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INTERACTION BETWEEN BENDING MOMENT AND SHEAR FORCE IN STEEL PLATED STRUCTURAL ELEMENTS

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Abstract: *Resistance of steel plated structural elements under interaction between bending moment and shear force according to EN 1993 compared with former national Slovak standard STN 73 1401: 1998 (Czech standard ČSN 73 1401: 1998). Parametric study. Comparison of different interaction formulae.*

Keywords: Resistance, Local buckling, Interaction, Bending moment, Shear force, Steel structure.

1. Introduction

Design requirements of stiffened and un-stiffened steel plates which are subject to in-plane forces are given in Eurocode EN 1993-1-5 and EN 1993-1-1. Interaction between the bending moment and the shear force are treated:

a) in EN 1993-1-1, clause 6.2.8, related to the resistance of the cross-section. Where the shear force is present allowance should be made for its effect on the moment resistance. Where the shear force is less than half the plastic shear resistance its effect on the moment resistance may be neglected except where shear buckling reduces the section resistance, see EN 1993-1-5. Otherwise the reduced moment resistance should be taken as the design resistance of the cross-section, calculated using a reduced yield strength

$$(1-\rho)f_y \tag{1}$$

for the shear area A_v, where the reduction factor reads

$$\rho=\left(\frac{2V_{Ed}}{V_{pl,Rd}}-1\right)^2, \quad V_{pl,Rd}=\frac{A_v\left(f_y/\sqrt{3}\right)}{\gamma_{M0}} \tag{2a, b}$$

For welded I or H sections A_v may be taken as follows $A_v = A_w = h_w t_w$ (area of the cross-section web).

f_y is yield strength,

γ_{M0} partial factor for resistance of cross-sections whatever the class is,

V_{Ed} design shear force,

$V_{pl,Rd}$ plastic design shear resistance.

b) in EN 1993-1-1, clause 6.2.10, related to interaction between bending moment, shear and axial force. Influence of the axial force is not taken into account in this paper.
Where the shear and the axial forces are present, allowance should be made for the effect of both shear force and the axial force on the resistance of the moment. Provided that the design value of the shear force V_{Ed} does not exceed 50% of the shear resistance $V_{pl,Rd}$ no reduction of the resistances defined for the bending moment and the axial force in 6.2.9 is needed, except where the shear buckling reduces the section resistance, see clause 7.1 in EN 1993-1-5. Where V_{Ed} exceeds 50% of $V_{pl,Rd}$ the design

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resistance of the cross-section to combinations of the bending moment and the axial force should be reduced using a reduced yield strength according to formula (1) for the shear area A_v, where the reduction factor is defined by formula (2a).

NOTE: Instead of reducing the yield strength f_{yw} also the plate thickness t_w of the relevant part of the cross section may be reduced.

c) in EN 1993-1-5, clause 7.1, related to plated structural components subject to in-plane loads.

Provided that $V_{Ed} / V_{b,Rd}$ does not exceed 0,5 , the design resistance to bending moment and axial force need not be reduced to allow for the shear force. If $V_{Ed} / V_{b,Rd}$ is more than 0,5 the combined effects of bending and shear in the web of an I or box girder should satisfy:

$$\frac{M_{Ed}}{M_{pl,Rd}} + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right)\left(2\frac{V_{Ed}}{V_{bw,Rd}} - 1\right)^2 \leq 1{,}0 \quad \text{for} \quad \frac{M_{Ed}}{M_{pl,Rd}} \geq \frac{M_{f,Rd}}{M_{pl,Rd}} \tag{3}$$

where

$M_{f,Rd}$ is the design plastic moment of resistance of the section consisting of the effective area of the flanges;

$M_{pl,Rd}$ the design plastic resistance of the cross section consisting of the effective area of the flanges and the fully effective web irrespective of its section class.

In addition the following requirements should be met

$$\frac{M_{Ed}}{\frac{f_y\, W_{eff}}{\gamma_{M0}}} \leq 1{,}0 \; , \qquad \frac{V_{Ed}}{V_{b,Rd}} \leq 1{,}0 \tag{4a, b}$$

where

$$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \leq \frac{\eta\, f_{yw}\, h_w\, t}{\sqrt{3}\,\gamma_{M1}} \tag{5}$$

Contributions of the web and flanges to the total buckling shear resistance are as follows

$$V_{bw,Rd} = \frac{\chi_w\, f_{yw}\, h_w\, t}{\sqrt{3}\,\gamma_{M1}} \, , \qquad V_{bf,Rd} = \frac{b_f\, t_f^2\, f_{yf}}{c\,\gamma_{M1}}\left[1 - \left(\frac{M_{Ed}}{M_{f,Rd}}\right)^2\right] \tag{6a, b}$$

Contribution of the flanges is usualy negligible. All relevant quantities used in above formulae are defined in Eurocodes EN 1993-1-5 and EN 1993-1-1.

2. Comparison of EN and STN Procedures and Parametric Study Based on EN Procedure

2.1. Comparison of EN 1993 and STN 73 1401 procedures

Graphical interpretation of Eurocode resistance formulae compared with former national STN 731401: 1998 (ČSN 731401: 1998) ones is shown in Fig. 1. Relative resistances valid for the web of the welded I-section calculated in the following numerical example for European and national standard are indicated in the diagram by symbols and their “coordinates”.

Formula (3) may be rewritten in the following form

$$\frac{V_{Ed}}{V_{bw,Rd}} = \frac{1}{2}\left[\sqrt{\frac{1 - \frac{M_{Ed}}{M_{pl,Rd}}}{1 - \frac{M_{f,Rd}}{M_{pl,Rd}}}} + 1\right] \tag{7}$$

Numerical example.

Graph in Fig. 1 is valid for:

material: $E = 210GPa$, $\nu = 0{,}3$, $f_y = 355MPa$, $\gamma_{M0} = 1{,}0$, $\gamma_{M1} = 1{,}0$,

welded I-section (class 4):

$h = 1550mm$, flange (class 1): $b = 300mm$, $t_f = 25mm$, web (class 4): $h_w = 1500mm$, $t_w = 10mm$,

fillet weld: $a_w = 5mm$.

Ratio of bending moment resistances of the flanges and the gross I-section $M_{f,Rd} / M_{pl,Rd} = 0{,}67$.

For simply supported girder loaded in the midspan by the transverse force F_{Ed}, with a transverse stiffener under F_{Ed}, we obtain for girder geometry $L = 5h_w$ the following values of the resistances F_{Rd}:

a) according to EN 1993-1-5: $F_{Rd,EN} = 2955$ kN from formula (3 or 7),

$M_{Ed,EN} / M_{pl,Rd} = 0{,}732$, $V_{Ed,EN} / V_{bw,Rd,EN} = 0{,}953$, relate to formula (7),

$M_{eff,Rd,EN} / M_{pl,Rd} = 0{,}799$, relates to formula (4a).

From formulae (4a) and (4b) we obtain $F_{Rd,EN} = 3227{,}8$ kN and $F_{Rd,EN} = 3101{,}8$ kN, respectively.

b) according to STN 73 1401: $F_{Rd,STN} = 2765$ kN,

$M_{Ed,STN} / M_{pl,Rd} = 0{,}685$, $V_{Ed,STN} / V_{bw,Rd,EN} = 0{,}891$,

$M_{eff,Rd,STN} / M_{pl,Rd} = 0{,}812$, $V_{ba,Rd,STN} / V_{bw,Rd,EN} = 0{,}913$,

relate to formulae given in STN 73 1401 (not given in this paper).

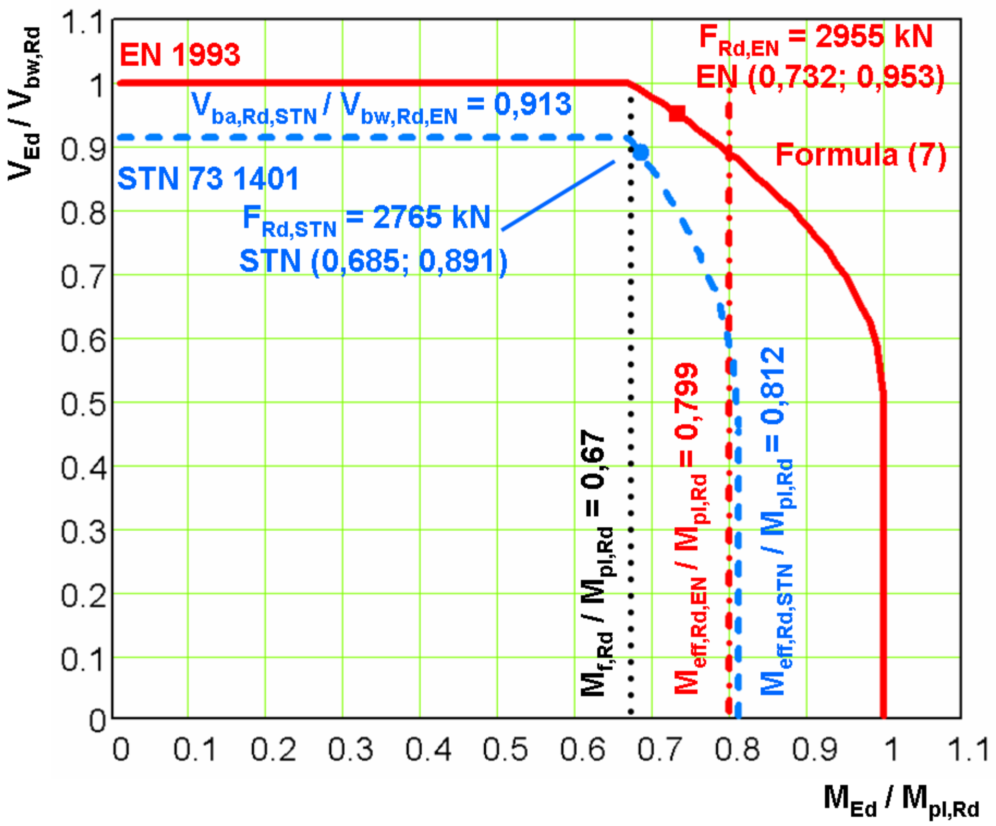

Fig. 1: Comparison of relative resistances calculated according to EN 1993-1-5 and STN 73 1401. Values of relative resistances of the web of the welded steel I-section (300 mm x 25 mm + 1500 mm x 10 mm + 300 mm x 25 mm, S355) loaded by combination of bending moment M_{Ed} and shear force V_{Ed} are indicated by "coordinates" of symbols ● and ■. Influence of M_{Ed} and V_{Ed} is characterised by the relationships $M_{Ed} = F_{Ed}\, L / 4$, $V_{Ed} = F_{Ed} / 2$, $L = 5\, h_w$, $M_{Ed} / V_{Ed} = 2.5 h_w = 3.75$ m.

2.2. Parametric study based on EN 1993-1-5 and EN 1993-1-1 procedures

Only the results of parametric study based on Eurocode procedures and formulae are presented here.

The values taken into account in the parametric study: material properties E = 210 GPa, G = 81 GPa,

ν = 0,3, steel grade S355, the safety factors γ_{M0} = 1.0, γ_{M1} = 1.0, the factor for shear area η = 1.2, the throat thickness of the fillet weld a_w = 5 mm (taken into account in calculating part of the web c, see Table 5.2, sheet 1 of 3 in EN 1993-1-1). The investigated cases are described in Table 1.

The simple supported girders with fictitious spans L = 2.5h_w, 5h_w, 7.5h_w and 10 h_w loaded in the midspan by the point load F_{Ed} were taken into account to obtain different ratios M_{Ed} / V_{Ed} for the purpose of parametric study and to have one parameter problem concerning loading M_{Ed} / V_{Ed} = f(F_{Ed}).

The numerical results may be directly used for any value γ_{M1} > 1.0 (see 3.2 conclusions).

Tab. 1: Investigated cases. Cross-section dimensions. Classes of web and flange.

No	Case	h [mm]	h_w [mm]	t = t_w [mm]	h_w / t_w	class of web in bending	web in shear is	b = b_f [mm]	t_f [mm]	class of flange
1	I.	1850	1800	10	180	4	slender	360	25	1
2	II.	1550	1500		150	4	slender	300		1
3	III.	1250	1200		120	4	slender	280		1
4	IV.	950	900		90	3	slender	250		1
5	V.	650	600		60	2	*)	220		1

*) non-slender for subcases a. and b.; slender for subcases c. and d. (see *Table 6*).

Tab. 2: Case I: h_w / t_w =180, web in bending: class 4, flange: class 1, web in shear: slender, γ_{M1} = 1.

No	Quantity	Subcases a	b	c	d
1	girder span L	2.5h_w	5h_w	7.5h_w	10h_w
2	aspect ratio $\alpha = a / h_w = 0.5L / h_w$	5	3.75	2.5	1.25
3	reduction factor χ_w	0.488	0.439	0.429	0.425
4	reduction factor ρ	0.584			
5	$M_{Ed} / V_{Ed} = 0.5(L - h_w)$ [m]	1.35	3.6	5.85	8.1
6	$F_{Ed,max} = F_{Rd}$ [kN]	3822.9	3240.9	2313	1672
7	$V_{Ed} / V_{b,Rd}$	1	1	0.731	0.533
7b	$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd}$ [kN]	1911.5	1620.5	1582.3	1568.5
7c	$V_{bf,Rd}$ [kN], $M_{f,Rd}$ = 5830.3 kNm	109.3	0	0	0
8	$(m_{Ed} / m_{fRd})^2 + (v_{Ed} - 1) / [(b t_f^2 f_{yf} / c \gamma_{M1}) / V_{bw,Rd}]$	1	1	-	-
9	$m_{Ed} + (1 - m_{f,Rd})(2v_{Ed} - 1)^2$, $m_{f,Rd} = M_{f,Rd} / M_{pl,Rd}$	-	1	0.848	0.779
10	$m_{Ed} / m_{c,Rd}$, $m_{c,Rd} = M_{c,Rd} / M_{pl,Rd}$	0.381	0.862	1	1
11	$v_{Ed} = V_{Ed} / V_{bw,Rd}$	1.061	1	0.731	0.533
12	$m_{Ed} = M_{Ed} / M_{pl,Rd}$, $M_{pl,Rd}$ = 8706.4 kNm	0.296	0.67	0.777	0.778

Simply supported girder: $V_{Ed} = 0.5F_{Ed}$, $M_{Ed} = 0.5(L - h_w)0.5E_{Ed}$, $M_{Ed,max} = 0.25F_{Ed}L$

Tab. 3: Case II: $h_w / t_w = 150$, web in bending: class 4, flange: class 1, web in shear: slender, $\gamma_{M1} = 1$.

No	Quantity	Subcases			
		a	b	c	d
1	girder span L	$2.5h_w$	$5h_w$	$7.5h_w$	$10h_w$
2	aspect ratio $\alpha = a / h_w = 0.5L / h_w$	5	3.75	2.5	1.25
3	reduction factor χ_w	0.558	0.504	0.493	0.489
4	reduction factor ρ	0.691			
5	$M_{Ed} / V_{Ed} = 0.5(L - h_w)$ [m]	1.125	3	4.875	6.75
6	$F_{Ed,max} = F_{Rd}$ [kN]	3634.3	2952	1986	1435
7	$V_{Ed} / V_{b,Rd}$	1	0.952	0.655	0.477
7b	$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd}$ [kN]	1817.1	1550.9	1515.9	1503.2
7c	$V_{bf,Rd}$ [kN], $M_{f,Rd} = 4060.3$kNm	100.6	0	0	0
8	$(m_{Ed} / m_{fRd})^2 + (v_{Ed} - 1) / [(b t_f^2 f_{yf} / c \gamma_{M1}) / V_{bw,Rd}]$	1	-	-	-
9	$m_{Ed} + (1 - m_{f,Rd})(2v_{Ed} - 1)^2$, $m_{f,Rd} = M_{f,Rd} / M_{pl,Rd} = 0.67$	-	1	0.831	0.8
10	$m_{Ed} / m_{c,Rd}$, $m_{c,Rd} = M_{c,Rd} / M_{pl,Rd} = 0.799$	0.422	0.915	1	1
11	$v_{Ed} = V_{Ed} / V_{bw,Rd}$	1.059	0.952	0.655	0.477
12	$m_{Ed} = M_{Ed} / M_{pl,Rd}$, $M_{pl,Rd} = 6057.2$ kNm	0.337	0.731	0.799	0.799

Simply supported girder: $V_{Ed} = 0.5F_{Ed}$, $M_{Ed} = 0.5(L - h_w)0.5E_{Ed}$, $M_{Ed,max} = 0.25F_{Ed}L$

Tab. 4: Case III: $h_w / t_w = 120$, web in bending: class 4, flange: class 1, web in shear: slender, $\gamma_{M1} = 1$.

No	Quantity	Subcases			
		a	b	c	d
1	girder span L	$2.5h_w$	$5h_w$	$7.5h_w$	$10h_w$
2	aspect ratio $\alpha = a / h_w = 0.5L / h_w$	5	3.75	2.5	1.25
3	reduction factor χ_w	0.651	0.592	0.58	0.575
4	reduction factor ρ	0.844			
5	$M_{Ed} / V_{Ed} = 0.5(L - h_w)$ [m]	0.9	2.4	3.9	5.4
6	$F_{Ed,max} = F_{Rd}$ [kN]	3432.8	2755	1871	1352
7	$V_{Ed} / V_{b,Rd}$	1	0.945	0.656	0.477
7b	$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd}$ [kN]	1716.3	1457	1426	1414.8
7c	$V_{bf,Rd}$ [kN], $M_{f,Rd} = 3044.1$kNm	114.1	0	0	0
8	$(m_{Ed} / m_{fRd})^2 + (v_{Ed} - 1) / [(b t_f^2 f_{yf} / c \gamma_{M1}) / V_{bw,Rd}]$	1	-	-	-
9	$m_{Ed} + (1 - m_{f,Rd})(2v_{Ed} - 1)^2$, $m_{f,Rd} = M_{f,Rd} / M_{pl,Rd} = 0.704$	-	1	0.873	0.845
10	$m_{Ed} / m_{c,Rd}$, $m_{c,Rd} = M_{c,Rd} / M_{pl,Rd} = 0.844$	0.423	0.906	1	1
11	$v_{Ed} = V_{Ed} / V_{bw,Rd}$	1.071	0.945	0.656	0.478
12	$m_{Ed} = M_{Ed} / M_{pl,Rd}$, $M_{pl,Rd} = 4322.1$ kNm	0.357	0.765	0.844	0.845

Simply supported girder: $V_{Ed} = 0.5F_{Ed}$, $M_{Ed} = 0.5(L - h_w)0.5E_{Ed}$, $M_{Ed,max} = 0.25F_{Ed}L$

Tab. 5: Case IV: h_w / t_w =90, web in bending: class 3, flange: class 1, web in shear: slender, γ_{M1} = 1.

No	Quantity	Subcases			
		a	b	c	d
1	girder span L	$2.5h_w$	$5h_w$	$7.5h_w$	$10h_w$
2	aspect ratio $\alpha = a / h_w = 0.5L / h_w$	5	3.75	2.5	1.25
3	reduction factor χ_w	0.789	0.717	0.704	0.699
4	reduction factor ρ	1			
5	$M_{Ed} / V_{Ed} = 0.5(L – h_w)$ [m]	0.675	1.8	2.925	4.05
6	$F_{Ed,max} = F_{Rd}$ [kN]	3165.8	2475	1677.8	1210.9
7	$V_{Ed} / V_{b,Rd}$	1	0.935	0.646	0.47
7b	$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd}$ [kN]	1582.9	1323.5	1297.9	1288.6
7c	$V_{bf,Rd}$ [kN], $M_{f,Rd}$ = 2052.3 kNm	128	0	0	0
8	$(m_{Ed} / m_{fRd})^2 + (v_{Ed} – 1) / [(b t_f^2 f_{yf} / c \gamma_{M1}) / V_{bw,Rd}]$	1	-	-	-
9	$m_{Ed} + (1 – m_{f,Rd})(2v_{Ed} -1)^2$, $m_{f,Rd} = M_{f,Rd} / M_{pl,Rd}$ = 0.741	-	1	0.908	0.886
10	$m_{Ed} / m_{c.Rd}$, $m_{c.Rd} = M_{c.Rd} / M_{pl.Rd}$ = 0.885	0.436	0.908	1	1
11	$v_{Ed} = V_{Ed} / V_{bw.Rd}$	1.088	0.935	0.646	0.47
12	$m_{Ed} = M_{Ed} / M_{pl.Rd}$, $M_{pl.Rd}$ = 2771.2.4 kNm	0.386	0.804	0.885	0.885

Tab. 6: Case V: h_w / t_w =60, web in bending class 2, flange class 1, γ_{M1} = 1.

No	Quantity	Subcases			
		a	b	c	d
	web in shear	non-slender		slender	
1	girder span L	$2.5h_w$	$5h_w$	$7.5h_w$	$10h_w$
2	aspect ratio $\alpha = a / h_w = 0.5L / h_w$	5	3.75	2.5	1.25
3	reduction factor χ_w	1	1	0.998	0.987
4	reduction factor ρ	1			
5	$M_{Ed} / V_{Ed} = 0.5(L – h_w)$ [m]	0.45	1.2	1.95	2.7
6	$F_{Ed.max} = F_{Rd}$ [kN]	2460.5	2220	1556	1140
7	$V_{Ed} / V_{b,Rd}$	0.881	0.903	0.634	0.47
7b	$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd}$ [kN]	1395.9	1229.8	1227.7	1214
7c	$V_{bf,Rd}$ [kN], $M_{f,Rd}$ = 1220.3 kNm	166.1	0	0	0
8	$(m_{Ed} / m_{fRd})^2 + (v_{Ed} – 1) / [(b t_f^2 f_{yf} / c \gamma_{M1}) / V_{bw,Rd}]$	-	-	-	-
9	$m_{Ed} + (1 – m_{f,Rd})(2v_{Ed} -1)^2$, $m_{f,Rd} = M_{f,Rd} / M_{pl,Rd}$ = 0.793	-	1	1	1
10	$m_{Ed} / m_{c,Rd}$, $m_{c,Rd} = M_{c,Rd} / M_{pl,Rd}$ = 1	0.36	0.865	0.985	1
10b	$M_{Ed} / M_{V,c,Rd}$	0.373	1	-	-
10c	reduction factor for shear area $1- (2V_{Ed}/V_{pl,Rd} – 1)^2$	0	0.352	-	-
10d	$M_{V,c,Rd}$ [kNm]	1220.3	1332.8	-	-
11	$v_{Ed} = V_{Ed} / V_{bw,Rd}$	1.061	0.903	0.634	0.470
12	$m_{Ed} = M_{Ed} / M_{pl,Rd}$, $M_{pl,Rd}$ = 1539.8 kNm	0.296	0.865	0.985	1

Simply supported girder: $V_{Ed} = 0.5F_{Ed}$, $M_{Ed} = 0.5(L – h_w)0.5E_{Ed}$, $M_{Ed,max} = 0.25F_{Ed}L$

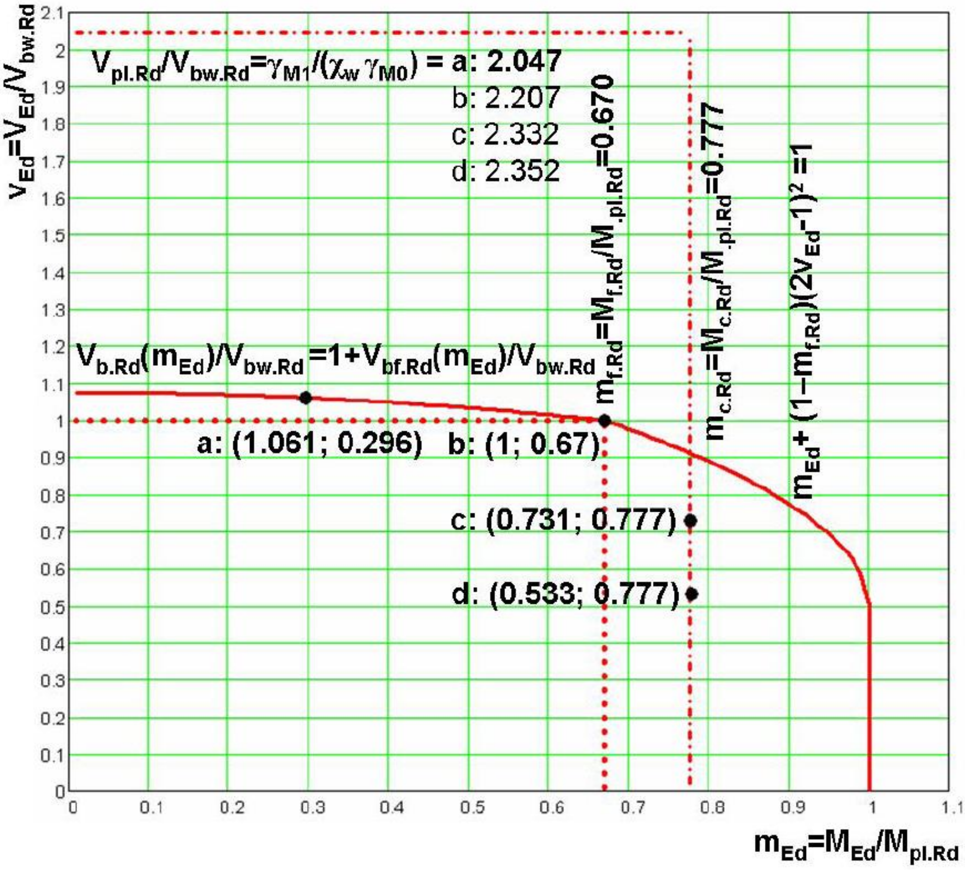

Fig. 2: Case I: h_w / t_w =180, web in bending: class 4, flange: class 1, web in shear: slender, γ_{M1} = 1.

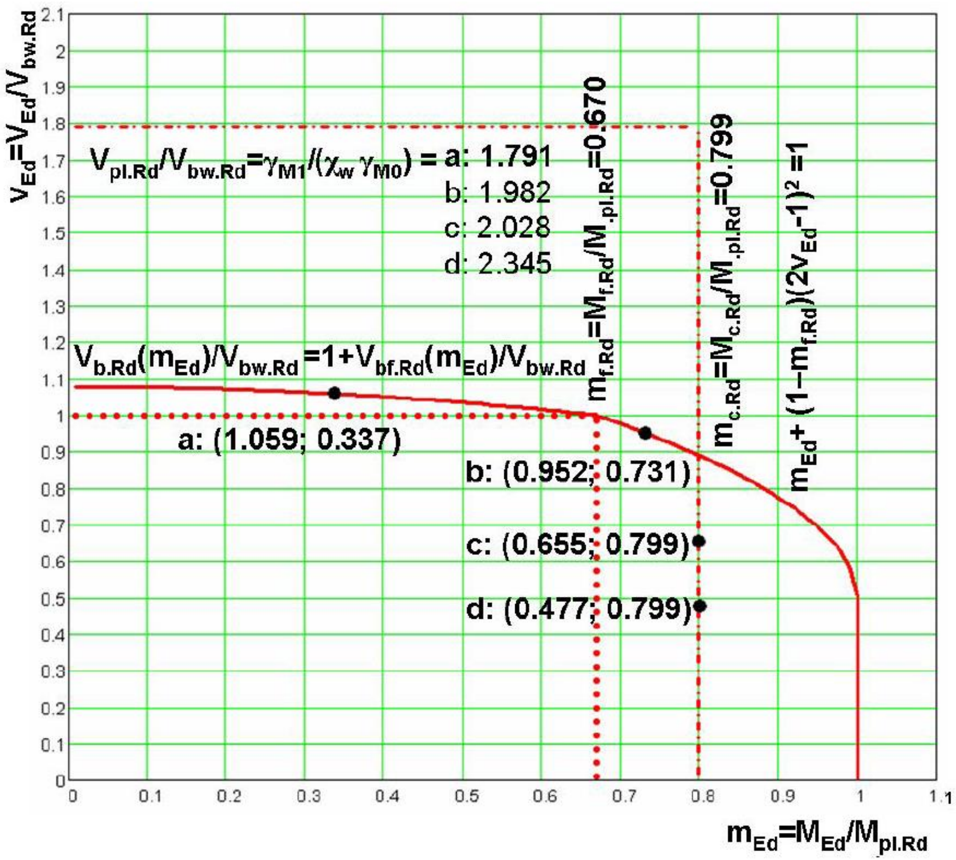

Fig. 3: Case II: h_w / t_w =150, web in bending: class 4, flange: class 1, web in shear: slender, γ_{M1} = 1.

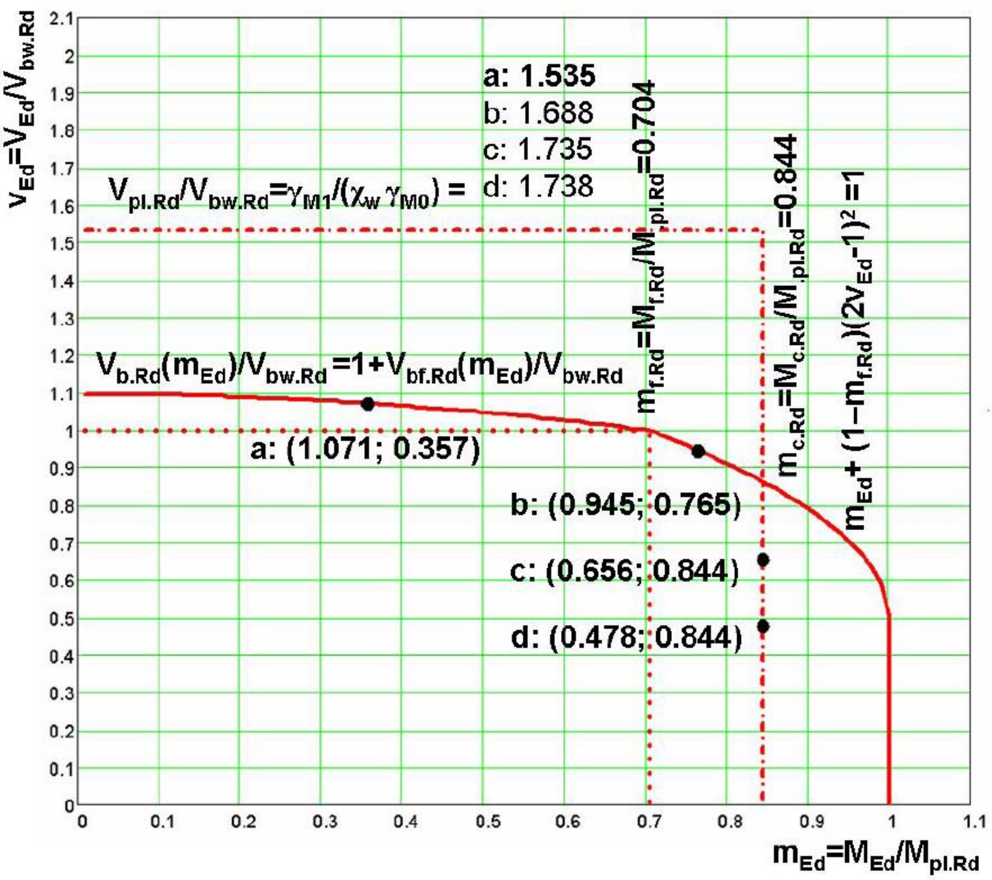

Fig. 4: Case III: h_w / t_w =120, web in bending: class 4, flange: class 1, web in shear: slender, γ_{M1} = 1.

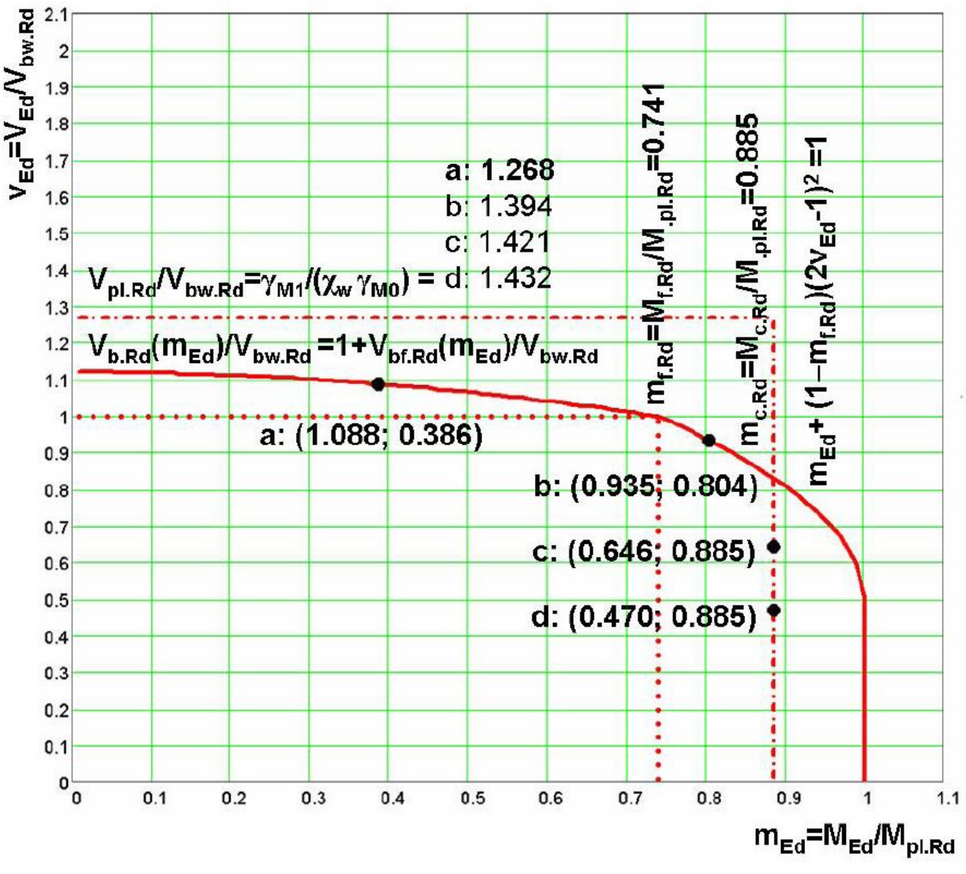

Fig. 5: Case IV: h_w / t_w =90, web in bending: class 3, flange: class 1, web in shear: slender, γ_{M1} = 1.

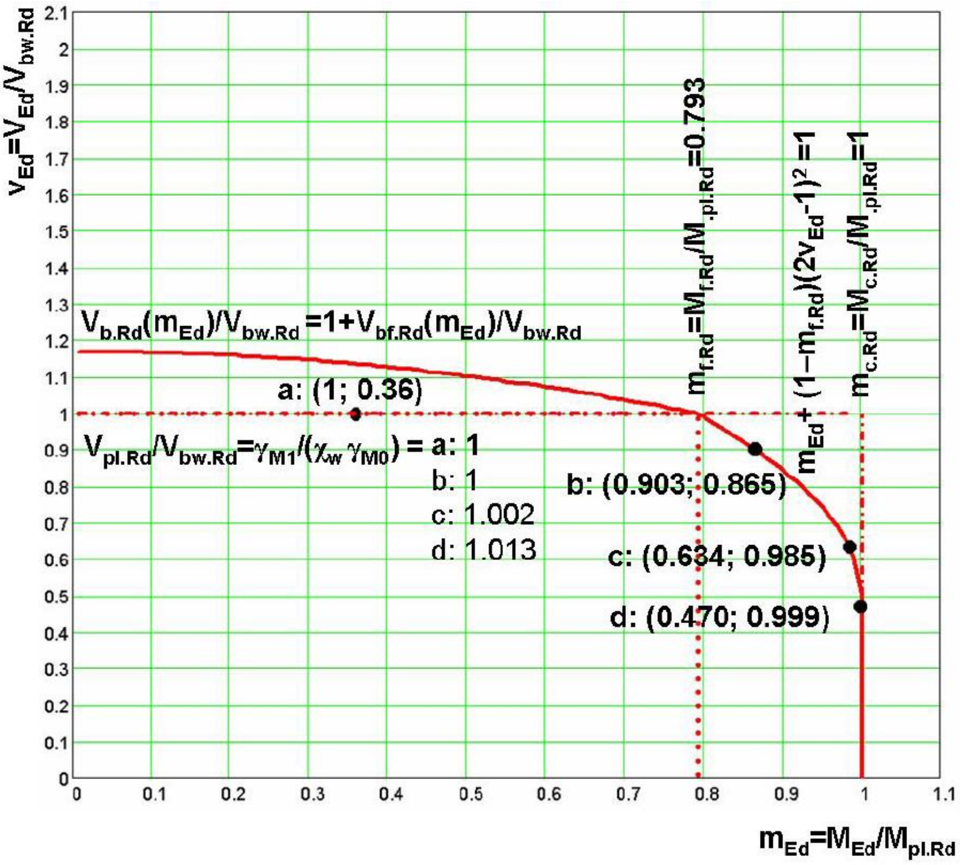

Fig. 6: Case V: h_w / t_w =60, web in bending: class 2, flange: class 1, web in shear: non-slender for subcases a. and b., slender for subcases c. and d., γ_{M1} = 1.

3. Conclusions

3.1. Conclusions related to comparison of of EN 1993 and STN 73 1401 procedures

Local buckling of metal structural elements is important topic in agenda of working groups preparing new generation of Eurocodes, which will be available in 2019. Problems of web girder resistance to transverse forces were outlined in (Baláž Koleková, 2013a, b) and (Koleková, Baláž, 2013a, b).

In this paper problems of steel plated structural elements under combination of the bending moment and the shear force are analysed. Twin paper (Koleková, Baláž, 2014a, b) is focused on the same problem but for aluminium structures. Comparison of results of these two papers enable to improve both Eurocodes EN 1993 for steel and EN 1999 for aluminium structures and to prepare their harmonization.

Differences between EN and STN procedures: a) difference in the shear resistance definition; b) STN interaction formula is valid only for class 4 cross-sections. STN interaction formulae for classes 1, 2 and 3 cross-sections have different form. See clause 6.6.7 and Annex D in STN 73 1401; c) EN interaction formula is simpler; d) EN 1993 gives greater resistances comparing with STN 731401: 1998 (ČSN 731401: 1998).

Generally the following is valid: a) for the greater web slendernesses and for the steels with greater strengths the shear resistance is decisive. For such cases EN resistance may be greater up to 17 %; b) for opposite cases the bending moment resistance is decisive and difference in the resistances calculated according to EN and STN is negligible.

In above numerical example, which is part of the large parametric study, the web slenderness is 150, steel grade S355 and the ratio of resistances $F_{Rd,EN}$ / $F_{Rd,STN}$ = 2955 kN /2765 kN = 1.069. The difference in resistances is 6.9 %.

3.2. Conclusions related to parametric study

The numerical results of parametric study were obtained for the safety factor γ_{M1} = 1.0, but they are valid also for any value $\gamma_{M1} > 1.0$. Only the following modification is necessary to do: the numerical values F_{Ed} given in tables should be divided by γ_{M1} (consequently also M_{Ed} and V_{Ed} values) and the numerical values of ratios $V_{pl,Rd}$ / $V_{bw,Rd}$ given in diagrams should be multiplied by γ_{M1}. In some special rare cases the change of γ_{M1} value may lead to the change of cross-section classification.

The functions $v_{Ed} = f(m_{Ed})$ drawn in Fig.1 – 6 consist of the following three parts:

1. parts, which are described in the interval

$$0 \leq m_{Ed} = M_{Ed} / M_{pl,Rd} \leq m_{f,Rd} = M_{f,Rd} / M_{pl,Rd} \qquad (8)$$

by the horizontal lines

$$v_{Ed} = V_{Ed} / V_{bw,Rd} = 1 \qquad (9)$$

or by the curves, if also contribution of the flanges in shear resistance is taken into account, described by the functions (see also formulae (6a, b))

$$V_{b,Rd}\,(m_{Ed}^{)} / V_{bw,Rd} = [V_{bw,Rd} + V_{bf,Rd}(m_{Ed})] / V_{bw,Rd} = 1 + V_{bf,Rd}(m_{Ed}) / V_{bw,Rd} \qquad (10)$$

The equation for the curve taking into account also contribution of the flanges in shear resistance

$$v_{Ed} = V_{Ed} / V_{b,Rd} = 1 \qquad (11)$$

may be after inserting formulae (5), (6a, b) rewritten in the form

$$(m_{Ed} / m_{f,Rd})^2 + (v_{Ed} - 1) / [bt_f^2 f_{yf}/(c\,\gamma_{M1}) / V_{bw,Rd}] = 1 \qquad (12)$$

where

$$c = a\,[0.25 + (1.6 b_f t_f^2 f_{yf}) / (t_w h_w^2 f_{yw})] \qquad (13)$$

For drawing the following form of (13) is useful

$$v_{Ed} = 1 + [bt_f^2 f_{yf}/(c\,\gamma_{M1}) / V_{bw,Rd}]\,[1 - (m_{Ed} / m_{f,Rd})^2] \qquad (14)$$

In all calculation should be verified if

$$V_{b,Rd} \leq \eta\, h_w t_w f_{yf} / (\sqrt{3}\,\gamma_{M1}) \qquad (15)$$

The curves in Fig. 2 – 6 were drawn for γ_{M1} = 1.0 and only for subcases a. The shapes of curves for subcases b., c., d. and $\gamma_{M1} > 1.0$ would be only slightly changed, because the contribution of the flanges to the shear resistance is usually not very important.

2. parts, which are described in the interval $m_{f,Rd} = M_{f,Rd} / M_{pl,Rd} \leq m_{Ed} = M_{Ed} / M_{pl,Rd} \leq 1$ by the formula (7), may be rewritten in the form

$$m_{Ed} + (1 - m_{f,Rd})\,(2v_{Ed} - 1)^2 = 1 \qquad (16)$$

The validity of the (7) and (9) is restricted by the condition

$$m_{Ed} = M_{Ed} / M_{pl,Rd} \leq m_{c,Rd} = M_{c,Rd} / M_{pl,Rd} \qquad (17)$$

where $M_{c,Rd} = M_{pl,Rd}$ for class 1 and 2, $M_{c,Rd} = M_{el,Rd}$ for class 3 and $M_{c,Rd} = M_{eff,Rd}$ for class 4 cross-sections.

3. parts are vertical lines, for which $m_{Ed} = 1$, if $v_{Ed} \leq 0.5$.

Note the difference between the shapes of EN 1993-1-5 and STN 73 1401 curves and their intervals in Fig. 1.

Among several important tasks in creating of new generation of metal Eurocodes there are: to shorten Eurocodes contents, to harmonise different parts of relevant Eurocode and to harmonise different Eurocodes (e.g. EN 1993 for design of steel structures with EN 1999 for design of aluminium structures0. The results of analysis done in this paper gives us the opportunity to do in the case of investigated problem.

The part EN 1993-1-1 is primary valid for class 1, 2 and 3 beam cross-sections in compression, bending or their combination and for beams with non-slender cross-section webs in shear. The part EN 1993-1-5 focuses on the local buckling of class 4 cross-sections in compression, bending or their combination and for beams with slender cross-section webs in shear. There are often discussions related to the question in which part of EN 1993 the rules used in parts EN 1993-1-1 and EN 1993-1-5 as well concerning investigated problems should be located. It seems, that the best solution would be to join the both parts into one document. This would be much more convenient also for Eurocode users in practice. For example the rules given in paragraphs a) and b) (see Introduction) could be deleted from EN 1993-1-1, because rules given in c) paragraph taken from EN 1993-1-5, which are primary for class 4 cross-sections and slender cross-section webs in shear, cover also class 1 and 2 cross-sections and non-slender webs in shear. In similar way as in EN 1993 also the rules in EN 1999 could be harmonized. Eurocodes would be shortened and more clear written. And this not everything, what we can do in this area. Generally it is possible to say that EN 1993 and EN 1999 contain for the same topics different formulae and procedures in some cases without any important reason. The resistance of cross-section loaded by combination of bending moment and shear force is an example for it. All relevant rules in EN 1999 [the formula (6.90b) is even incorrect **there, see Fig. 1 in Koleková, Baláž (2014a) or Fig. 3 in Koleková, Baláž (2014b)] concerning the resistance of cross-section loaded by combination of bending moment and shear force could be replaced by the modified (harmonized) rules of EN 1993.**

For the class 1, 2 and 3 cross-sections loaded by combination of bending moment and shear force there will be the following changes in the previous formulae taken from EN 1993-1-5, clause 7.1

$$V_{bf,Rd} = 0 \text{ kN},\ V_{b,Rd} = V_{bw,Rd} = V_{pl,Rd} = h_w t_w f_{yf} / (\sqrt{3}\,\gamma_{M0}) \qquad (18)$$

The rules given in EN 1993-1-1, clause 6.2.8 and in clause 6.2.10 (for N_{Ed} = 0 kN) valid for class 1, 2 and 3 cross-sections with non-slender webs under combination of M_{Ed} and V_{Ed} may be expressed in the following way

$$M_{Ed} = M_{c,Rd} \text{ if } V_{Ed} / V_{pl,Rd} \leq 0.5 \qquad (19)$$

or after dividing by $M_{pl,Rd}$ and $V_{pl,Rd}$, respectively, in non-dimension form

$$m_{Ed} = m_{c,Rd} \text{ if } v_{Ed} \leq 0.5 \qquad (20)$$

and

$$M_{Ed} = M_{f,Rd} + M_{w,Rd}\,[1 - (2V_{Ed}/V_{pl,Rd} - 1)^2],\ \ M_{w,Rd} = M_{c,Rd} - M_{f,Rd}\ ,\ \text{if}\ V_{Ed} / V_{pl,Rd} > 0.5 \qquad (21)$$

which may be written in the form

$$m_{Ed} = m_{f,Rd} + (m_{c,Rd} - m_{f,Rd})\,[1 - (2V_{Ed}/V_{pl,Rd} - 1)^2],\ \text{if}\ v_{Ed} > 0.5 \qquad (22)$$

or

$$v_{Ed} = \{[(m_{c,Rd} - m_{Ed}) / (m_{c,Rd} - m_{f,Rd})]^{0.5} + 1\}/2 \qquad (23)$$

For class 1 and 2 cross-sections

$$M_{c,Rd} = M_{pl,Rd},\ \text{that is}\ m_{c,Rd} = 1 \qquad (24)$$

After inserting $m_{c,Rd}$ =1 in (23), the formula (7) is obtained. This is evidence that for class 1 and 2 cross-sections the formula (7.1) in clause 7.1 of EN 1993-1-5 gives identical results as the rules given in clauses 6.2.8 and 6.2.9 (for N_{Ed} = 0 kN) of EN 1993-1-1. Fig. 7 shows the comparison of these rules and their graphical interpretation.

For class 3 cross-sections

$$M_{c,Rd} = M_{el,Rd} \qquad (25)$$

and from Fig. 8 it may be seen that in this case clauses 6.2.8 and 6.2.9 (for N_{Ed} = 0 kN) of EN 1993-1-1 give results, which differ a little bit from results of clause 7.1 of EN 1993-1-5, because relevant formula corresponds to STN 73 1401 formula (compare Fig. 1 with Fig. 8).

Note that in clauses 6.2.8 and 6.2.9 (for N_{Ed} = 0 kN) of EN 1993-1-1 only safety factor γ_{M0} is used, not γ_{M1}. The safety factor γ_{M1} is used only for class 4 cross-sections, slender webs in shear and slender webs under transverse force F_{Ed}, if cross-section resistance is analysed.

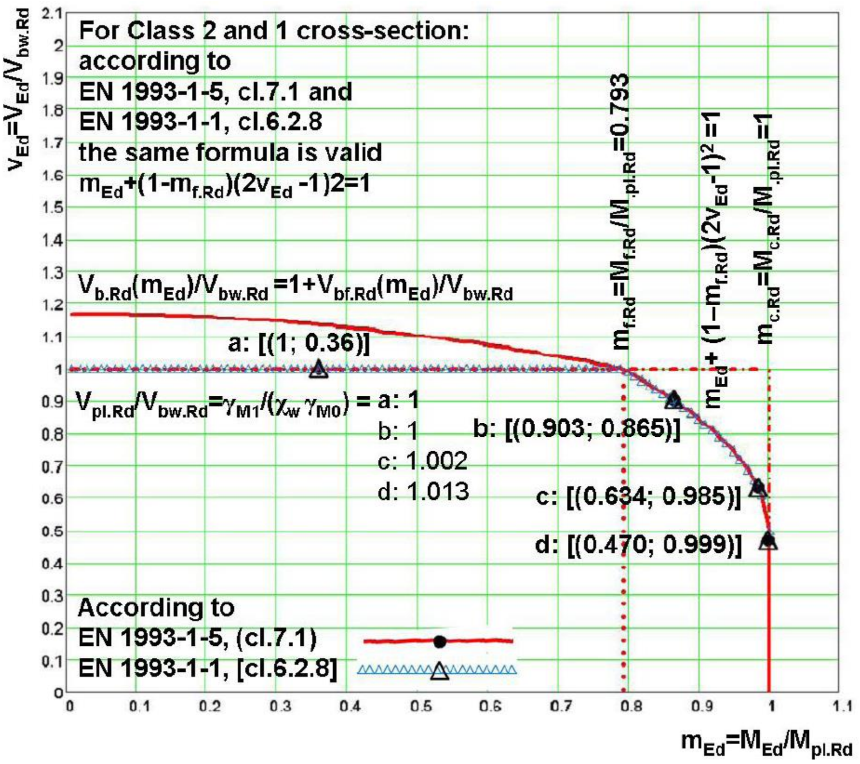

Fig. 7: Case V: hw / tw =60, web in bending: class 2, flange: class 1, web in shear: non-slender for subcases a. and b., slender for subcases c. and d., γM1=1. Comparison of the results based on the rule given in the clause 6.2.8 and 6.2.10 (for NEd = 0 kN) of EN 1993-1-5 with the results based on the rule given in the clause 7.1 of EN 1993-1-1.

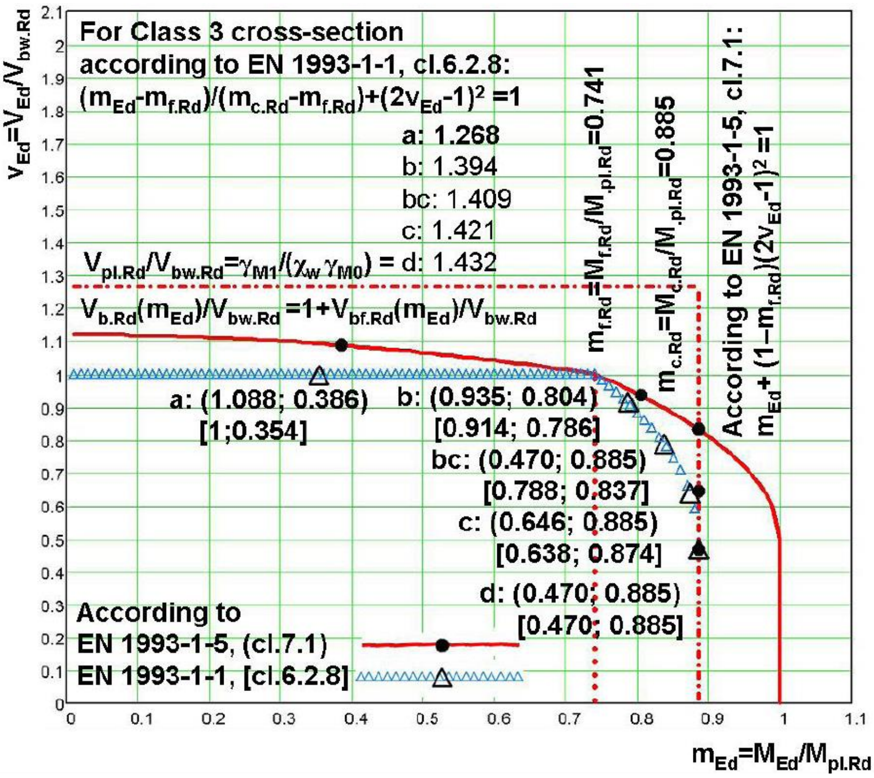

Fig. 8: Case IV: h_w / t_w =90, web in bending: class 3, flange: class 1, web in shear: slender, γM1 = 1. Comparison of the results based on the rule given in the clause 6.2.8 and 6.2.10 (for NEd = 0 kN) of EN 1993-1-5 with the results based on the rule given in the clause 7.1 of EN 1993-1-1. New subcase bc. was calculated for L = 6hw, for which the difference between the results is the biggest (5.8 %).

Tab. 7: Case IV: h_w / t_w =90, web in bending: class 3, flange: class 1, web in shear: slender, γM1 = 1. Comparison of the results based on the rule given in the clause 6.2.8 and 6.2.10 (for NEd = 0 kN) of EN 1993-1-5 with the results based on the rule given in the clause 7.1 of EN 1993-1-1.

Case IV	Subcase				
$h_w / t_w = 90$	a ($L = 2.5h_w$)	b ($L = 5h_w$)	bc ($L = 6h_w$)	c ($L = 7.5h_w$)	d ($L = 10h_w$)
$F_{Ed.EN\ 1993\text{-}1\text{-}1}$ [kN] EN 1993-1-1, cl.6.2.8 and 10	3165.8	2475	2181	1677.8	1210.9
$F_{Ed.EN\ 1993\text{-}1\text{-}5}$ [kN] EN 1993-1-5, cl.7.1	2475	2420	2062.6	1656.3	1210.5
$\frac{F_{Ed.EN\ 1993\text{-}1\text{-}1}}{F_{Ed.EN\ 1993\text{-}1\text{-}5}}$	1.279	1.023	1.058	1.013	1

The biggest difference between the results is for the subcase bc.: 5.8 % (see also Fig. 8). The difference 27.9 % for the subcase a. is not caused by different procedures of EN 1993-1-1 and EN 1993-1-5. It is caused by the fact that according to clause 7.1 in EN 1993-1-5 it is possible to take into account also the contribution of flanges $V_{bf.Rd}$ to the shear resistance $V_{b.Rd}$. This is not possible to do according to the clauses 6.2.8 and 6.2.9 (for $N_{Ed} = 0$ kN) in EN 1993-1-1.

The problem of the Eurocode EN 1993 procedures is that in the following special limiting cases there is a jump between numerical values, if the safety factor $\gamma_{M1} > 1.0$:

- in the case of local buckling the value of shear buckling resistance

$$V_{b.Rd} = V_{bw.Rd} + V_{bf.Rd} = \frac{\chi_w f_{yw} h_w t}{\sqrt{3}\gamma_{M1}} + \frac{b_f t_f^2 f_{yf}}{c\gamma_{M1}}\left[1-\left(\frac{M_{Ed}}{M_{f,Rd}}\right)^2\right] \quad (26)$$

will become in the moment when $\chi_w = 1$ the following shear resistance of the cross-section

$$V_{pl,Rd} = \frac{f_{yw} h_w t}{\sqrt{3}\gamma_{M0}} \quad (27)$$

- in the case of global buckling the value of flexural buckling resistance

$$N_{b.Rd} = \chi A f_y / \gamma_{M1} \quad (28)$$

will become in the moment when $\chi = 1$ the following resistance of the cross-section

$$N_{c.Rd} = A f_y / \gamma_{M0} \quad (29)$$

- in the case of global buckling the value of lateral torsional buckling resistance

$$M_{b.Rd} = \chi_{LT} A f_y / \gamma_{M1} \quad (30)$$

will become in the moment when $\chi_{LT} = 1$ the following resistance of the cross-section

$$M_{c.Rd} = W f_y / \gamma_{M0} \quad (31)$$

The jump between (26) and (27) is caused not only by $\gamma_{M1} > 1.0$, but also by the fact that (27) does not take into account the contribution of flanges $V_{bf.Rd}$ to the shear resistance.

There is not such problem in Eurocode EN 1999-1-1, because it uses only one safety factor γ_{M1} with recommended value 1.1. The safety factor γ_{M0} is not used in EN 1999 for design of aluminium structures.

Remark 1:

The symbol ρ is used in Eurocodes as the reduction factor, which enables in the clause 4.4 in EN 1993-1-5 to calculate in the case of local buckling the properties A_{eff} and/or W_{eff} of the effective cross-section under compression, bending or their combination (numerical values of ρ are given in Tables 1 – 6). The same symbol ρ is used in the clause 6.2.8 and 6.2.10 in EN 1993-1-1 for another purpose, see paragraph a) in the Introduction. Generally, it is not convenient to use the same symbol for different phenomena. It is therefore proposed to use in the new generation of Eurocodes the following formula (32) instead of formulae (1) and (2a)

$$\rho_V = 1 - (2V_{Ed}/V_{pl,Rd} - 1)^2 \qquad (32)$$

The formula (32) has comparing with the formulae (1) and (2a) two advantages: another symbol ρ_V is used, which is really the reduction factor taking into account the negative influence of shear force in decreasing of bending moment resistance of cross-section. The meaning of ρ in (1) and (2a) is not the reduction factor.

Remark 2:

The numerical example in (Koleková, Baláž, 2013b) contains mistakes, which were corrected in (Baláž, Koleková, 2013c): the quantities g and g_p were exchanged and m_2 was calculated in incorrect way in (Koleková, Baláž, 2013b).

Acknowledgement

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EN 1993-1-5 (2006) and Corrigendum AC (2009) *Eurocode 3 - Design of steel structures. Part 1-5: Plated structural elements.*

EN 1999-1-1 (2007) and Amendment A1 (2009) and Amendment A2 (2013) *Eurocode 9 - Design of aluminium structures. Part 1-1: General structural rules.*

STN 73 1401 (1998) *Navrhovanie oceľových konštrukcií.*

ČSN 73 1401 (1998) *Navrhování ocelových konstrukcí.*

ANNEX: Procedure and formulae of STN 73 1401: 1998

The Slovak standard STN 73 1401: 1998 Design of steel structures was valid from March 1998 till 1st April 2010, when it was withdrawn. STN 73 1401: 1998 was partly based on prestandard Eurocode ENV 1993-1-1: 1992.

Verification of the bending moment resistance of class 4 cross-section

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1{,}0\,, \qquad M_{c,Rd} = \frac{W_{eff}\, f_y}{\gamma_{M1}} \qquad (A.1)$$

Note γ_{M1} in STN 73 1401 and ENV 1993-1-1 instead of γ_{M0} used in EN 1993-1-1, compare (A.1) with (4a).

Reduction factor for the calculation the effective web width in bending

$$\rho_M = \frac{\beta_{1,M}}{\beta_{r,M}} \leq 1 \qquad \text{(A.2)}$$

where $$\beta_{1,M} = 130\sqrt{\frac{235\text{MPa}}{f_y}}, \qquad \beta_{r,M} = 0.8\frac{h_w}{t_w} + 0.2\beta_{1,M} \qquad \text{(A.3)}$$

and effective width

$$b_{c,eff} = b_c \rho_M, \qquad b_{e1} = 0.4 b_{c,eff}, \qquad b_{e2} = 0.6 b_{c,eff} \qquad \text{(A.4)}$$

Webs with

$$\frac{h_w}{t_w} > \beta_{1,V} = 100(0.7 + \frac{0.3}{\alpha^2})\sqrt{\frac{235\text{MPa}}{f_y}}, \qquad \alpha = \max(a / h_w;\ h_w / a) \geq 1 \qquad \text{(A.5)}$$

shall be verified for resistance to shear buckling.

Verification of the shear buckling resistance

$$\frac{V_{Ed}}{V_{b.Rd}} \leq 1, \qquad V_{b,Rd} = \rho_V \frac{h_w\, t_w\, f_{yw}}{\sqrt{3}\,\gamma_{M1}} \qquad \text{(A.6)}$$

where reduction factor for shear

$$\rho_V = \frac{\beta_{1,V}}{\beta_w} \leq 1, \qquad \beta_w = \min(h_w / t_w;\ a / t_w) \qquad \text{(A.7)}$$

The simple post-critical method can be used for the webs of I-section girders with or without intermediate transverse, provided that web has transverse stiffeners at the supports.

$$V_{ba,Rd} = \rho_{V,a} \frac{h_w\, t_w\, f_{yw}}{\sqrt{3}\,\gamma_{M1}} \qquad \text{(A.8)}$$

where reduction factor for shear in the simple post-critical method

$$\rho_{V,a} = \frac{\beta_{1,V}}{\beta_{r,V}}, \qquad \beta_{r,V} = 0.8\beta_w + 0.2\beta_{1,V} \qquad \text{(A.9)}$$

The complete post-critical method takes into account the contribution of the flanges to shear buckling resistance, provided that web has transverse stiffeners with aspect ratio of the web field $1 \leq \alpha \leq 3$. This method cannot be used if the flange is directly loaded by the moveable loading.

$$V_{bb,Rd} = \rho_{V,b} \frac{h_w\, t_w\, f_{yw}}{\sqrt{3}\,\gamma_{M1}} \qquad \text{(A.10)}$$

where reduction factor for shear in the complete post-critical method

$$\rho_{V,b} = \rho_{V,a} + \rho_{V,f} \qquad \text{(A.11)}$$

The reduction factor taking into account the contribution of the flanges $\rho_{V,f}$ is defined in STN 73 1401 by the formula (59):

$$\rho_{V,f} = \varphi_f \left(\sqrt{\frac{M_{N,fc}}{M_{pl,w}}} + \sqrt{\frac{M_{N,ft}}{M_{pl,w}}}\right) \qquad \text{(A.12)}$$

The relevant quantities used in (A.12) are defined in STN 73 1401 in the clause 6.7.3.2.

Verification of the resistance of cross-section under combination of compression force N_{Ed}, bending moment M_{Ed} and shear force V_{Ed} according to STN 73 1401:1998.

The simple post-critical method (here for N_{Ed} = 0 kN)

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1{,}0 , \quad \text{if} \quad \frac{V_{Ed}}{V_{ba,Rd}} \leq 0.5 \tag{A.13}$$

$$M_{Ed} = M_{f,Rd} + (M_{c,Rd} - M_{f,Rd})\,[1 - (2V_{Ed}/V_{ba,Rd} - 1)^2], \quad \text{if} \quad \frac{V_{Ed}}{V_{ba,Rd}} > 0.5 \tag{A.14}$$

Instead of $M_{f,Rd}$ and $M_{c,Rd}$ reduced resistances $M_{fN,Rd}$ and $M_{cN,Rd}$ are used in (A.14) if N_{Ed} > 0 kN (Fig. A.1).

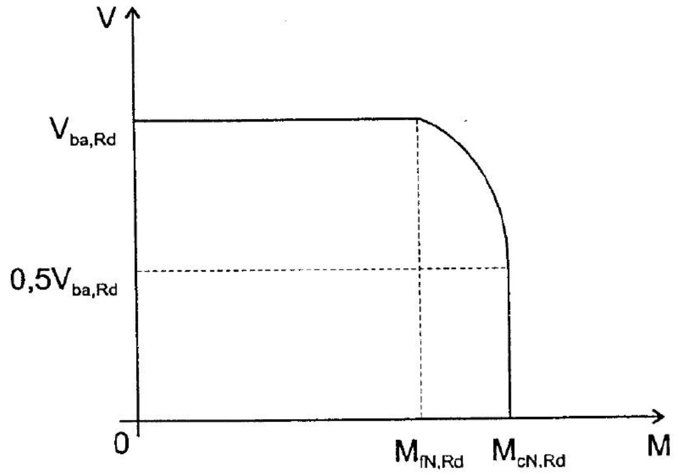

Fig. A.1: Interaction of shear force and bending moment according to the simple post-critical method of STN 73 1401:1998.

The complete post-critical method (here for N_{Ed} = 0 kN)

The same as simple post-critical method if $V_{Ed} \leq V_{ba,Rd}$.

$V_{Ed} \leq V_{bb,Rd}$ if $V_{Ed} > V_{ba,Rd}$ and $M_{Ed} \leq M_{f,Rd}$ (or $M_{fN,Rd}$ if N_{Ed} > 0 kN), see Fig. A.2.

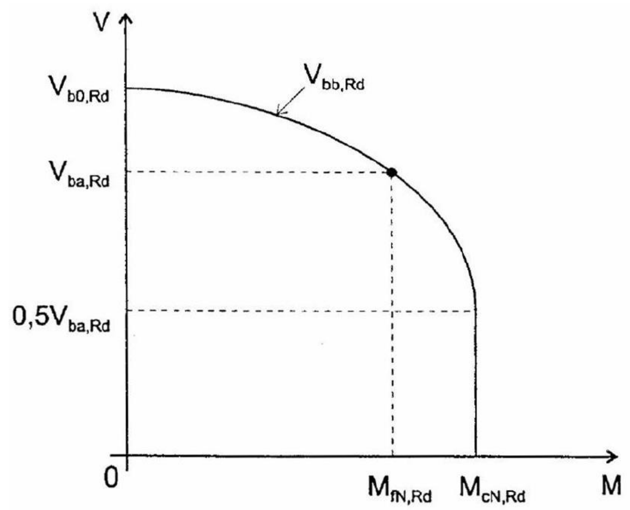

Fig. A.2: Interaction of shear force and bending moment according to the complete post-critical method of STN 73 1401:1998.

The procedure of STN 73 1401:1998 is applied (for $V_{V,f}$ = 0 kN) in numerical example showed in Fig. 1.