999-1-1 compared with various interaction formulae. Critical analysis of the current interaction formulae given in Eurocode EN 1999-1-1 and proposals for their improvement, which may be used in the new Eurocode generation available in the year 2019.*

Keywords: Resistance, Local buckling, Interaction, Bending moment, Shear force, Aluminium structure.

1. Introduction

Design requirements of stiffened and unstiffened aluminium plates which are subject to in-plane forces are given in Eurocode EN 1999-1-1. Interaction between bending moment and shear force are treated in EN 1999-1-1 in the following clauses:

a) the clause 6.2.8 related to resistance of class 1, 2 and 3 cross-section (formulae (6.38) and (6.39)). Where a shear force is present allowance should be made for its effect on the moment resistance. If the shear force V_{Ed} is less than half the shear resistance V_{Rd} its effect on the moment resistance may be neglected except where shear buckling reduces the section resistance, see 6.7.6. Otherwise the reduced moment resistance should be taken as the design resistance of the cross-section, calculated using a reduced strength.

$$f_{o,V} = f_o\left[1-(2V_{Ed}/V_{Rd}-1)^2\right] \tag{1}$$

where for non-slender sections with $h_w/t_w < 39\sqrt{250\text{MPa}/f_o}$ the shear resistance is (formula (6.29))

$$V_{Rd} = A_v \frac{f_o}{\sqrt{3}\gamma_{M1}} \tag{2}$$

The shear area A_v may be taken as (formula (6.30)):

$$A_v = \sum_{i=1}^{n}\left[(h_w-\sum d)(t_w)_i-(1-\rho_{o,haz})b_{haz}(t_w)_i\right] \tag{3}$$

b_{haz} is the total depth of HAZ (Heat Affected Zone) material occurring between the clear depth of the web between flanges. For sections without welds, $\rho_{o,haz}$ = 1. If the HAZ extends the entire depth of the web panel $b_{haz} = h_w - \sum d$.

d the diameter of holes along the shear plane

n the number of webs.

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In the case of an equal-flanged I-section classified as class 1 or 2 in bending, the resulting value of the reduced moment resistance $M_{v,Rd}$ is (formula (6.39)):

$$M_{v,Rd} = t_f b_f (h - t_f) \frac{f_o}{\gamma_{M1}} + \frac{t_w h_w^2}{4} \frac{f_{o,V}}{\gamma_{M1}} \qquad (4)$$

where h is the total depth of the section and h_w is the web depth between inside flanges.
In the case of an equal-flanged I-section classified as class 3 in bending, the resulting value of $M_{v,Rd}$ is given by expression (4) but with the denominator 4 in the second term replaced by 6.
For slender webs and stiffened webs, for sections classified as class 4 in bending or affected by HAZ softening, see 6.7.6.

b) the clause 6.2.10 related to interaction between bending moment, shear and axial force for class 1, 2 and 3 cross-sections. Influence of axial force is not taken into account in this paper.

Where shear and axial force are present, allowance should be made for the effect of both shear force and axial force on the resistance of the moment. Provided that the design value of the shear force V_{Ed} does not exceed 50% of the shear resistance V_{Rd} no reduction of the resistances defined for bending and axial force in 6.2.9 need be made, except where shear buckling reduces the section resistance, see 6.7.6. Where V_{Ed} exceeds 50% of V_{Rd} the design resistance of the cross-section to combinations of moment and axial force should be reduced using a reduced yield strength

$$(1-\rho)f_o \qquad (5)$$

for the shear area A_v, where reduction factor $\rho = \left(\frac{2V_{Ed}}{V_{Rd}} - 1\right)^2$ (6)

NOTE: Instead of applying reduced yield strength, the calculation may also be performed applying an effective plate thickness.

c) the clause 6.5.6 related to resistance of unstiffened plates under combined in-plane loading (formula (6.90b) in Amendment A1 to EN 1999-1-1).

If the combined action includes the effect of a coincident shear force, V_{Ed}, then V_{Ed} may be ignored if it does not exceed 0,5 V_{Rd} (see 6.5.8). If $V_{Ed} > 0{,}5V_{Rd}$ the following condition should be satisfied:

$$\frac{N_{Ed}}{N_{c,Rd}} + \frac{M_{Ed}}{M_{c,Rd}} + \left(\frac{2V_{Ed}}{V_{Rd}} - 1\right)^2 \leq 1{,}00 \qquad (7)$$

d) the clause 6.7.6 related to plate girders. Interaction between bending moment, shear force and axial force for class 4 cross-sections and slender webs in shear (formula (6.147)). In this paper N_{Ed} = 0 kN is supposed.
Provided that the flanges can resist the whole of the design value of the bending moment and axial force in the member, the design shear resistance of the web need not be reduced to allow for the moment and axial force in the member, except as given in 6.7.4.2(10).

If $M_{Ed} > M_{f,Rd}$ the following two expressions should be satisfied:

$$\frac{M_{Ed} + M_{f,Rd}}{2M_{pl,Rd}} + \frac{V_{Ed}}{V_{w,Rd}}\left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right) \leq 1{,}00 \qquad (8)$$

$$M_{Ed} \leq M_{c,Rd} \qquad (9)$$

where: $M_{c,Rd}$ is the design bending moment resistance according to 6.7.2 (4).

$M_{f,Rd}$ the design bending moment resistance of the flanges only, see 6.7.5(9).

$M_{pl,Rd}$ the plastic design bending moment resistance.

2. Interaction Formula Development for Cross-Section Resistance to Combination of M_{Ed} and V_{Ed}

Plate girders with slender webs are common structural element used in metal (steel or aluminium) structures. The bending and shear cross-section resistance depends on the following parameters: structural material, flange and web geometry, arrangement and geometry of stiffeners, residual stresses, initial geometric imperfections, etc. Bending and shear resistance of a girder depends also on bending moment-shear force ratio. The parametric study and influence of M_{Ed}-V_{Ed} ratio see in (Baláž, Koleková, 2014b). Where bending load is higher than bending resistance of flanges ($M_{Ed} > M_{f,Rd}$), the reduction of shear resistance has to be considered.

2.1. Plastic resistance for class 2 and 1 cross-sections

The lower bound theorem of plastic theory can be used for deriving a theoretical interaction formula. Two possible states of stress are shown in Fig. 1, both compatible with von Mises yield criterion. The stress distributions are valid for class 1 or 2 cross sections.

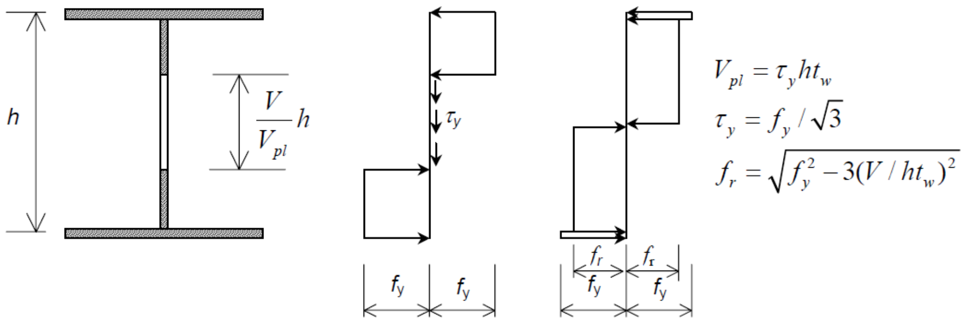

Fig. 1: I-section of class 1 or 2 subject to combined bending and shear and two possible stress distributions compatible with the von Mises yield criterion (Johansson et al., 2007).

With the simplification that the flange thickness is small compared to the beam depth the left hand stress distribution results in a bending moment:

$$M_{Ed} = M_{pl,Rd} - \frac{f_y t_w}{4}\left(\frac{V_{Ed}}{V_{pl,Rd}} h\right)^2 \qquad (10)$$

After rearrangement and introduction of the notation $M_{f.Rd} = f_y h A_f$ for the bending moment, that the flanges can carry alone, and $h^2 t_w f_y / 4 = M_{w,Rd} = M_{pl,Rd} - M_{f,Rd}$ the equation (10) becomes:

$$\frac{M_{Ed}}{M_{pl,Rd}} + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)\left(\frac{V_{Ed}}{V_{pl,Rd}}\right) = 1, \quad \text{if} \quad M_{Ed} > M_{f,Rd} \qquad (11)$$

If $M_{Ed} < M_{f,Rd}$, a statically admissible direct stress distribution σ in the flanges only can carry the bending moment and the web can be fully mobilized for resisting shear, and there is no interaction. The formula (11) should be understood such that it gives a set of M_{Ed} and V_{Ed} representing the limit of the resistance of the cross section. For design purpose the equal sign "=" is changed to "$\leq$". The stress distribution to the right in Fig. 1 was used by Horne (1951) in a study of the influence of shear on the bending resistance. It gives the following interaction formula:

$$\frac{M_{Ed}}{M_{pl,Rd}} + \left(1 - \frac{M_{f.Rd}}{M_{pl,Rd}}\right)\left(1 - \sqrt{1 - \left(\frac{V_{Ed}}{V_{pl,Rd}}\right)^2}\right) = 1, \quad \text{if} \quad M_{Ed} > M_{f,Rd} \qquad (12)$$

The equation (12) gives always higher values than (11), which means that (12) is the best estimate of the plastic resistance in accordance with the static theorem of the theory of plasticity.
The interaction formula (6.30) in clause 6.2.8(5) of EN 1993-1-1 $M_{Ed} \leq M_{y,V,Rd}$ is similar to (11) but starts the reduction of bending resistance when $V_{Ed} > 0.5V_{pl,Rd}$.

$$M_{Ed} \leq M_{y,V,Rd} \tag{13}$$

where $$M_{y,V,Rd} = \left[W_{pl} - \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1 \right)^2 \frac{(h_w t_w)^2}{4t_w} \right] \frac{f_y}{\gamma_{M0}}, \quad \text{but} \quad M_{y,V,Rd} \leq M_{c,Rd} \tag{14}$$

where $M_{c,Rd} = M_{pl,Rd}$ for class 1 and 2, $M_{c,Rd} = M_{el,Rd}$ for class 3 cross-sections.
Formula (13) can be rewritten as:

$$\frac{M_{Ed}}{M_{pl,Rd}} + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)\left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2 = 1, \quad \text{if} \quad \frac{V_{Ed}}{V_{pl,Rd}} > 0.5 \tag{15}$$

this interaction formula is more favourable in the part of the range than the theoretically determined formulae. The justification for this is empirical. There are test results available on rolled sections showing no interaction at all. This is even true if the increased plastic shear resistance $\eta\ f_y\ A_w$ is used, which can be resisted at the same time as the full plastic bending moment. The reason for this is mainly strain hardening of the material and it has been documented for steel grades up to S355. In this case the strain hardening can be utilised without excessive deformations. The reason is that the presence of high shear leads to a steep moment gradient, which in turn means that the plastic deformations are localised to a small part of the beam. Formulae (13, 15) can be seen as a cautious step in direction to utilise this fact. The cautiousness can be justified by lack of evidence for higher steel grades and because of the relatively lower strain hardening for higher grades it can be expected that such steel would show a less favourable behaviour.

2.2. Local buckling resistance for class 4 and slender webs in shear

When it comes to slender webs for which buckling influences the resistance there are no useful theories for describing the interaction. The first bending-shear (M_{Ed}-V_{Ed}) interaction formula for plate girders was proposed in (Basler, 1961), who developed an empirical model based on observations from tests. The model is similar to the lower bound theorem of plastic theory but it is here applied to a problem where instability governs, which is outside the scope of the lower bound theorem. The model is shown in Fig. 2. and it can be seen that the assumed state of stress is very similar to the left one in Fig. 1. The only difference is that the strength in shear is not the yield strength but the reduced value $\chi_w f_y/\sqrt{3}$. Basler used his own model for the shear resistance, which does not coincide with the one in EN 1993-1-5. Based on shear tests many researchers have proposed their own mechanical models to assess the panel's post-critical capacity with the formation of an appropriate diagonal tension field and in the position of the plastic frame mechanism (e.g. Höglund, 1971). The rotated stress field model in (Höglund, 1971; 1997) has proved to be very general and accurate. For the M_{Ed}-V_{Ed} interaction Höglund adopted the Basler model and added the contribution of flanges to shear resistance when available.

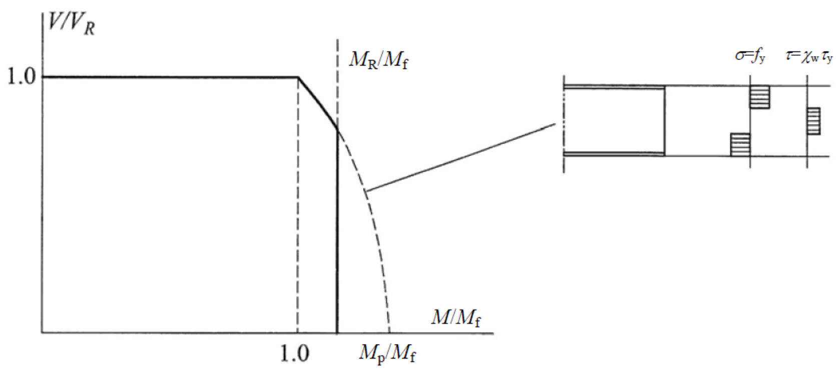

Fig. 2: Interaction between bending moment and shear according to Basler with parameters M_{Ed} / $M_{f,Rd}$, $M_{c,Rd}$ / $M_{f,Rd}$ and $M_{pl,Rd}$ / $M_{f,Rd}$ on horizontal axis (Johansson et al., 2007).

As in the case of the plastic resistance, there is no interaction if $M_{Ed} \leq M_{f,Rd}$ and for larger values the interaction curve is a parabola. In this case the cross section is in class 3 or 4 and the cross-section bending resistance $M_{c,Rd} \leq M_{pl.Rd}$. This is represented by a cut-off in Fig. 2 with a vertical line at

$M_{c,Rd}$ / $M_{f,Rd}$. The interaction formula used in EN 1993-1-5 is a modification of Basler's model as the reduction starts at V_{Ed} / $V_{bw,Rd}$ = 0,5:

$$\frac{M_{Ed}}{M_{pl,Rd}} + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)\left(\frac{2V_{Ed}}{V_{bw,Rd}} - 1\right)^2 = 1, \quad \text{if} \quad \frac{V_{Ed}}{V_{bw,Rd}} > 0.5 \tag{16}$$

The difference compared with (15) is that $V_{bw,Rd}$ is the resistance to shear buckling of the web according to chapter 5 in EN 1993-1-5. Equation (16) goes continuously over in (15) when the web slenderness is decreasing. $M_{pl,Rd}$ is used also for class 4 sections. It means that the formula has to be supplemented with a condition that:

$$M_{Ed} \leq M_{c,Rd} = M_{eff,Rd} \tag{17}$$

where $M_{eff,Rd}$ is the resistance calculated with effective cross section. Formula (16) is evaluated in (Baláž, Koleková, 2014b) for different geometries of girders. The girders are simply supported and loaded in three point bending and fitted with vertical stiffeners at supports and under the load. The webs are mainly of class 4. The curves above V_{Ed} / $V_{bw,Rd}$ = 1 are examples of the contribution from flanges, which may be added if $M_{Ed} \leq M_{f,Rd}$. The formula (16) was verified by the tests (Veljkovic, Johansson, 2001). Its applicability for higher strength steels is discussed in (Johansson et al., 2007).

2.3. EN 1993 and EN 1999 procedures

2.3.1. EN 1993

The design rules for interaction between shear force and bending moment in Eurocode 3 are found in EN 1993-1-1 for class 1 and 2 sections and in EN 1993-1-5 for class 3 (according to Johansson et al., 2007) and class 4 cross-sections. The rules in EN 1993-1-1 given in 6.2.8 are based on the plastic shear resistance and if shear buckling reduces the resistance, they refer to EN 1993-1-5. The slenderness limit for which shear buckling starts to reduce the resistance in an unstiffened plate girder is h_w / t_w = $72\sqrt{235\text{MPa}/f_y}\,/\eta$ and that limit is normally somewhere between the limits for class 2 and 3. The value η = 1.2 is recommended for steel grades up to and including S460. It may be taken conservatively equal 1.0. For higher steel grades η = 1.0 is recommended. 72 / 1.2 = 60. The rules are stated differently because different models for the interaction are used.

The interaction formula is used in EN 1993-1-5:2006 and BSI 5400-3:2000. In EN 1993-1-5 the interaction formula is applied to unstiffened and longitudinally stiffened girders and needs to be checked at a distance 0,5 h_w from the support with a vertical stiffener, while in BSI 5400- 3:2000 this interaction is applied only for longitudinally unstiffened girders. In EN 1993-1-5 another interaction formula based on reduced stress method is proposed too.

EN 1993-1-5 gives two independent interaction rules. The first rule (chapter 7 in EN 1993-1-5) is based on the calculation of characteristics of effective plate girder cross-section through effective widths and thickness of the web-plate. The second rule (reduced stress method in the chapter 10 of EN 1993-1-5) is based on determination of stress limits in unstiffened or stiffened plates. To consider the influence of moment gradient, this check is performed at a distance 0,5 h_w from the support with vertical stiffeners. Except the buckling verification, gross sectional resistance needs to be checked at the end of the panel (clause 4.6 (3) in EN 1993-1-5:2006). Effective width method used in EN 1993-1-5 is analyzed in (Baláž, Koleková, 2014b).

In the reduced stress method the interaction is given for stress states, calculated on gross cross-section characteristics, and is defined with the following equation:

$$\left(\frac{\sigma_{x,Ed}}{\rho_x f_y/\gamma_{M1}}\right)^2 + 3\left(\frac{\tau_{Ed}}{\chi_w f_y/\gamma_{M1}}\right)^2 \leq 1 \tag{18}$$

where $\sigma_{x,Ed}$ is maximum design value of normal pressure stress, τ_{Ed} is design shear stress, ρ_x and χ_w are the corresponding reduction factors. This interaction could be much more severe than the first one because no stress distribution is accounted for. The reduced stress method is not analyzed in this paper.

2.3.2. EN 1999

The formulae of EN 1999-1-1 are described in the introduction. The formulae in the clause 6.2.8 of EN 1999-1-1 and in the clause 6.2.8 of EN 1993-1-1 are based on the identical principles and they have the same form. The same formulae are also in the clause 6.2.10 of EN 1999-1-1 and in the clause 6.2.10 of EN 1993-1-1.

In the clause 6.5.6 of EN 1999-1-1 there is formula (6.90b) valid for unstiffened plates under combined in-plane loading (see Amendment A1). This formula is incorrect. EN 1993 does not contain such formula.

For class 4 cross-sections and for slender girder webs in shear there is formula (7.1) in the chapter 7 of EN 1993-1-5 and formula (6.147) in the clause 6.7.6.1 of EN 1999-1-1. The formula (6.147) is modified

Basler formula. Thanks to print error it is written in (Höglund, Tindall, 2012) in incorrect form. The formulae (7.1) and (6.147) differ. EN 1999-1-1 and EN 1993-1-1 should use the same formula to harmonize them. Fig.3 illustrates the fact that formula (6.90b) is incorrect and shows difference between formulae (7.1) and (6.147). It is proposed to use in EN 1999-1-1 the formula (7.1) of EN 1993-1-5 instead of formula (6.147).

The reduced stress method is not used in EN 1999.

3. Comparison of Various Procedures and Parametrical Study

The graphical interpretations of EN 1999-1-1 interaction formulae are shown in Fig. 3. Relative resistances valid for the I-section calculated in the following numerical example are indicated in the diagram by symbols and numerical values.

Formula (7) for $N_{Ed} = 0$, and formula (8) may be rewritten in the following forms

$$\frac{V_{Ed}}{V_{bw,Rd}} = \frac{1}{2}\left[\sqrt{1-\frac{M_{Ed}}{M_{c,Rd}}}+1\right], \qquad \frac{V_{Ed}}{V_{bw,Rd}} = \frac{1}{2}\left[\frac{1-\dfrac{M_{Ed}}{M_{pl,Rd}}}{1-\dfrac{M_{f,Rd}}{M_{pl,Rd}}}+1\right] \qquad (19a, b)$$

3.1. Numerical example

Graph in the Fig. 1 is valid for:

material: aluminium alloy EN AW-7020 T651, buckling class A:
$E = 70\text{GPa},\ \nu = 0{,}3,\ f_o = 280\text{MPa},\ f_u = 350\text{MPa},\ \gamma_{M1} = 1{,}1$,

extruded I-section (class 4):
$h = 650\text{mm},\ \text{flange (class 1)}:\ b = 120\text{mm},\ t_f = 25\text{mm},\ \text{web (class 4)}:\ h_w = 600\text{mm},\ t_w = 10\text{mm}$,

radius of fillet: $r = 5\text{mm}$.

Such high profile would be welded. For the sake of simplification we suppose that aluminium I-section is extruded profile. Influence of welds is not investigated in this paper. This enable us to compare pocedures used in Eurocode EN 1993-1-5 for steel and in EN 1999-1-1 for aluminium structures.

Ratio of bending moment resistances of the flanges and the gross I-section $M_{f,Rd} / M_{pl,Rd} = 0{,}676$.

For simply supported girder loaded in the midspan by the transverse force F_{Ed}, with a transverse stiffener under F_{Ed}, we obtain for girder geometry $L = 5h_w$ the following values of the resistances F_{Rd}:

a) according to EN 1993-1-5, formula (7.1) (formula (7) in Baláž, Koleková, 2014a, b): $F_{Rd,EN} = 1058$ kN,

$M_{Ed,EN} / M_{pl,Rd} = 0{,}899$, $V_{Ed,EN} / V_{bw,Rd,EN} = 0{,}779$,

$M_{eff,Rd,EN} / M_{pl,Rd} = 0{,}783$,

b) according to EN 1999-1-1, formula (6.147), here the formula (10b): $F_{Rd,EN} = 997{,}5$ kN,

$M_{Ed,EN} / M_{pl,Rd} = 0{,}847$, $V_{Ed,EN} / V_{bw,Rd,EN} = 0{,}735$,

$M_{eff,Rd,EN} / M_{pl,Rd} = 0{,}783$,

c) according to EN 1999-1-1, formula (6.90b), here the formula (10a): $F_{Rd,EN} = 858$ kN,

$M_{Ed,EN} / M_{pl,Rd} = 0{,}729$, $V_{Ed,EN} / V_{bw,Rd,EN} = 0{,}632$,

d) according to EN 1999-1-1, from the formula (9): $F_{Rd,EN} = 2\ M_{eff,Rd,EN}/(0{,}5L-0{,}5d) = 922{,}3$ kN.

e) according to EN 1999-1-1, from the formula: $F_{Rd,EN} = 2\ V_{bw,Rd,EN} = 1357$ kN.

Comparison of the final resistances F_{Rd} calculated according to:

EN 1993-1-5, formula (7.1): min(1058 kN; 922,3 kN; 1357 kN) = 922,3 kN,

EN 1999-1-1, formula (6.147): min(997,5 kN; 922,3 kN; 1357 kN) = 922,3 kN,

EN 1999-1-1, formula (6.147): min(858kN; 922, kN; 1357 kN) = 858 kN.

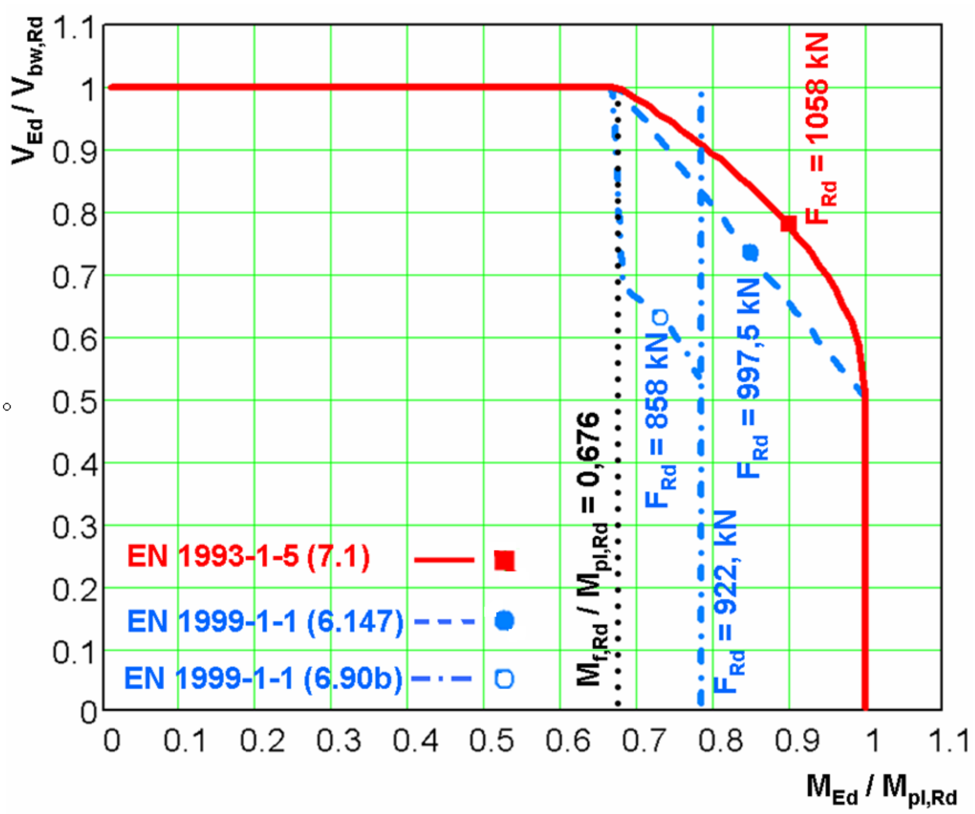

Fig. 3: Resistances calculated according to EN 1999-1-1 for aluminium class 4 cross-section under combination of bending moment M_{Ed} and shear force V_{Ed}. Values of resistances F_{Rd} for extruded I-profile (125 mm x 25 mm + 600 mm x 10 mm + 125 mm x 25 mm, EN AW-7020 T651) are indicated by coordinates of symbols ○, ● and ■. Influence of M_{Ed} and V_{Ed} is characterised by the relationships $M_{Ed} = F_{Ed}\ L / 4$, $V_{Ed} = F_{Ed} / 2$, $L = 5\ h_w$, $M_{Ed} / V_{Ed} = 2.5h_w = 3.75$ m.

3.2. Part of large parametrical study

The results in Fig. 3 relate to class 4 cross-section. Two more numerical examples were calculated for the same input values as in Fig. 3, only height of I-profiled was changed in 500 mm (to have class 3 cross-section) and in 400 mm (to have class 2 cross-section). The results are given in Fig. 4 and Fig. 5, respectively.

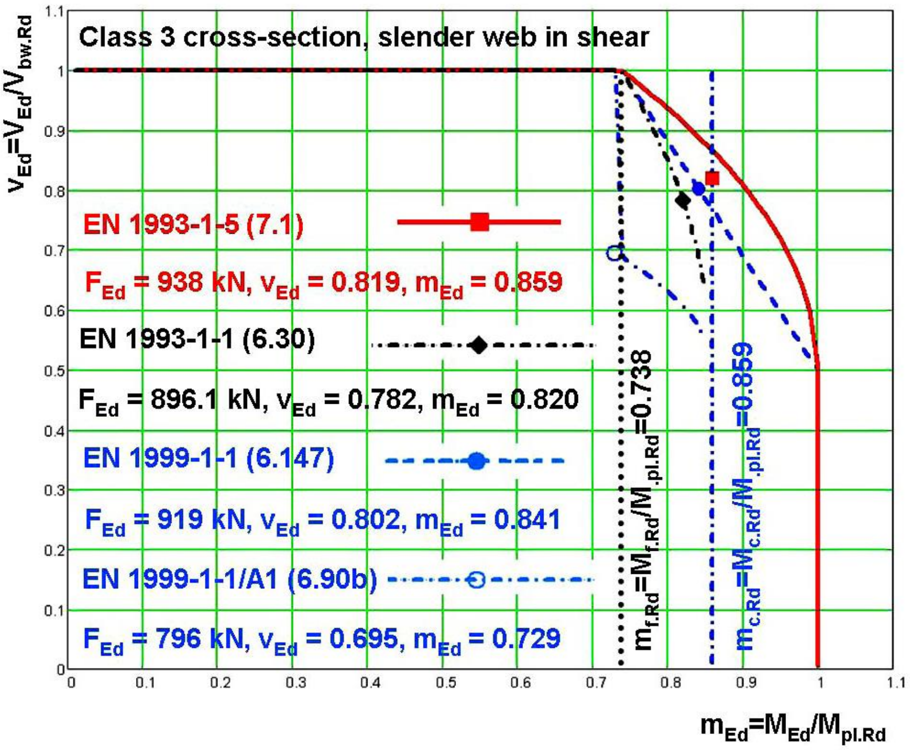

Fig. 4: The same input values as in Fig. 2, except the height profile h = 500 mm.

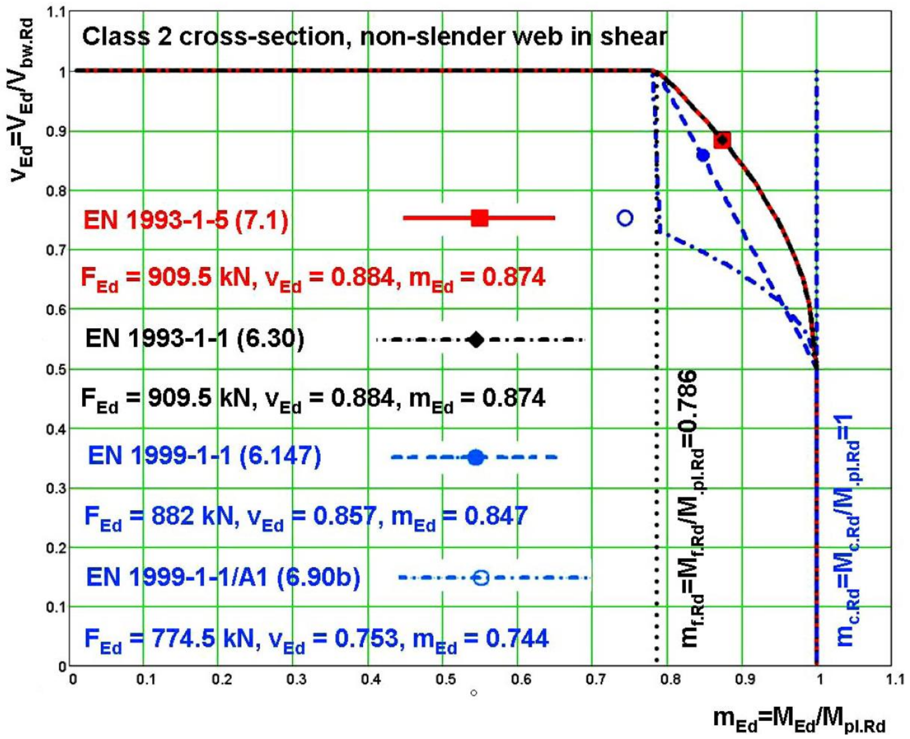

Fig. 5: Fig. 4 The same input values as in Fig. 2, except the height profile h = 400 mm.

Tab. 1: Comparison of formulae valid in interval $\frac{M_{f,Rd}}{M_{pl,Rd}} \le \frac{M_{Ed}}{M_{pl,Rd}} \le 1$ but $\frac{M_{Ed}}{M_{pl,Rd}} \le \frac{M_{c,Rd}}{M_{pl,Rd}}$

No.	Eurocode, clause	Formula No.	Formula
1	EN 1993-1-1 EN 1999-1-1 6.2.8 6.2.10	(6.30) (6.39)	Class 3 and girder web slender in shear $\frac{\frac{M_{Ed}}{M_{pl,Rd}} - \frac{M_{f,Rd}}{M_{pl,Rd}}}{\frac{M_{c,Rd}}{M_{pl,Rd}} - \frac{M_{f,Rd}}{M_{pl,Rd}}} + \left(\frac{2V_{Ed}}{V_{bw,Rd}} - 1\right)^2 \le 1,$ $M_{c,Rd} = M_{el,Rd}$
2	EN 1993-1-1 EN 1999-1-1 6.2.8 6.2.10	(6.30) (6.39)	Class 2 and girder web slender or non-slender in shear $\frac{M_{Ed}}{M_{pl,Rd}} + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)\left(\frac{2V_{Ed}}{V_{bw,Rd}} - 1\right)^2 \le 1,$ $M_{c,Rd} = M_{pl,Rd}$
3	EN 1993-1-5 7.1	(7.1)	Class 4 and girder web slender in shear $\frac{M_{Ed}}{M_{pl,Rd}} + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)\left(\frac{2V_{Ed}}{V_{bw,Rd}} - 1\right)^2 \le 1,$ $M_{c,Rd} = M_{eff,Rd}$
4	EN 1999-1-1 6.7.6	(6.147)	Class 4 and girder web slender in shear $\frac{M_{Ed} + M_{f,Rd}}{2M_{pl,Rd}} + \frac{V_{Ed}}{V_{w,Rd}}\left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right) \le 1,$ $M_{c,Rd} = M_{eff,Rd}$
5	EN 1999-1-1 6.5.6	(6.90b)	$\frac{N_{Ed}}{N_{c,Rd}} + \frac{M_{Ed}}{M_{c,Rd}} + \left(\frac{2V_{Ed}}{V_{Rd}} - 1\right)^2 \le 1$

From the Tab. 1 and from the Fig. 3, 4, and 5 it is clear:

a) the formula No. 5 in Table 1 is incorrect and it should be removed from EN 1999-1-1. See the big jump in $M_{Ed} = M_{f,Rd}$,.

b) the formulae No. 1, 2, 3, 4 in the Table 1 could be harmonized,

c) the formula No. 1 valid for class 3 cross-section will become formula No. 2 valid for class 2 cross-section if $M_{c,Rd} = M_{pl,Rd}$,

d) the form of formula No. 2 valid for class 2 cross-section is identical with the form of formula No. 3 valid for class 4 cross-section.

4. Conclusion

An empirical model based on observations from tests was first developed by Basler. His formula was later modified by Höglund and others. The Eurocode formulae are based on these results.

Comparison of the resistances in Fig. 1 calculated according to EN 1993-1-5, formula (7.1), EN 1999-1-1, formula (6.147) and EN 1999-1-1, formula (6.90b) shows that:

- formula (6.90b) should be deleted from EN 1999-1-1. Use of the formula in its partial form (for $N_{Ed} = 0$ kN) leads to unrealistic small cross-section resistances. Use of the formula in its full form (for $N_{Ed} \neq 0$ kN) leads to incorrect results.
- formula (6.147) from EN 1999-1-1 gives more conservative results than formula (7.1) from EN 1993-1-5. The formulae of these two Eurocodes should be harmonised.

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