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EVALUATION OF THE AREAL TEXTILE THINNING

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Abstract: *The areal textile with outstanding structure is approximated by the areal continuum with the same mechanical properties. This allows to use the equations of the continuum mechanics and to define the basic mechanical properties of the textile. Till this time no suitable experimental method for continuous measurement of textile fabric thinning during straining exists. This is the reason, why the mechanical properties of the textiles are expressed through the force per unit length in N/m. The main aim of the presented paper is to theoretically define the areal textile thinning during its straining through the continuum mechanics.*

Keywords: Mechanics of continuum, Conjugate pairs, Specific forces, Cauchy conjugate pair, Thinning of areal textile.

1. Mechanics of the areal textiles

The identification of the mechanical properties of the areal textiles under the uniaxial and biaxial straining is physical problem, which leads to a task with seven unknowns. The textile fabric is very specific formation; therefore it is necessary to describe its mechanical properties (in contrast to solids) for particular state of stress and strain.

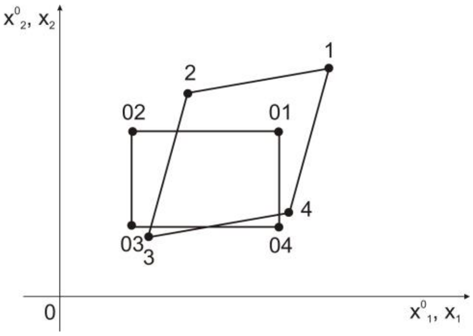

Fig. 1: The geometry of original and deformed sample.

The dependence between Euler's and Lagrange's point coordinates (according to Striz (2001) (Part I)) can be evaluated by measuring the movements of observed points of areal fabric, Fig. 1:

$$x_i^p = x_i^{op} + w_i^p, \quad i = 1,2, \tag{1}$$

where p is point number, the circle denotes the Lagrange coordinate. The work by Striz and Vysanska (2011) describes the deformation process of the material using deformation gradient F and the jacobian J. Tensor F is expressed as

$$F = \begin{pmatrix} 1+v_{11} & v_{12} & 0 \\ v_{21} & 1+v_{22} & 0 \\ 0 & 0 & 1+v_{33} \end{pmatrix} \tag{2}$$

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and jacobian J

$$J = (1+v_{33})[(1+v_{11})(1+v_{22}) - v_{12}v_{21}]. \tag{3}$$

The parameters ν_{ij} are the differential movements of the vertices of areal fabric on Fig. 1. Let's define tensor of extension U and tensor of rotation R using of the material deformational gradient F.

$$\begin{aligned} U^2 &= F^T F, \\ F &= R\,U. \end{aligned} \tag{4}$$

Tensor of extension U can be determined, for example, by so called method of the projectors according to Striz (2001) (Part I). Till this time, the unpublished method for determination of tensors U and R is presented below. The tensors U, R have following structure

$$U = \begin{pmatrix} u_{11} & u_{12} & 0 \\ u_{12} & u_{22} & 0 \\ 0 & 0 & 1+v_{33} \end{pmatrix}, \tag{5}$$

$$R = \begin{pmatrix} r_{11} & r_{12} & 0 \\ -r_{12} & r_{11} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

The individual components can be expressed as

$$\begin{aligned} u_{11} &= \frac{(1+v_{11})(2+v_{11}+v_{22}) + v_{21}(v_{21}-v_{12})}{\sqrt{4(1+v_{11}+v_{22}) + (v_{11}+v_{22})^2 + (v_{12}-v_{21})^2}}, \\ u_{22} &= \frac{(1+v_{22})(2+v_{11}+v_{22}) + v_{12}(v_{12}-v_{21})}{\sqrt{4(1+v_{11}+v_{22}) + (v_{11}+v_{22})^2 + (v_{12}-v_{21})^2}}, \\ u_{12} &= \frac{(1+v_{11})v_{12} + (1+v_{22})v_{21}}{\sqrt{4(1+v_{11}+v_{22}) + (v_{11}+v_{22})^2 + (v_{12}-v_{21})^2}}. \end{aligned} \tag{7}$$

$$\begin{aligned} r_{11} &= \frac{2+v_{11}+v_{22}}{\sqrt{4(1+v_{11}+v_{22}) + (v_{11}+v_{22})^2 + (v_{12}-v_{21})^2}}, \\ r_{12} &= \frac{v_{12}-v_{21}}{\sqrt{4(1+v_{11}+v_{22}) + (v_{11}+v_{22})^2 + (v_{12}-v_{21})^2}}. \end{aligned} \tag{8}$$

It is easy to find, that equation (2) is satisfied.

The definition of the unit-less quantity v_{33} is based on the Cauchy's relative force Σ:

$$\Sigma = \begin{pmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{9}$$

and corresponding deformation $\varepsilon_{ij}(m)$, which is work-conjugate to Σ. The components of the tensor Σ are determined from the static balance equations of forces $Q_1(x)$, $Q_2(x)$ in the fabric axes 11 and 22 according to Striz (2001) (Part II).

The deformation tensor $\varepsilon_{ij}(m)$ is defined using the tensor of extension U and meanwhile unknown parameter "m":

$$\varepsilon_{ij}(m) = \frac{1}{m}\left[U^m - I\right]. \tag{10}$$

The exponent “*m*” characterizes the conjugate pair. The presented work is looking for the particular value of “*m*” for the textile fabric. The conjugate pair has to fulfill condition of equality of mechanical work (energy), obtained from known conjugate pairs with parameters of: 2, 1, 0, -1, -2.

$$I(m) = \int_0^{x_{\max}} \left[s_{11}\frac{d\varepsilon_{11}}{dx} + s_{22}\frac{d\varepsilon_{22}}{dx} + 2s_{12}\frac{d\varepsilon_{12}}{dx} \right] dx = D, \tag{11}$$

where $x = \frac{Q_1(x)}{Q_{1\max}}$, and Q_1 is outer force along axis 1.

To determine the quantity D it is necessary to use two conjugate pairs at minimum. The method presented in Striz and Vysanska (2011) defines the relative forces and deformations in equation (11). Parameter “*m*” in Cauchy conjugate pair is assessed from equation (11). Then the conjugate pair is set for given fabric.

2. Evaluation of the Fabric Thinning

The method described in Striz and Vysanska, (2011) together with parameter “*m*” from equation (11) can be used to express the values of six mechanical modules $\overline{E}_{ij}$. One can express the invariant shear modulus using $\overline{E}_{ij}$ modules according to

$$\tilde{E}_4 = \frac{1}{2}\left[\frac{1}{4}\left(\overline{E}_{11} + \overline{E}_{22} - 2\overline{E}_{12}\right) + \overline{E}_4\right]. \tag{12}$$

Further we have determined τ_i and γ_i, which are also invariant:

$$\begin{aligned} \tau_i &= \frac{1}{\sqrt{6}}\sqrt{(s_{11} - s_{22})^2 + s_{11}^2 + s_{22}^2 + 6s_{12}^2}, \\ \gamma_i &= \sqrt{\frac{2}{3}}\sqrt{(\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2 + 6\varepsilon_{12}^2}. \end{aligned} \tag{13}$$

In linear mechanics this relation is valid:

$$G = \frac{\tau_i}{\gamma_i},$$

where G is shear modulus. We determine similar equation for non-linear mechanics:

$$\tilde{E}_4 = \frac{\tau_i}{\gamma_i}. \tag{14}$$

After substitution of the equations (14), (13) in (12) and adjustment we get

$$\frac{1}{4}\left(\overline{E}_{11} + \overline{E}_{22} - 2\overline{E}_{12}\right) + \overline{E}_4 = \frac{\sqrt{(s_{11} - s_{22})^2 + s_{11}^2 + s_{22}^2 + 6s_{12}^2}}{\sqrt{(\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2 + 6\varepsilon_{12}^2}}. \tag{15}$$

We define unknown ε_{33} from equation (11) and on the basis of the equations (4) and (9) we can express it like

$$\varepsilon_{33} = \frac{1}{m}\left[(1 + v_{33})^m - 1\right]. \tag{16}$$

Unit-less quantity v_{33} can be expressed in the following form

$$h = h_0(1 + v_{33}), \tag{17}$$

where h_0 is initial measured thickness of the areal fabric and h is running thickness of fabric and together with the quantity v_{33} is dependent on established coordinate x in a relation (11). Then the problem of fabric thinning is solved.

The components of conjugate pairs s_{ij} and ε_{ij} are stated from equations in Striz and Vysanska (2011):

$$\begin{aligned} S_B &= \frac{J}{2}\left(F^{-1}\Sigma R + R^T \Sigma \left(F^{-1}\right)^T\right), \\ s_{ij} &= \frac{1}{2}\left(S_B U^{1-n} + U^{1-n} S_B\right), \\ \varepsilon_{ij} &= \frac{1}{n}\left(U^n - I\right), \end{aligned} \tag{18}$$

where S_B is Biott tensor of variable forces (n = 1). By solving the problem for various exponents (n = 2, 1, 0, -1, -2, m) one can determine the thickness of the fabric $h(x, n)$ from the equations mentioned above. For different areal textiles and different conjugate pairs we get unsuitable values of h (e.g. $h > h_0$). If we substitute exponent $n = m$ in equations (18), we can gain for value s_{ij} the quantity Σ, and so verify calculated value of „*m*“.

3. Conclusions

Till this time no experimental method for running measurement of textile thinning during fabric loading exists. The present work allows validating calculated values of (m, h) and excluding unsuitable conjugate pairs, when such method will be available.

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