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MODELING AND SIMULATION OF THE RECUPERATIVE HEAT EXCHANGER

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Abstract: *The paper demonstrates mathematic model of the recuperative heat exchanger. System response in analytical form was designated by solving differential equations system. Transfer function of analysing exchanger was also designated. Based on transfer function, there was performed a simulation of tank treated as controlled plant.*

Keywords: Modelling, Dynamic characteristic, Heat exchanger, Simulation, SCILAB.

1. Introduction

The main issue in control system design is an identification of controlled plant. Only for well-recognised controlled plant, settable time constants of the control system can be selected in the way that the constants will be close to optimum from a specific quality control criteria point of view (e.g. minimum time, aperiodic order etc.). The paper demonstrates an example of mathematical modelling of shell tank heat exchanger used in Spirits Factories for heat recovery for domestic purposes from produced alcohol.

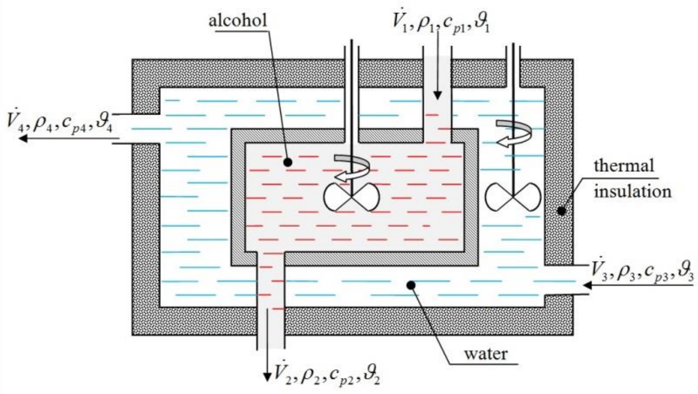

Fig. 1: Schematic representation of analysing shell tank heat exchanger.

2. Mathematical Modelling

2.1. Simplifying assumptions

Below are presented the basic simplifying assumptions which have to be supplemented in the next stage of the modelling process (Cengel et al., 2001):

- uniform temperature distribution in chambers (lumped parameter), the assumption is made possible by using the stirrers (Fig. 1),
- negligibly small changes in ρ_i mass density (dealing with liquids),
- negligibly small heat capacity of the wall of inner and outer tank,
- system is perfectly isolated from the environment (negligible heat losses).

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2.2. Initial mathematical description

The coolant (water) flows through outer tank and the cooled liquid (alcohol) flows through inner tank. For each tank the motion equations can be determined. Based on above simplifying assumptions, the equations can be written as:

$$V_1\rho_2 c_{p2}\frac{d\vartheta_2}{dt} = \dot{V}_1\rho_1 c_{p1}\vartheta_1 - \dot{V}_2\rho_2 c_{p2}\vartheta_2 - kA\left(\vartheta_2 - \vartheta_4\right) \tag{1}$$

$$V_3\rho_4 c_{p4}\frac{d\vartheta_4}{dt} = \dot{V}_3\rho_3 c_{p3}\vartheta_3 - \dot{V}_4\rho_4 c_{p4}\vartheta_4 - kA\left(\vartheta_2 - \vartheta_4\right) \tag{2}$$

where: A – cooling surface – inner tank surface $\left(\mathrm{m}^2\right)$, k - heat transfer coefficient $\left(\frac{\mathrm{W}}{\mathrm{m}^2\cdot\mathrm{K}}\right)$, ϑ_1, ϑ_2, ϑ_3, ϑ_4 - temperature $\left(°\mathrm{C}\right)$ in accordance with Fig. 1, V_1, V_2 – liquid volume in each tank $\left(\mathrm{m}^3\right)$.

During the following calculations it is assumed that $\dot{V}_1 = \dot{V}_2 = \mathrm{const}$, $\dot{V}_3 = \dot{V}_4 = \mathrm{const}$. V_1 and V_3 are also constant. Then $k = k(\dot{V}) = \mathrm{const}$, $\rho_1 = \rho_2$, $\rho_3 = \rho_4$. Therefore, it can be written:

$$m_1 c_{p2}\frac{d\vartheta_2}{dt} = \dot{m}_1 c_{p1}\vartheta_1 - \dot{m}_1 c_{p2}\vartheta_2 - kA\left(\vartheta_2 - \vartheta_4\right) \tag{3}$$

$$m_3 c_{p4}\frac{d\vartheta_4}{dt} = \dot{m}_3 c_{p3}\vartheta_3 - \dot{m}_3 c_{p4}\vartheta_4 + kA\left(\vartheta_2 - \vartheta_4\right) \tag{4}$$

where: $m_1 = V_1\rho_2$ – mass of cooled alcohol $\left(\mathrm{kg}\right)$, $\dot{m}_1 = \dot{V}_1\rho_2$ – mass flow of cooled factor by the exchanger ($\mathrm{kg}\cdot\mathrm{s}^{-1}$).

The result is a system of two simplified differential equations that describe the dynamic properties of shell tank heat exchanger. To obtain one differential equation that is used for describing the impact of changes in the disturbance variables ϑ_1 and ϑ_3 on the output temperature ϑ_2, first there must be determined the temperature ϑ_4 from the equation (3) and then this equation must be differentiated in terms of time and substituted into equation (4) instead of ϑ_4 and $\dot{\vartheta}_1$. Thus after appropriate transformations, the second-order differential equation was obtained in the form of:

$$\begin{aligned}&\frac{m_1 m_2 c_{p2} c_{p4}}{kA}\ddot{\vartheta}_2 + \left[m_2 c_{p4}\left(1 + \frac{\dot{m}_1 c_{p2}}{kA}\right) + m_1 c_{p2}\left(1 + \frac{\dot{m}_3 c_{p4}}{kA}\right)\right]\dot{\vartheta}_2 + \\ &+\left[\dot{m}_1 c_{p2} + \dot{m}_3 c_{p4}\left(1 + \frac{\dot{m}_1 c_{p2}}{kA}\right)\right]\vartheta_2 = \dot{m}_1 c_{p1}\left(1 + \frac{\dot{m}_3 c_{p4}}{kA}\right)\vartheta_1 + \frac{\dot{m}_1 c_{p1} m_2 c_{p4}}{kA}\dot{\vartheta}_1 + \dot{m}_3 c_{p3}\vartheta_3\end{aligned} \tag{5}$$

Initial conditions for $t = 0$ are $\vartheta_2 = \vartheta_{2o}$, $\dot{\vartheta}_2(0) = 0$.

2.3. Analytical solution

Analysed recuperative heat exchanger (Peszyński et al., 2011) is a very good example of device for energy recovery. Into exchanger flows $\dot{m}_1 = 10\,\mathrm{kg}\cdot\mathrm{s}^{-1}$ of warm ethyl alcohol and $\dot{m}_3 = 15\,\mathrm{kg}\cdot\mathrm{s}^{-1}$ of cold water. The mass of cooled liquid equals $m_1 = 564\,\mathrm{kg}$ and the mass of coolant is $m_2 = 200\,\mathrm{kg}$. Mean specific heat of cooled liquid is $c_{p2} = 2.85\,\mathrm{kJ}\cdot\mathrm{kg}^{-1}\cdot\mathrm{K}^{-1}$ while flowing C_2H_5OH equals $c_{p1} = 2.97\,\mathrm{kJ}\cdot\mathrm{kg}^{-1}\cdot\mathrm{K}^{-1}$ and cooling water is $c_{p3} = c_{p4} = 4.19\,\mathrm{kJ}\cdot\mathrm{kg}^{-1}\cdot\mathrm{K}^{-1}$. Temperature of cooling water is 5°C while heat transfer coefficient equals $k = 5.02\,\mathrm{kW}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-1}$. The size of humidified surface is $A = 3\,\mathrm{m}^2$. Based on these data the transient function was designated. The function determines the temperature of cooled ethyl alcohol if the temperature of the flowing medium increases to 60°C.

To speed up the calculations, in the first place the constants of differential equation (5) were determined and the initial conditions in a steady state were assumed. After substituting the appropriate values, the equation was obtained:

$$92547.25\ddot{\vartheta}_2 + 10680.45\dot{\vartheta}_2 + 210.29\vartheta_2 = 153.65\vartheta_1 + 1652.63\dot{\vartheta}_1 + 62.85\vartheta_3 \ \mathrm{kJ}\cdot\mathrm{s}^{-1} \tag{6}$$

where: $t = 0$, $\vartheta_2 = \vartheta_{2o}$, $\dot{\vartheta}_2(0) = 0$.

To determine the steady state temperature $\vartheta_2(0)$ the equation (6) has been used wherein it is assumed that the derivatives occurring in the differential equation are zero, therefore:

$$210.29\vartheta_{2o} = 153.65\vartheta_{1o} + 62.85\vartheta_{3o} \text{ hence } \vartheta_{2o} = \frac{153.65\cdot 50 + 62.85\cdot 5}{210.29} = 38°\mathrm{C}.$$

Because of the differential equation (6) contains derivative $\dot{\vartheta}_1$ on the right side, therefore the derivative has to be eliminated and a new forced initial condition has to be designated

$$\dot{\vartheta}_2(0_t),\ \dot{\vartheta}_2(0_+) = \frac{1652.63}{92547.25}\left[\vartheta_1(0_+) - \vartheta_1(0_-)\right] = 0.018\cdot(60-50) \doteq 0.18°\mathrm{C}\cdot\mathrm{s}^{-1}.$$

The equation (6) can be written as:

$$92547,25\ddot{\vartheta}_2 + 10680.45\dot{\vartheta}_2 + 210.29\vartheta_2 = 153.65\vartheta_1 + 62.85\vartheta_3 \tag{7}$$

assumed initial conditions: $t = 0$, $\vartheta_2(0) = 38°\mathrm{C}$, $\vartheta_2(0_+) = 0.18°\mathrm{C}\cdot\mathrm{s}^{-1}$.

After solving the characteristic equation $92547.25\lambda^2 + 10680.45\lambda + 210.29 = 0$, two roots were obtained:

$$\lambda_1 = -0.025,\ \lambda_2 = -0.090.$$

Particular solution of the differential equation in constant form of X for $\vartheta_1 = 60°\mathrm{C}$, $\vartheta_3 = 5°\mathrm{C}$ equals: $210.29X = 153.65\cdot 60 + 62.85\cdot 5$, $X = 45.3$. The general integral of the differential equation (7) is:

$$\vartheta_2 = C_1 e^{-0.025t} + C_2 e^{-0.090t} + 45.3$$

C_1 and C_2 constants have been determined from the initial conditions of the equation (7):

$$\vartheta_2(0) = 38 = C_1 + C_2 + 45{,}3;\ \vartheta_2(0_+) = 0{,}18 = -0{,}025C_1 - 0{,}090C_2$$

Solving this system of algebraic equations allows to calculate constants: $C_1 = -7{,}34$, $C_2 = 0{,}038$.

Function that determines variations of temperature ϑ_2 of time was obtained, i.e. motion equation (dynamic characteristic of analysing plant).

$$\vartheta_2(t) = 45.3 - 7.34\mathrm{e}^{-0.025t} + 0.038\mathrm{e}^{-0.090t} \tag{8}$$

The value of the temperature in the new steady state was achieved through the calculation of equation limit (8) for $t \to \infty$.

$$\lim_{t\to\infty} \vartheta_2(t) = 45{,}3 = \overline{\vartheta_{2o}}$$

3. Simulation

Simulation of heating processes in exchanger was carried out in SCILAB environment, exactly in Xcos graphical editor (Bartoszak, 2012). For this purpose, the mathematical model described above was converted to differential form. The initial steady state before the disturbance has been adopted as a working point. It is the most common way of properties description used in automatics because it allows to create a transfer function of an object. The initial values for the working point are as follows: $\vartheta_{1o} = 50°\mathrm{C}$, $\vartheta_{2o} = 38°\mathrm{C}$, $\vartheta_{3o} = 5°\mathrm{C}$, $\vartheta_{4o} = 11{,}6°\mathrm{C}$.

The differential equation (7) takes the form:

$$92547.3\Delta\ddot{\vartheta}_2 + 10680.45\Delta\dot{\vartheta}_2 + 210.3\Delta\vartheta_2 = 153.65\Delta\vartheta_1 \tag{9}$$

where initial conditions for $t = 0$ are: $\Delta\vartheta_2(0) = 0$ and $\Delta\dot{\vartheta}_2(0_+) = 0.18°\mathrm{C}\cdot\mathrm{s}^{-1}$.

In order to solve a task in differential form, the Laplace transform was used. By applying the principles of the transform, equation was obtained in the Laplace transform form

$$92547,3\left[s^2\Delta\vartheta_2(s)-\Delta\dot{\vartheta}_2(0_+)\right]+10680.45s\Delta\vartheta_2(s)+210.3\Delta\vartheta_2(s)=153.65\Delta\vartheta_1(s) \qquad (10)$$

From equation (10) the image of output function $\Delta\vartheta_2(s)$ was obtained

$$\Delta\vartheta_2(s)=\frac{153.65\Delta\vartheta_1(s)+92547.3\Delta\dot{\vartheta}_2(0_+)}{92547.3s^2+10680.45s+210.3} \qquad (11)$$

After substituting numerical value for $\Delta\dot{\vartheta}_2(0_+)$ and taking into account that the size of input changes in step form, transfer function was obtained

$$\frac{\Delta\vartheta_2(s)}{\Delta\vartheta_1(s)}=\frac{1665,85s+153,65}{92547,35s^2+10680,45s+210,3} \qquad (12)$$

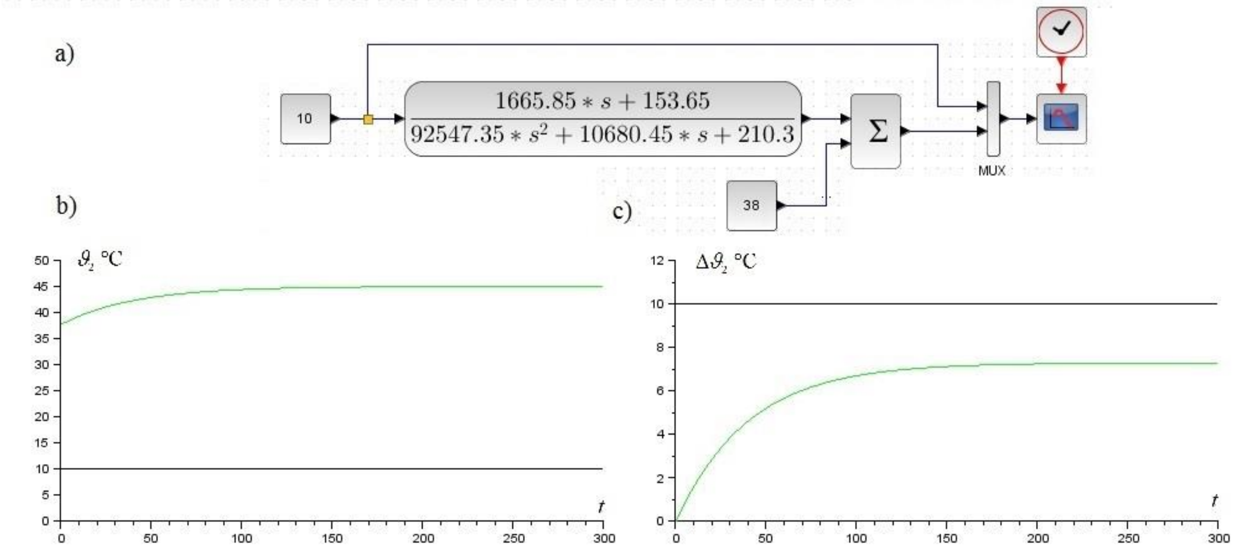

Fig. 2: SCILAB simulation: a) flowchart; b) ϑ_2 system response; c) differential response of $\Delta\vartheta_2$ system.

The equation (12) was moved to SCILAB numerical environment (Fig. 2a). Because of abrupt change of alcohol temperature was $\Delta\vartheta_1(s)=10°\mathrm{C}$, at the input of system it was applied the conversion block CONST_m = 10. The value defined as the working point was also added to the output value using block CONST_m = 38.0. The time course of output changes in the output is shown in Fig. 2b.

4. Conclusions

Obtained mathematical model of recuperative heat exchanger was very useful to determine the time constants of designed control system. Designation of mathematical model was necessary because ordering party, in the view of type and continuity of production, did not allow its determination by using experimental methods. Diagram shown on Fig. 2b was used to determine the time constants by Ziegler method. The only verification form of the mathematical model was PID controller tuning. The PID controller tuning provided based on the mathematical model created established the assumed quality control.

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