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PROPOSITION OF CONSTITUTIVE MODEL FOR FIBRE-REINFOCED HYPERELASTIC MATERIALS

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Abstract: *In the paper the class of relatively simple constitutive models of hyperelastic non-homogeneous composite materials with isotropic matrix reinforced with continuous fiber families is proposed. The model was formulated on the basis of strain energy additivity assumption. Proposed class of constitutive models reduce in approximation to the classical for linear theory models of fibrous composites, where full bounding between matrix and fibers is assumed. The strain energy potential for proposed model is a poly-convex function, what ensure existence of solution of boundary value problems for hyperelasticity and good numerical conditioning. Constitutive relationships expressed in objective incremental form are implemented in FORTRAN in user procedure UMAT of FEM system ABAQUS. The numerical tests were carried out to check correctness of the implementation, and two types of boundary value problems of tubes tension/compression tests are solved.*

Keywords: Anisotrophy, Hyperelasticity, Fibre-reinforced materials, Constitutive models.

1. Introduction

Composite materials in the form of isotropic matrices reinforced with fibers are commonly used in technical applications because of their desirable mechanical properties. The main goal of fiber insertion into matrix is to obtain needed mechanical properties understood as desired stiffness and assumed strength. The fundamental condition to obtain planed mechanical properties of composite is a good coupling between components (in this case between fibers and matrix). And this assumption was a starting point several years ago for proposition of fiber composite theoretical model based on mixture theory, cf. Spencer (1972), Boehler (1987). Such models implemented in small deformation theory are very usefull in case of many engineering problems from geotechnical applications for geosynthetic grid modeling to aircraft skins modeling. The fundamental motivation for development of anisotropic hyperelastic constitutive models was and still is biomechanics and mechanics of woven materials. In case of biomechanics and soft tissue constitutive modeling, the application of large deformation theory is well-founded, cf. Bonet & Wood (1997). Unfortunately, in other mechanic disciplines the need for large deformation theory application is not always well understood. For example, geosynthetic grid becomes reinforcement for soil when soil deformation is large, when differences between configurations are significant. Then it follows that application of theory with nonlinear geometry is needed. On the other hand it is inadmissible in continuum mechanics to use so called “physically linear” constitutive models, because they are not fulfilling all basic requirements resulting from objectivity and energy conservation rules. The simplest theory in which all requirements are fulfilled is theory of hyperelastic materials, and a class of constitutive models for fiber reinforced materials considered herein is situated in group of anisotropic hyperelastic constitutive models (especially orthotropic and transversally isotropic hyperelastic materials, cf. Jemioło & Telega (2001)). The main goal of this paper is to extend previously presented constitutive model (Gajewski & Jemioło, 2007) in such a way that allows analysis of composites in which matrix is reinforced with many fiber families and its illustration on some non-trivial numerical examples.

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2. Basic Assumptions and Constitutive Model of Anisotropic Hyperelasticity

Strain energy function (SEF) of hyperelastic material reinforced with several fiber families can be postulated in the following additive form:

$$\Psi = \left(1 - \sum_{n=1}^{N} p_n\right)\Psi_M + \sum_{n=1}^{N} p_n \Psi_{Rn} , \tag{1}$$

where Ψ_M is a strain energy function for matrix (SEFM), Ψ_{Rn} are elastic strain energy functions of fiber families (SEFR), and p_n stands for volume ratio of fibers in material volume unit. Matrix material is an isotropic material, for which SEF function is isotropic with respect to right deformation tensor $\mathbf{C} = \mathbf{F}^T\mathbf{F}$, and left deformation tensor $\mathbf{B} = \mathbf{F}\mathbf{F}^T$, where $\mathbf{F}$ is so called deformation gradient tensor. The tensor $\mathbf{F}$ has a positive determinant $J = \det\mathbf{F} > 0$, and symbol "T" in above relations stand for tensor transposition. According to the above assumption (isotropic function) the SEFM $\Psi_M = W(I_1, I_2, J)$ is a function of three non-reducing invariants of deformation tensor: $I_1 = \mathrm{tr}\mathbf{C}$, $I_2 = \mathrm{tr}(\mathrm{cof}\mathbf{C})$ and $J = \det\mathbf{F} = \sqrt{\det\mathbf{C}}$. The SEFR function of n-th fiber family, which "works" in direction described with vector $\mathbf{m}_n(\mathbf{X})$, is approximated as:

$$\Psi_{Rn} = \frac{E_{Rn}}{4}\left(I_{4n} - 1\right)^2 , \tag{2}$$

where $I_{4n} = \mathrm{tr}\hat{\mathbf{M}}_n$. The $\hat{\mathbf{M}}_n$ tensors represent parametric tensors $\mathbf{M}_n = \mathbf{m}_n \otimes \mathbf{m}_n$ in actual configuration (e.g. $\hat{\mathbf{M}}_n = \mathbf{F}\mathbf{M}_n\mathbf{F}^T$). In (2) the E_{Rn} parameter have an interpretation of Young modulus of n-th fiber family. In this paper we are considering a special case of SEFM function, so called Ciarlet model for compressible materials:

$$\Psi_M = \frac{\mu_o}{2}\left[f\left(I_1 - 3\right) + \left(1 - f\right)\left(I_2 - 3\right)\right] + \mathrm{A}J^2 - \left[\frac{\lambda_o}{2} + \mu_o\right]\ln J - \mathrm{A} , \tag{3}$$

also discussed in Jemioło (2002), where $\mathrm{A} = \frac{1}{4}\left[\lambda_o - 2\mu_o\left(1 - f\right)\right]$. The parameters: μ_o and λ_o can be interpreted as Lame constants. Function (3) is a poly-convex one and fulfills appropriate conditions of growth of elasticity potential if and only if $\mu_o > 0$, $f \in (0,1)$ and $\lambda_o > 2\mu_o\left(1 - f\right)$, cf. Jemioło (2002). From local energy and mass conservation laws (altogether with balance equations of linear and angular momentum) one can obtain the Kirchhoff stress tensor for matrix made of Ciarlet model in the following form:

$$\boldsymbol{\tau}_M = \mu_0 f\,\mathbf{B} + \mu_0\left(1 - f\right)\left(I_1\mathbf{B} - \mathbf{B}^2\right) + \left[\frac{1}{2}\left(\lambda_0 - 2\mu_0\left(1 - f\right)\right)J^2 - \frac{1}{2}\lambda_0 - \mu_0\right]\mathbf{I} . \tag{4}$$

From (2) one can obtain the following relation for Kirchhoff stress in n-th fiber family:

$$\boldsymbol{\tau}_{Rn} = E_{Rn}\left(I_{4n} - 1\right)\hat{\mathbf{M}}_n . \tag{5}$$

So, the constitutive relationship for material reinforced with fiber families is as follows:

$$\boldsymbol{\tau} = J\boldsymbol{\sigma} = \left(1 - \sum_{n=1}^{N} p_n\right)\boldsymbol{\tau}_M + \sum_{n=1}^{N} p_n \boldsymbol{\tau}_{Rn} , \tag{6}$$

where $\boldsymbol{\sigma}$ is a Cauchy's stress tensor. The proposed constitutive model was implemented in the finite element method program ABAQUS through UMAT procedure for which constitutive relationship (6) has been rearranged into an incremental form, cf. ABAQUS (2000a and b).

3. Compression and Tension Tests of an Elastic Reinforced Tube

The problem of compression/tension of elastic reinforced tube in the direction of it's axis by applying displacement boundary conditions at the bottom and top base is considered. On the other parts of pipe outside surface the zero stress boundary conditions are assumed. The pipe is made of hyperelastic material with fiber reinforcement, characterized by constitutive relationship expressed by (6). The four cases are considered, i.e. lack of reinforcement, the reinforcement overlaps with direction 3, reinforcement is placed circumferentially and reinforced is placed helicoidaly, see Fig. 1. It is worth emphasizing that fiber placement directions are given in reference configuration, and during deformation undergo local changes.

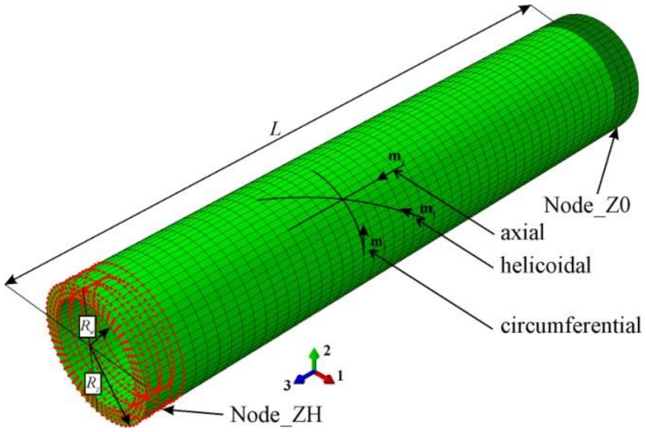

Fig. 1: FEM mesh for pipe of 100 mm length (R_w=8 mm, R_z=10 mm) with indication of node groups where boundary displacement conditions were assumed.

For the node group marked as Node_ZH (all nodes belonging to the top base of the pipe together with nodes laying on interior and exterior side surfaces but not further from the edge than 6 mm) the zero displacement boundary conditions for all three displacement components are assumed. Next, all nodes marked as Node_Z0, cf. Fig. 1, were joined with some reference node by applying multi-point constraints (MPC) option which gives the possibility to assume displacement boundary conditions for all nodes through reference node. In that case, in reference node the zero displacement boundary conditions for components u_1 and u_2 (preserving circular shape) and for all rotation angles were assumed. The pipe compression is obtained by assuming non-zero displacement u_3 =-20 mm, and in case of tension test u_3 = 50 mm. The following material data were assumed: p = 0.05, $\mu_0 = 1.0\, E_M$, $\lambda_0 = 1.5\, E_M$, $E_Z = 26\, E_M$, where E_M is an initial Young's modulus of matrix material, and $f = 0.2$.

All tasks were solved using standard Newton-Raphson incremental algorithm. The graphs of resulting cumulative compression and tension forces (denoted as F_{cc} and F_{ct}, respectively) in reference node as a function of displacement u_3 are shown in Fig. 2. In Fig. 3 the contour graphs of Mises stresses on deformed compressed tubes configurations are presented for minimum obtained forces (beginning of local or global buckling). The main goal of this example is a comparison of solutions obtained for hyperelastic isotropic material with solutions obtained for implemented constitutive models of hyperelastic materials reinforced with differently placed fiber families.

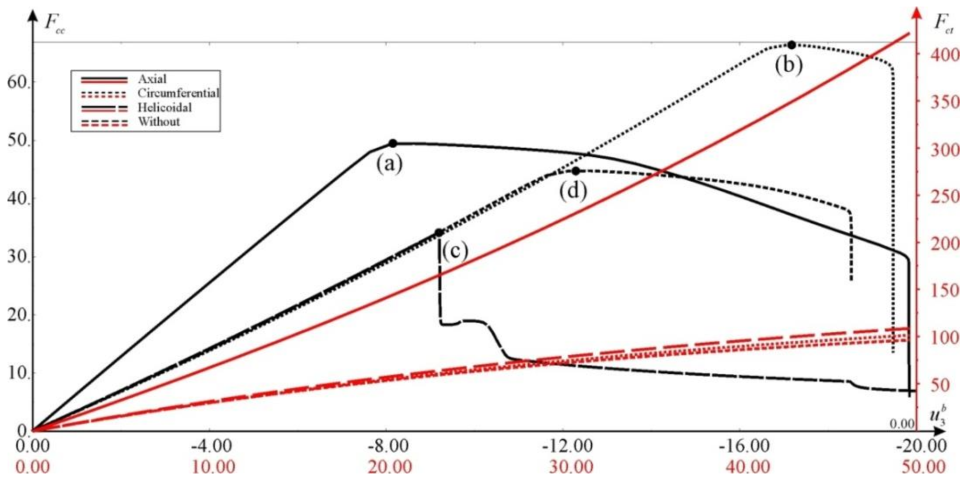

Fig. 2: The compression/tension cumulative force as a function of displacement u_3 for different types of fibre reinforcement.

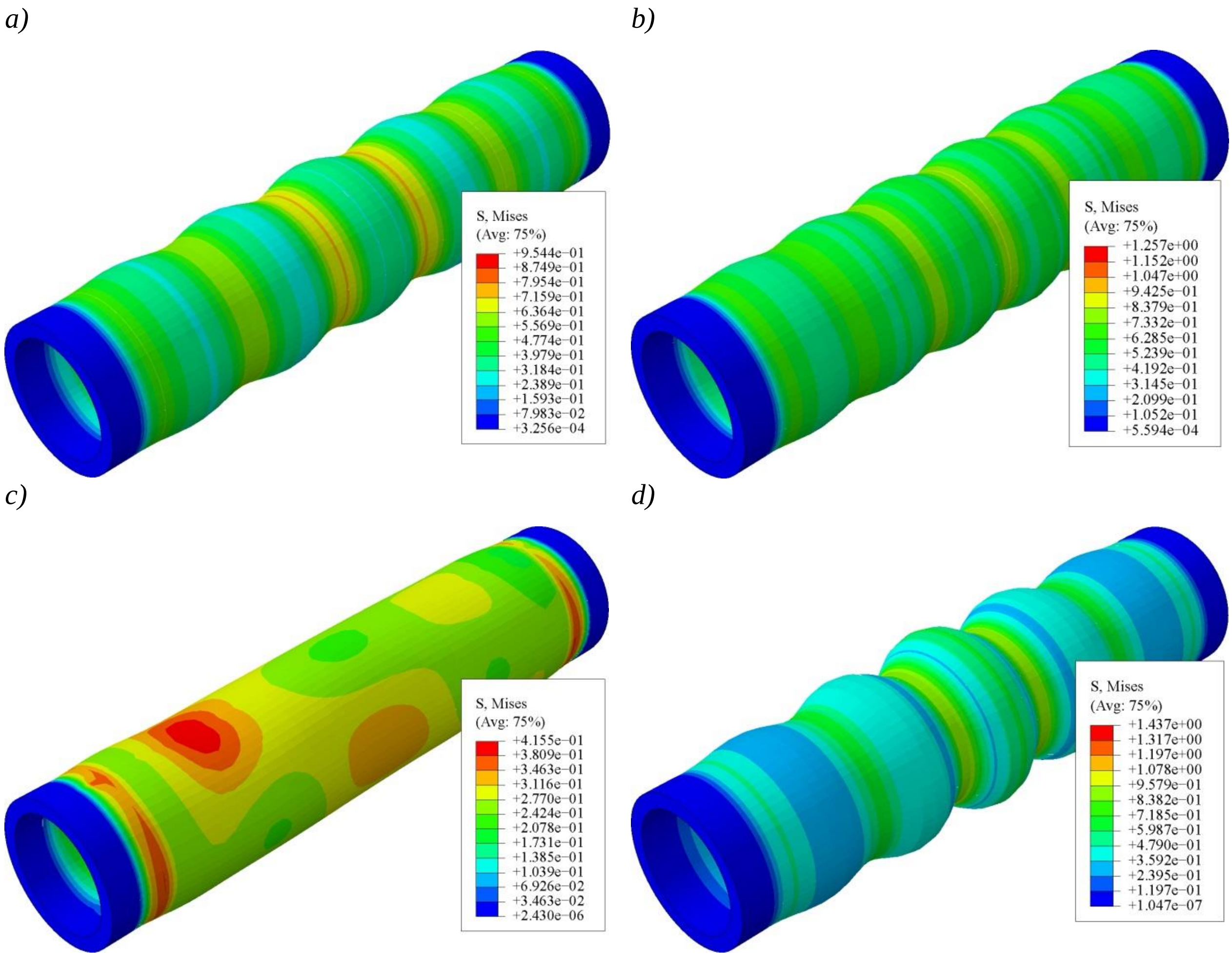

Fig. 3: Contour graphs of Mises stresses for compression tests in the configuration corresponding to minimum force, as indicated in Fig. 2.

4. Final Remarks

In the paper the constitutive relationship for anisotropic hyperelasticity in case of fibre reinforced materials has been shown altogether with its numerical implementation in FEM system ABAQUS. Some example applications illustrating the compression/tension of the differently reinforced tube is presented proving correctness and robustness of the implementation. The most suitable application area for proposed relationship is probably biomechanics, and modeling of vasculature consisting of a complex system of arteries, arterioles, capillaries and veins.

References

ABAQUS (2000a) Theory manual, Version 6.1, Hibbitt, Karlsson and Sorensen, Inc., Pawtucket.

ABAQUS (2000b) Standard User's manual, Version 6.1, Hibbitt, Karlsson and Sorensen, Inc., Pawtucket.

Boehler, J. P. (ed) (1987) Applications of tensor functions in solid mechanics. CISM Courses and Lectures, No. 292, Wien-New York, Springer-Verlag.

Bonet, J., Wood R.D. (1997) Nonlinear continuum mechanics for finite element analysis, Cambridge University Press.

Gajewski, M., Jemioło, S. (2007) FEM implementation of hyperelastic constitutive models of reinforced materials, XVI Slovak-Polish-Russian Seminar Theoretical Foundation of Civil Engineering, Žilina, pp. 119-126, Moscow, (in Polish).

Jemioło, S. (2002) A study of the hyperelastic properties of isotropic materials. Modelling and numerical implementation, Prace Naukowe, Budownictwo z. 140, pp. 1-308, OWPW, (in Polish).

Jemioło, S., Telega, J. J. (2001) Modelling elastic behaviour of soft tissues, Part II. Transverse isotropy, Eng. Trans., Vol. 49, No. 2-3, pp. 241-281.

Spencer, A. J. M. (1972) Deformations of fibre-reinforced materials, Oxford, Clarendon Press.