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DETERMINATION OF MATERIAL PARAMETERS OF RUBBER AND RUBBER COMPOSITES BY BIAXIAL LOADING

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Abstract: *This paper describes a method, designed for measurement of material characteristics of rubber like materials, using biaxial loading device. We present the experimental measurement and FEA simulation in Comsol Multiphysics of the response of rubber specimens from conveyor belt used for the transport of overburden in a coal mine. The results of numerical simulation are verified by experimental measurements. This work is part of a broader project that aims to increase service life of the conveyor belts.*

Keywords: Rubber, Digital Image Correlation, Shear Modulus, Rubber material model, FEA.

1. Introduction

Rubber is a material capable of large deformation (100% at least) under relatively small load. Rubber, after load removal, quickly returns to almost its original state. Specimens of styrene-butadiene rubber, investigated in this work, were cut out of a covering layer of the conveyor belt. The material parameters of this covering layer are needed for the numerical simulation of the behaviour of the complete belt. Specimens were loaded in uniaxial and biaxial tension. Applied forces were measured by force sensors and large 2D displacements were measured by a non-contact, optical method - Digital Image Correlation - using Dantec Dynamics device and 2D strains were evaluated by Istra4D. Mooney-Rivlin hyperelastic material model with two parameters was chosen. The parameters were calculated by optimization methods in Matlab. The shear modulus was computed from these parameters and FEA of loading states specified above was performed in Comsol Multiphysics.

2. Experimental

Measurements are performed at room temperature using both special biaxial testing machine and universal testing machine TIRA for uniaxial loading. Measured variables are deformation and force. Force is measured by load cells placed on machine's grippers. Deformation is measured using Dantec Dynamics Q-400 system with Istra 4D software. As a result deformation is plane only one camera was used so the whole measurement is simplified to 2D. It is acceptable due to relatively thin specimens. All specimens were cut off from the same cover layer of belt. Specimens for uniaxial measurements are thin strips 300 x 28 x 3 mm. Specimens for biaxial measurement had a cruciform shape of 140 x 140 x 7 mm, with the width of cross arms 60 mm.

2.1. Uniaxial load

Uniaxial measurements were performed on 5 specimens of the same dimensions. The determined stress-strain diagram is represented in Fig. 1. These data were used for shear modulus determination. Mooney-

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Rivlin parameters were computed in Comsol Multiphysics by optimization method C_{01} = 1.72 MPa, C_{10} = 0.15 MPa and shear modulus G = 3.73 MPa.

2.2. Biaxial load

Biaxial measurements were performed on special biaxial testing machine which can be seen on Fig. 3. Specimens were received from covering layer of conveyor belt. Measured data can be seen on Fig. 2. Mooney-Rivlin parameters were computed in Matlab by chapter 2.2.1. So C_1 = 0.49 MPa , C_2 = 0.56 MPa and shear modulus G = 2.11 MPa.

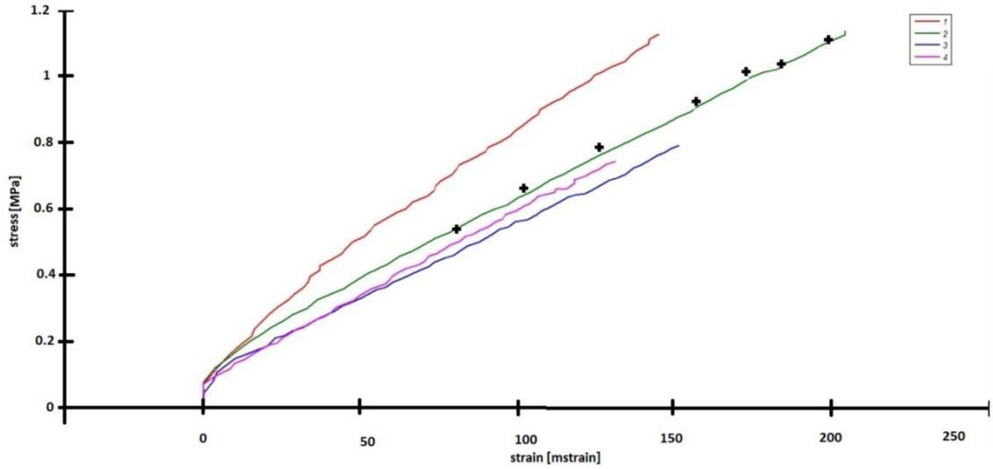

Fig. 1: Uniaxial tensile test. Full lines - measured data, points - FE simulation.

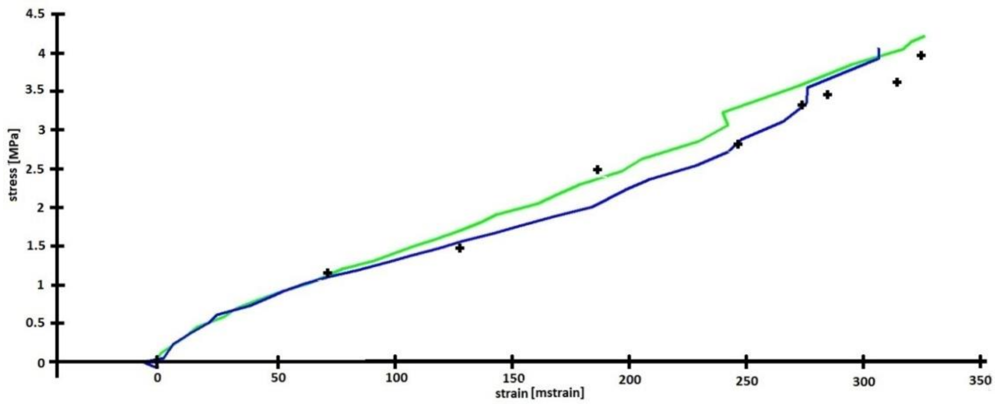

Fig. 2: Biaxial test. Full lines - measured data, points - FE simulation.

2.2.1. Shear Modulus

Elastomers are described by hyperelastic materials e.g. Ogden, Mooney-Rivlin or Neo-Hookean (Petríková, 2008). Each hyperelastic material is expressed by the strain energy density function. To determine shear modulus we use Mooney-Rivlin hyperelastic material with two parameters due to achieved elongation, using more parameters would lead to better fitting at higher deformations. Expression for strain energy density function in terms of the stretch ratios for Mooney-Rivlin material is represented by equation (1) and (2)

$$\Psi = C_1(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) + C_2(\lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2} - 3) \quad (1)$$

$$\lambda_i^2 = 2E_{ij} + 1 \quad (2)$$

where C_1, C_2 are Mooney-Rivlin parameters, λ_i are the stretch ratios and E_{ij} are components of Green Langrange strain tensor. Equation (3) defines the relation between stress (4) and deformation.

$$\sigma_a = \lambda_a \frac{\partial \Psi}{\partial \lambda_a} - p \tag{3}$$

$$\sigma_a = \frac{F_a \lambda_a}{A_{0a}} \tag{4}$$

where F_a is measured load force and A_{0a} is cross-sectional area of undeformed specimen. If according to (Holzapfel, 2000) the incompressibility of the rubber is assumed, then

$$\lambda_1 \lambda_2 \lambda_3 = 1 \tag{5}$$

so

$$\lambda_3 = \frac{1}{\lambda_1 \lambda_2} \tag{6}$$

So the first and second principal stresses are determined as equations (7, 8).

$$\sigma_1 = 2C_1\left(\lambda_1^2 - \frac{2}{\lambda_1^2 \lambda_2^2}\right) + 2C_2(-\lambda_1^{-2} + 2\lambda_1^2\lambda_2^2) \tag{7}$$

$$\sigma_2 = 2C_1\left(\lambda_2^2 - \frac{2}{\lambda_1^2 \lambda_2^2}\right) + 2C_2(-\lambda_2^{3} + 2\lambda_1^2\lambda_2^2) \tag{8}$$

Parameters C1 and C2 are determined by solving overdetermined system of equations (7 and 8) in Matlab. Lamé parameter is determined by equation (9), whereas according to (Holzapfel, 2000) Lamé parameter is equal to shear modulus G.

$$\mu = 2(C_1 + C_2) = G \tag{9}$$

The shear modulus of uni-axially loaded sample can be computed from optimization module in Comsol with more accuracy (COMSOL). Computation performed in Comsol is based on equivalent expression for strain energy density of equation (1), and Mooney-Rivlin material model. The inputs were measured stress- strain data to perform the computation of shear modulus. Output of the process are computed Mooney- Rivlin parameters, the fitting is displayed in the Fig. 4.

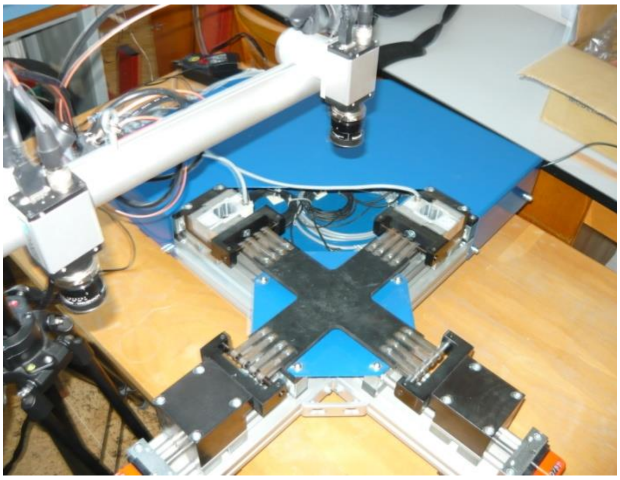

Fig. 3: Biaxial loading device.

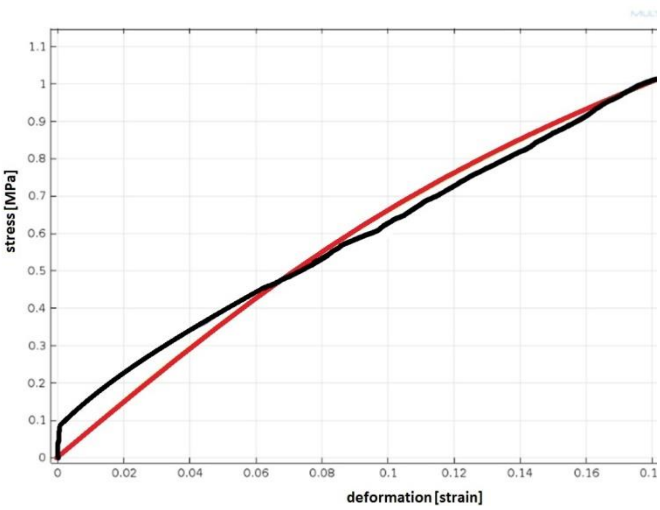

Fig. 4: Stress-strain diagram, -black experimental data, red - fitted curve.

2.2.2. FEA

Finite element analyses of both experiments were performed in Comsol Multiphysics. The comparison of measured and computed stress-strain fields is displayed in Figs. 5 and 6. Specific values from simulation can be found on Figs. 1 and 2. Simulations were performed in 2D with material parameters as computed in part 2.1. and 2.2., using Mooney-Rivlin material model.

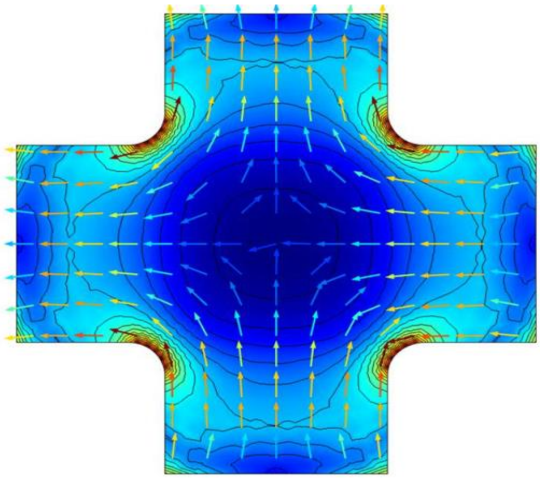

Fig. 5: FEA, displays first principal stress and first principal strain and its directions.

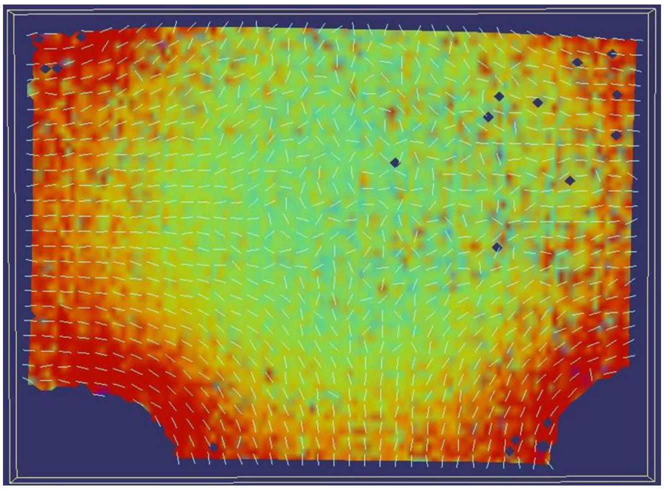

Fig. 6: Istra 4D image from deformation measurement, displays first principal strain.

3. Conclusions

This article includes measured stress-strain diagrams for two different types of experiments and five measured samples from covering layer of conveyor belt, calculation of shear modulus, and FEA of various experiments with the validity of described methods.

Material model used for uniaxial tension was Mooney-Rivlin. However, in the range of experimental deformation (under 100%) Neo-Hookean model with only one parameter could be sufficient to determine the shear modulus. On the contrary there was relatively higher deformation developed in the biaxial measurement, mainly due to stiffer clamping of the sample. This indicates a good fit of Mooney-Rivlin material model with two parameters. Homogeneous strain field is observed in the middle of the cruciform specimen designed for biaxial testing (Figs. 5 and 6). Similarly, first principal strain directions are the same both in FEA and measurement. The comparison of the values of stress calculated by FEA and determined by measurement showed a very small difference of 0.5 MPa. This indicates that methods used to determine shear modulus are valid for given material. The difference between computed shear modulus is given by measurement uncertainty, 2D simplification, numerical approximations and anisotropy that may arise in samples during the preparation. The work history of provided conveyor belt is unknown. Therefore computed values of shear modulus are valid just for this particular sample of conveyor belt and only to the amount of deformation achieved in experimental measurement with bi-axial or uni-axial tensile load.

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