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POST-CRITICAL MULTI-MODAL VIBRATION OF A CONTINUOUS INVERSE PENDULUM TYPE SYSTEM

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Abstract: *Auto-parametric effects are processes endangering structures during a seismic attack. Tall structures exposed to a strong vertical component of an earthquake excitation nearby the epicenter can collapse due to auto-parametric resonance effect. Vertical and horizontal response components being independent in the linear regime get into a complex interaction due to non-linear terms in post-critical regime. Two generally different types of the post-critical regimes are presented: (i) post-critical state with possible recovery; (ii) exponentially rising horizontal response leading to a collapse. A special attention is paid to processes of transition from semi-trivial to post-critical state in case of time limited excitation period as it concerns the seismic processes. Solution method combining analytical and numerical approaches is developed and used. Its applicability and shortcomings are commented. A few hints for engineering applications are given.*

Keywords: Auto-parametric systems, Dynamic stability, Semi-trivial solution, Multi-modal vibration, Post-critical states.

1. Outline of the System

Non-linear dynamic effects are the most dangerous processes endangering structures during a seismic attack. Among them auto-parametric non-linear vibration in state of post-critical auto-parametric resonance, see Tondl et al. (2000), Hatwal et al. (1983) or Bajaj et al. (1994), can lead to collapse particularly in case of high slender systems or large dynamically sensitive structures. Auto-parametric resonance caused in the past heavy damages or collapses of towers, bridges and other structures. The main cause of these effects is a strong vertical component of an earthquake excitation in epicenter area. In sub-critical linear regime vertical and horizontal response components are independent and therefore in such a case no horizontal response component is observed. If the amplitude of a vertical excitation in a structure foundation exceeds a certain limit, a vertical response component looses dynamic stability, e.g. Benettin et al. (1980) and dominant horizontal response component arises and can lead to failure of the structure.

The seismic type broadband random non-stationary excitation can be particularly dangerous and amplify these effects. From viewpoint of rational dynamics the problem is of the inverse pendulum type. Authors are involved in this topics related with earthquake engineering for a long time, see e.g. Náprstek and Fischer (2009, 2010, 2011) and others.

In principle easily deformable tall structures are the most sensitive regarding effects of auto-parametric resonance (chimneys, towers, etc.). Therefore the structure itself is modeled as a console with continuously distributed mass and stiffness in order to respect the whole eigen-value and eigen-form spectrum, see Fig. 1. The subsoil model enables to respect vertical and rocking component of the response including internal viscosity of the Voight type. Mathematical model is deduced by Hamiltonian functional including kinetic and potential energies as well as the Rayleigh function:

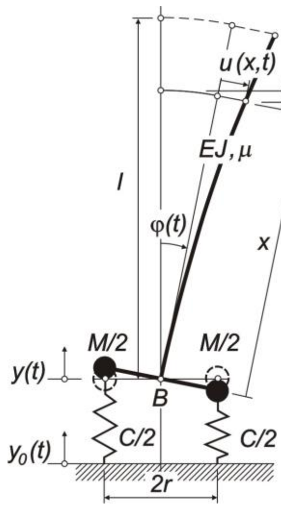

Fig. 1: Outline of the 3-DOF autoparametric system.

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$$T(t) = \frac{1}{2}M\big(\dot{y}^2(t) + r^2\dot{\varphi}^2(t)\big) + \frac{1}{2}\mu\int_0^l\big[\big(\dot{\varphi}(t)x + \dot{u}(x,t)\big)^2 + \dot{y}^2(t) - 2\dot{y}(t)\big(\dot{\varphi}(t)x + \dot{u}(x,t)\big)\sin\varphi(t)\big]\mathrm{d}x, \quad \text{(1a)}$$

$$U(t) = Mg\cdot y(t) + \frac{1}{2}C\big((y(t) - y_0(t))^2 + r^2\varphi^2(t)\big) + \mu g\int_0^l[y(t) - x(1-\cos\varphi(t)) - u(x,t)\sin\varphi(t)]\mathrm{d}x + \frac{1}{2}EJ\int_0^l u''^{\,2}(x,t)\mathrm{d}x. \quad \text{(1b)}$$

Hamiltonian functional provides Lagrangian differential system linking strongly non-linear two-degree of freedom (TDOF) part with multi-degree of freedom (MDOF) part modeling continuous console to the simultaneous governing system, see e.g. Náprstek and Fischer (2009,2010), or Náprstek and Fischer (2011). The material damping of the console is proportional. Therefore the deformation of that can be expressed in a form of a convergent series:

$$u(x,t) = \sum_{i=1}^{n}\alpha_i(t)\cdot\psi_i(x) \Rightarrow \psi(\xi,t) = \sum_{i=1}^{n}\alpha_i(t)\chi_i(\xi), \quad (2)$$

where $(x) = l\cdot\chi_i(\xi)$ and basis functions $\chi_i(\xi)$ are eigen functions (eigen forms) of the differential equation:

$$\chi_i''''(\xi) + \lambda_i\chi_i(\xi) = 0, \quad (\lambda_i/l)^4 = \mu\omega_i^2/EJ \quad (3)$$

with boundary conditions for a console beam: $\chi_i(0) = 0,\ \chi_i'(0) = 0,\ \chi_i''(1) = 0,\ \chi_i'''(1) = 0.$

The Lagrangian system for components $\zeta(t), \varphi(t)$ and components $\alpha_i(t)$ arithmetizing coordinates $\chi_i(\xi)$ reads:

$$\begin{aligned}
&\ddot{\zeta}(t) - \frac{1}{4}\kappa_0(\varphi^2(t))^{\cdot\cdot} - \kappa_0\sum_{i=1}^{n}\big[\big(\varphi(t)\dot{\alpha}_i(t)\big)^{\cdot}\,\Theta_{0,i}\big] + \omega_0^2\Big[\zeta(t) - \zeta_0(t) + \eta_c\Big(\dot{\zeta}(t) - \dot{\zeta}_0(t)\Big)\Big] = 0\\
&\ddot{\varphi}(t) - \frac{1}{2}\kappa_1\ddot{\zeta}(t)\varphi(t) + \kappa_1\sum_{i=1}^{n}\big[\ddot{\alpha}_i(t)\,\Theta_{1,i} + (\dot{\zeta}(t)\dot{\alpha}_i(t) - \omega_2^2\alpha_i(t))\,\Theta_{0,i}\big] + \omega_1^2[\varphi(t) + \eta_c\dot{\varphi}(t)] = 0 \qquad (4)\\
&\ddot{\alpha}_i(t)\cdot\Theta_{2,i} + \ddot{\varphi}(t)\cdot\Theta_{1,0} - [(\dot{\zeta}(t)\varphi(t))^{\cdot} + \omega_2^2\varphi(t)]\cdot\Theta_{0,i} + \omega_3^2[\alpha_i(t) + \eta_e\dot{\alpha}(t)]\Theta_{3,i} = 0
\end{aligned}$$

where

$$\zeta_0(t) = y_0(t)/l,\ \zeta(t) = y(t)/l,\ \varphi(t),\ u(x,t)/l = \psi(\xi,t), \xi = x/l,\ \rho = r/l,\ m = \mu l \quad (5)$$

Solution method combining analytical and numerical approaches is presented in the final text together with its applicability and shortcomings. A wide parametric analysis is provided and regular and special cases indicated, quantified and commented. A few hints for engineering applications being motivated by these results are given.

2. Post-Critical Response Types and Transition Effects

The system shows that horizontal and vertical response components are independent in the semi-trivial regime, which is linear in such a case. Their interaction takes place due to non-linear terms in post-critical regime only. Interaction of nonlinear modes provided by the console and subsoil is investigated, as it comes to light that sub- and super-harmonic resonances can produce a number of effects typical for nonlinear approach particularly when internal resonances arise, see Fig. 2, where the sub-harmonic resonance effect is obvious.

Complicated inter-resonance effects of quasi-periodic type are verified in order to detect an existence of possible beating processes related with a possible periodic energy transfer among degrees of freedom. A number of non-linear effects related with MDOF character of the rather weak console and its interaction with bottom part of the system is discussed, in the best knowledge of authors, for the first time. Various post-critical deterministic as well as chaotic types of the response are investigated as well. Attractive and repulsive post-critical limit cycles are typical and will be carefully discussed in the full text. Investigation being conducted using Lyapunov exponent testing makes possible to identify a number of interaction types of eigen-modes.

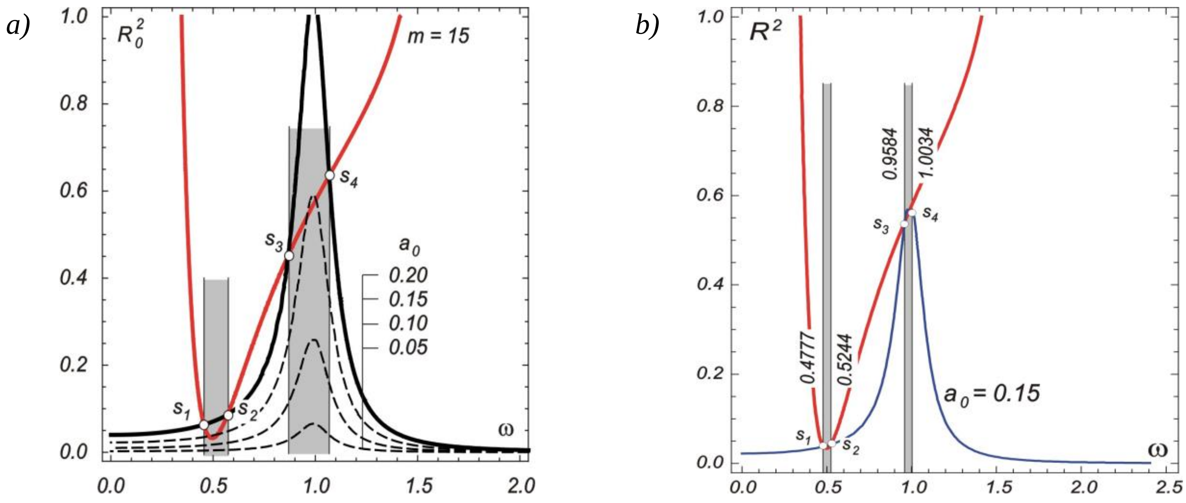

Fig. 2: Instability intervals: a) Large excitation amplitude (a_0 = 0.20);
b) Medium excitation amplitude (a_0 = 0.15).

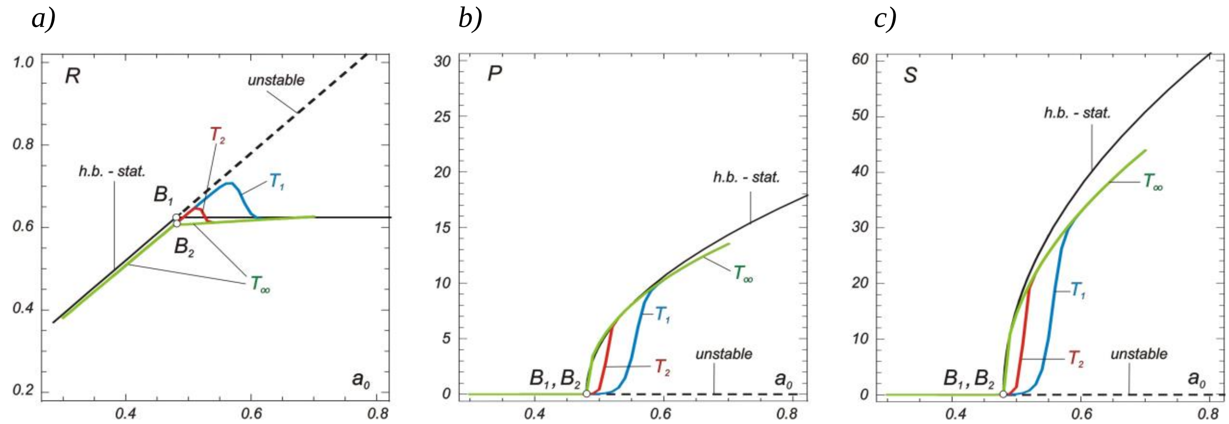

Fig. 3: Process of the stability loss and of the post-critical response:
a) Bifurcation diagram of the vertical response component P - amplitude R;
b) and c) Bifurcation diagrams of horizontal (rotation) components φ, ψ - amplitudes P, S.

Two generally different types of the post-critical regimes are presented in the paper: (i) the close neighbourhood of the stable state (area between the semi-trivial solution stability limit and the limit of irreversibility); despite the response is strongly non-linear, structure can regain the stable state, when excitation drops below a certain limit; (ii) if the response leaves beyond the limit of irreversibility, the rocking response component looses any periodic character and rises exponentially leading inevitably to a failure of the structure without possibility of any recovery. In such a case an avalanche process throughout eigen-modes can emerge releasing accumulated energy in the system before the post-critical processes occurs.

A special attention is paid to processes of transition from semi-trivial to post-critical state in case of time limited excitation period as it concerns the seismic processes, see Fig. 3. In the picture (a) the bifurcation point B_1 is an origin of two branches: stable and unstable. It is obvious that for a short excitation interval (blue curve) the response follows the unstable branch for higher excitation amplitudes than it corresponds to B_1 and only after a certain time it drops asymptotically to nearly horizontal stable branch. Similarly in pictures (b) and (c) we can observe that the response for a short excitation interval is nearly zero following the unstable branch and only later is rising to stable branch demonstrating large horizontal amplitudes. Due to this effect a significant increase of admissible excitation amplitude is provided contributing for the sake of the structure safety.

3. Conclusions

Authors deal with easily deformable tall structures, which are very sensitive to effects of auto-parametric resonance (chimneys, towers, etc.). If the amplitude of a vertical excitation in a structure foundation exceeds a certain limit, a vertical response component looses stability and dominant horizontal response component arises. This post-critical regime (auto-parametric resonance) follows from the non-linear interaction of vertical and horizontal response components and can lead to a failure of the structure.

In principle solution methods combining analytical and numerical approaches have been developed and used. Their applicability and shortcomings are commented. A few hints for engineering applications in a design practice are given. Some open problems are indicated.

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