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COMPUTER NUMERICAL SOLUTION OF VON MISES PLANAR TRUSS BY THE POTENTIAL ENERGY

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Abstract: *The task of the von Mises planar truss is to examine the effect of load located on top joint oriented in vertical direction. The mathematical concept of large displacement elastic analysis of the von Mises truss specified for computers is described. The model consists of finite nodes, tensile stiffness, and rotation stiffness. The formulas for the evaluation of displacements of nodes and rotations of segments were derived using geometric and physical conditions. Formulae for the determination of potential energy of the system are presented. Using search for the minimum potential energy, we can find the deformation of the model. The solution is searched step by step, using the Newton-Raphson iteration. The presented computational algorithm allows to model the von Mises truss using a finite amount of segments. Such solution is suitable for the load-deflection curve computation of a limit load model.*

Keywords: Von Mises truss, Nonlinear solution, Potential energy, Newton-Raphson method, Discrete model, Computational algorithm.

1. Introduction

The study of the two-bar truss, also known as the von Mises planar truss, is important to define the main stability characteristics of framed structures as well as flat arches, and of many other phenomena associated with bifurcation buckling. The von Mises planar truss is an example of a classical elastic system having numerous references in the literature (von Mises, 1923; von Mises & Ratzersdorfer, 1925; Kwasniewski, 2009).

The objective of the solution is an analysis of the load-deflection curve of top joint. The load-deflection curve of the top joint has been attempted with use of the computer programme which calculates the change of the potential energy for individual nodes of the model.

The deflection for the specific node, which was used in past, was attempted to calculated by a static method. Using this method for calculation of any node could be more difficult in terms of time and accuracy of the result. Contrastingly, using the potential energy is important to analyze the deflection of any nodes.

The solution is useful for plotting the load-deflection curves of asymmetrical trusses with random imperfection. Let us note that imperfections are generally random variables, realizations of which can be simulated by methods of type Monte Carlo; it can be seen in the following publications (Karmazínová et al., 2009; Gottvald & Kala, 2012; Kala, 2012; Kala, 2013). In the design of steel structure, it is important to count with the influence of imperfection, see, e.g., (Kala, 2008; Gottvald, 2010; Kala & Kala, 2010; Kala, 2012). Their effect has the significance for both static and economic designs of the structure, see, e.g., (Gottvald, 2010; Kala et al., 2012). The mathematical concept described in this article is suitable for the analysis of load-deflection curves of asymmetrical trusses with random imperfection.

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2. Geometry of the Model

Two slender struts connected by means of top joints form the model of von Mises truss, see Fig. 1. The geometry is characterized by the span *L*, angle α and Young's modulus of elasticity *E*. If the span *L* and angle α are known, it can be evaluated the height of the von Mises truss *H*, see (1):

$$H = \frac{1}{2} \cdot L \cdot tg\alpha \tag{1}$$

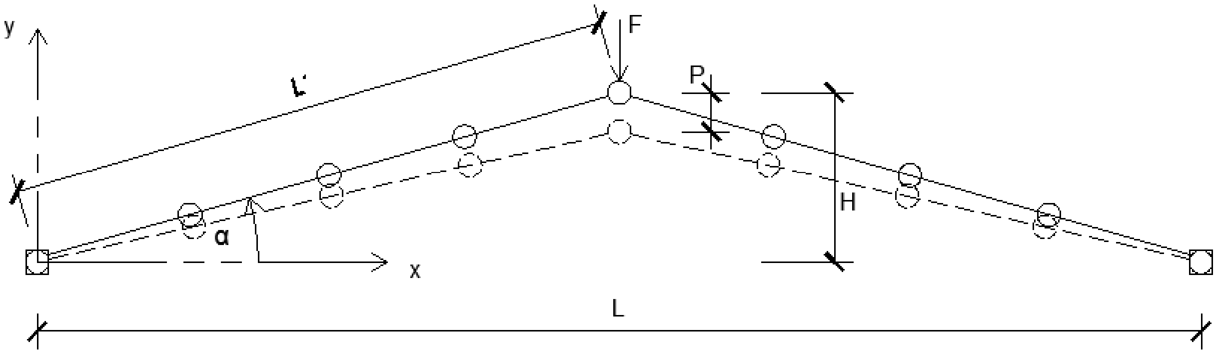

Fig. 1: Model of von Mises planar truss.

In the first step, the model is divided into a finite number of segments. The model is loaded with force *F* in the top joint, see Figure 1. Initial coordinates of individual mass points of the model are evaluated, see (2), (3).

$$x_i = L_i \cdot \cos\alpha \tag{2}$$

$$y_i = x_i \cdot tg\alpha \tag{3}$$

where L_i is the segment length (distance between the initial node and the i^{th} node). It is obtained according to the following equation:

$$L_i = \frac{2 \cdot i \cdot L'}{m} \tag{4}$$

where *i* is the index of the node, L' is the length of one strut, *m* is the finite number of segments into which the structure was divided (*m* is an even number for symmetrical models). If coordinate y_i is evaluated for the top joint, then (3) is adjusted to the following equation stemming from (1) and (2):

$$y_i = H - (x_i \cdot tg\alpha - H) \tag{5}$$

3. Solution by Potential Energy

The energy principle is applied to solve internal forces, taking into account geometrical non-linearity. The movement of individual points of the von Mises truss can be observed by means of the computer programme. At the beginning, coordinates of nodes are fixed as the initial state. Subsequently, a dislocation will be attributed to a chosen node, and potential energy of the system will be calculated for this state.

$$E_p = \frac{1}{2}\left(K_u \sum u_i^{\ 2} + K_\varphi \sum \varphi_i^{\ 2}\right) \tag{6}$$

where K_u is axial stiffness of the model, u_i is the matrix of the distance between actual and previous positions of the i-th node, K_φ is bending stiffness of the model, and φ_i is the rotation of individual parts which can be calculated as the difference between two angles formed by two segments. Calculation of axial stiffness will be obtained according to the following relation.

$$K_u = \frac{E \cdot A}{L'} \tag{7}$$

where E is Young's modulus, A is cross-section area, and L' is hypotenuse length formed by bending units. The calculation of bending stiffness will be determined according to the following formula:

$$K_\varphi = \frac{E \cdot I}{L'} \tag{8}$$

where E is Young' modulus, I is the second moment of the area, L' is the length of hypotenuse formed by bending units.

4. Newton Iteration Method

To search for the extreme values of the potential energy method, the Newton-Raphson iteration, for example, can be used as it was carried out in the journal; it is described in (Frantík, 2007). In the case of position change of initial coordinates, the equation will have the following forms:

$$J(x_i^n) \cdot v(x_i^n) = -f(x_i^n) \tag{9}$$

$$J(y_i^n) \cdot v(y_i^n) = -f(y_i^n) \tag{10}$$

where $J(x_i^n)$, $J(y_i^n)$ are matrices of partial derivations of vector functions f in points with coordinates x_i, y_i in steps n. The vector functions $f(x_i^n)$, $f(y_i^n)$ are numerical central derivations of the vectors of potential energies, see (11) , (12):

$$f(x_i^n) = \frac{E_{p_x}^U - E_{p_x}^L}{2s} \tag{11}$$

$$f(y_i^n) = \frac{E_{p_y}^U - E_{p_y}^L}{2s} \tag{12}$$

where s is parameter of solutions (1x10^-08), and E_{px}^U, E_{px}^L, E_{py}^U, E_{py}^L are potential energies with changes by parameter s. As the matrices $J(x_i^n)$, $J(y_i^n)$ are regular, the vector of unknown dislocations of coordinates will have just only one solution for the step $v(x_i^n)$, $v(y_i^n)$ searched for. This solution can be obtained from relations (9, 10), for example by means of the Gaussian elimination method, and subsequently, the positions of new nodes can be determined.

$$x_i^{n+1} = x_i^n + v(x_i^n) \tag{13}$$

$$y_i^{n+1} = y_i^n + v(y_i^n) \tag{14}$$

After having calculated (13, 14), the new coordinates will change by the parameter s, and the calculation of potential energy will be repeated. Checking the coordinates of individual points continues from the step 1 to n. In the course of calculation, the programme saves the values of coordinates of nodes.

5. Conclusions

The mathematical solution determined to create a computer programme based on finite numbers of segments is developed. The objective of the solution is an analysis of the load-deflection curve of top point. The solution is applicable to drawing the load-deflection curves of asymmetrical trusses with random imperfections. The axes of struts are, in general, curves which can be modelled by means of random quantities or random fields; it was applied (Kala, 2007). Geometrical and material characteristics, as well, should correspond with the real data obtained from experimental measurements as accurately as possible, as described in (Melcher et al., 2004; Strauss et al., 2006; Kala et al., 2009). Applying the Newton method of tangents, the computer programme can check the coordinates of any node. It represents a possibility of more detailed analysis enabling to take into consideration all important imperfections. The statistical analysis can apply advanced methods of reliability analyses based on the methods of type Monte Carlo, see, e.g., (Kala, 2010a; Kala, 2010b).

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