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ESTIMATION OF CRITICAL STRESS VALUES FOR CRACK INITIATION FROM SHARP V-NOTCHES

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Abstract: *The aim of the paper is to estimate a critical value of an applied stress for a crack initiation from sharp tip of the V-notch using a procedure based on linear elastic fracture mechanics (LELM). The V-notch represents singular stress concentrator with the stress singularity exponent different from 0.5, therefore the generalized form of LELM for stress distribution description in the vicinity of the sharp V-notch was used. The stress singularity exponent depends on the V-notch opening angle, in general. A stability criterion based on the average value of the tangential stress component across a critical distance d from the V-notch tip is used in the paper. Resultant values of the critical applied stress are compared with experimental data.*

Keywords: Failure initiation, Stress singularity, V-notch, Generalized stress intensity factor.

1. Introduction

V-notches are one of the most frequent stress concentrators occurring in engineering structures. They cause a decrease of the critical applied stress value which leads to the crack initiation. Therefore, stability criteria for the crack initiation from the tip of V-notch have been formerly developed. Also the problems of the brittle fracture of structural components with sharp V-notches have been considered in papers (e.g. Knésl, 1991; Knésl, 1993; Seweryn, 1994; Seweryn, 2002; Goméz, 2003).

V-notches are singular stress concentrators with stress singularity exponents different from 0.5, which depend on the V-notch opening angle. Therefore, the standard procedures of LELM should be generalized for stress singularity exponents in the range from 0 to 0.5 (Knésl, 1991; Seweryn, 2002).

The aim of this paper is to estimate the critical value of an applied stress for the crack initiation from sharp V-notch. Two different kinds of materials, different geometries, V-notch opening angles and notch depths are considered. The stability criterion used is based on the averaged value of the tangential stress component across a critical distance *d* from the notch tip. Results obtained are compared with experimental data taken from the literature (Seweryn, 1994; Goméz, 2003).

2. Theoretical Background

2.1. Stress distribution around the sharp tip of a V-notch

In the first step it is necessary to estimate the stress singularity exponent and stress distribution around the sharp tip of a V-notch. The singular stress field around the V-notch tip can be expressed by (Knésl, 1991):

$$\sigma_{ij} = \frac{H_I}{\sqrt{2\pi}} \cdot r^{-p} \cdot f_{ij}(\alpha, p, \theta), \tag{1}$$

where H_I is generalized stress intensity factor corresponding to mode I of loading, p is stress singularity exponent, f_{ij} (α,p,θ) are known shape functions, r and θ are polar coordinates with the origin at the

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V-notch tip and α is opening angle of the V-notch (see Fig. 1). The stress singularity exponent p depends on the angle α only and can be obtained by solving of characteristic equation (Knésl, 1991):

$$\sin(2(1-p)(\pi-\alpha)) + (1-p)\sin(2(\pi-\alpha)) = 0. \quad (2)$$

Generalized stress intensity factor H_I can be determined numerically using finite element (FE) method, e.g. by using of so-called direct method, see Fig. 2.

2.2. The criterion of stability

Conditions of the sharp V-notch stability depend on the stress state near the V-notch tip and on material resistance to the fracture. Criterion development following an idea that the mechanism of the crack initiation from pre-crack and the crack initiation from the sharp V-notch is the same and it is controlled by variable L, which has a clear physical meaning and is well defined in both cases, for body with a crack as well as for body containing V-notch. Then, the critical values L_C for the crack and V-notch can be compared:

$$L_C(\dots, K_{IC}, \dots) = L_C(\dots, H_{IC}, \dots), \quad (3)$$

where K_{IC} [MPa.m$^{0.5}$] is a fracture toughness and H_{IC} [MPa.m^{p}] is a critical value of generalized stress intensity factor. As the variable L an average stress calculated across the distance d from the notch tip can be used (Knésl, 1991). The average stress $\bar{\sigma}$ represents average value of the tangential stress component $(\sigma_{\theta\theta})_{max}$ acting in front of the notch tip and can be related to the quantity σ_{crit}, i.e. to the critical stress. The crack doesn´t propagate for:

$$\bar{\sigma} = \frac{1}{d}\int_0^d (\sigma_{\theta\theta})_{max} dr < \sigma_{crit} = \frac{2K_{IC}}{\sqrt{2\pi d}}. \quad (4)$$

The value of the critical stress σ_{crit} for the crack can be obtained from Eq. (4) as a function of K_{IC}. The critical stress in the case of the sharp V-notch can be obtained by similar way, i.e. by integration of Eq.(1) for H_I equal to H_{IC}:

$$\sigma_{crit} = \frac{H_{IC}}{\sqrt{2\pi}}\frac{(2-p)(1+q)}{d^p}, \quad (5)$$

where $q = -\cos\big(p(\pi-\alpha)\big)/\cos\big((2-p)(\pi-\alpha)\big)$. By comparison of Eq. (4) and (5):

$$H_{IC} = K_{IC}\frac{2d^{p-\frac{1}{2}}}{(2-p)(1+q)}. \quad (6)$$

Several expressions for estimation of the critical distance d exist (Zouhar, 2012). The expression published by Yosibash (2010), $d = \frac{2}{\pi}\left(\frac{K_{IC}}{\sigma_C}\right)^2$, was used in calculations. σ_C is a tensile strength.

The applied stress σ_{appl} represents a remote loading of the body with V-notch, see Fig. 3. The critical value of applied stress $\sigma_{appl,crit}$ necessary for the crack initiation from the V-notch can be expressed as:

$$\sigma_{appl,crit} = \frac{H_{IC}}{H_I}\sigma_{appl}. \quad (7)$$

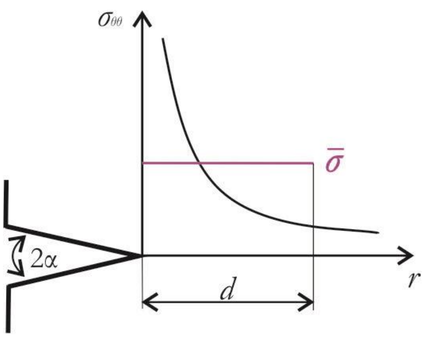


Fig. 1: Averaged tangential stress value in front of the V-notch.

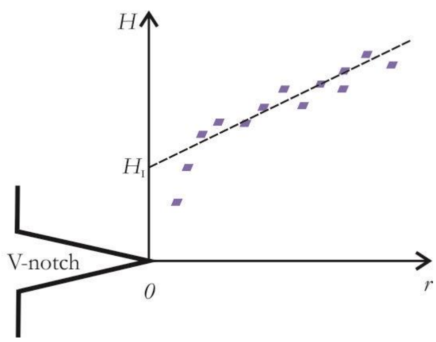


Fig. 2: Determination of H_I value by the direct method.

3. Numerical Model

Three different sets of experiments were modeled by finite element method according to Seweryn (1994) and Goméz (2003). Two sets were specimens with sharp V-notch in the middle of each side (DENT) (Fig. 3) made of polymethylmethacrylate (PMMA) and duraluminum, respectively. The third set represents single edge notch tensile (SENT) specimens (Fig. 3) made of PMMA.

The dimensions of the DENT specimens were: length $L = 192$ mm, width $4w = 109$ mm, notch depth $a = 27$ mm and V-notch opening angle $\alpha = 0 \div 70°$ (changing with step of 10°). The thickness of PMMA specimen was $t = 4$ mm and the thickness of duraluminum specimen was $t = 5$ mm. Both kinds of specimens were loaded by tension. The eighth of the specimen was modeled, because of the symmetry in geometry and loading conditions. Conditions of plane strain were considered in all cases.

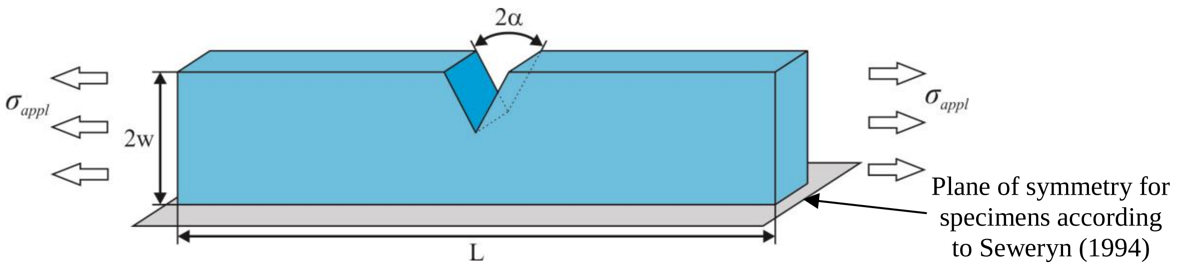


Fig. 3: Geometry of the model used for finite element calculations.

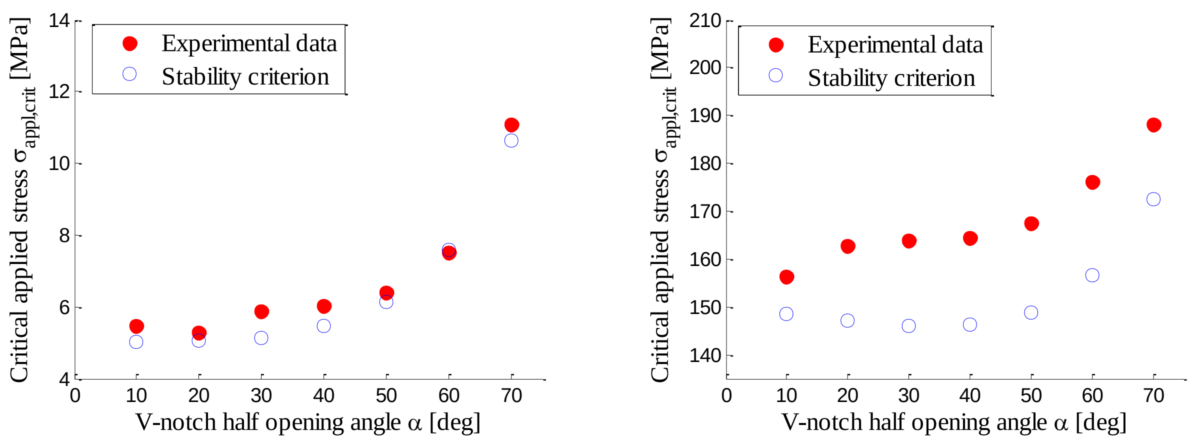
The dimensions of the SENT specimens were: length $L = 196$ mm, width $2w = 28$ mm, notch depths $a = 5$, 10, 14, 20 mm and V-notch opening angle $\alpha = 30°$, 45°, 60°, 75°, the thickness was $t = 14$ mm. The influence of the V-notch opening angle α and V-notch depth a on the critical applied stress was studied. The quarter of the specimen was modeled, because of the symmetry in geometry and loading conditions.

Following material properties were considered: PMMA - Seweryn (1994): Young´s modulus $E = 2.3$ GPa, Poisson´s ratio $\nu = 0.36$, fracture toughness $K_{IC} = 1.9$ MPa.m$^{1/2}$ and tensile strength $\sigma_c = 70$ MPa; duraluminum - Seweryn (1994): $E = 70$ GPa, $\nu = 0.33$, $K_{IC} = 54.3$ MPa.m$^{1/2}$ and $\sigma_c = 454$ MPa; PMMA - Goméz (2003): $E = 3.0$ GPa, $\nu = 0.40$, $K_{IC} = 1.0$ MPa.m$^{1/2}$ and $\sigma_c = 75$ MPa.

Both materials were considered as homogenous linear-elastic and isotropic. The critical distance d was 0.451 mm for PMMA according to Seweryn (1994), 0.113 mm in the case of PMMA according to Goméz (2003) and 9.026 mm in the case of duraluminum (Seweryn, 1994).

4. Results and Discussion

Using FE analysis the critical applied stresses for the crack initiation from sharp V-notch was estimated and compared with the experimental data. V-notch stability conditions depend on the applied load and on the material resistance to the fracture.

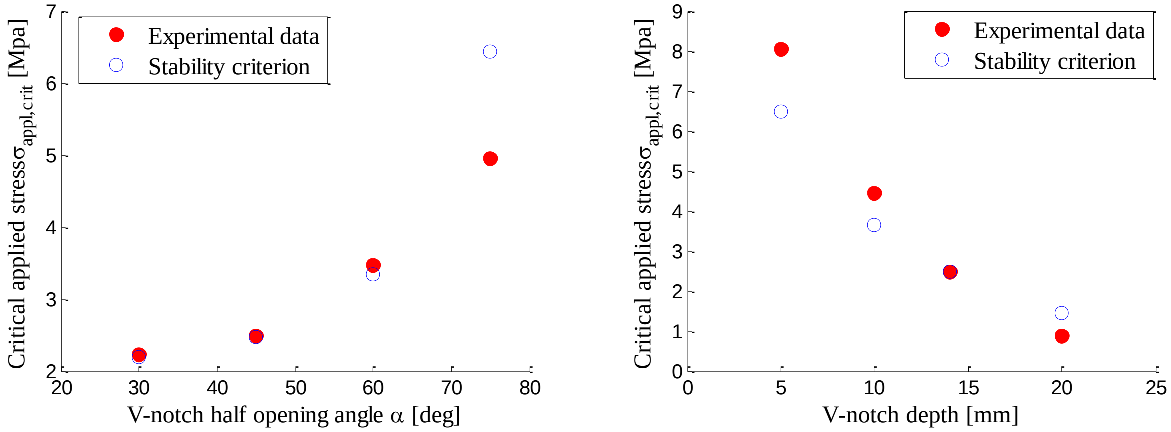


a) PMMA *b) Duraluminum*

Fig. 4: Comparison of estimated critical values of applied tensile stress $\sigma_{appl,crit}$ and experimental data taken from Seweryn (1994).

Very good agreement between calculated critical applied stress values and experimental data was found in the case of PMMA specimens for V-notch opening angle α less than 70° (see Fig. 4a and Fig. 5a), and for depth of the V-notch a higher than 10 mm (see Fig. 5b). Vicinity of the free surface influenced calculated results for V-notch depth *a* equals or less than 10 mm.

The criterion described above is not convenient for ductile materials (see Fig. 4b). The difference between calculated and experimental data is caused by substantial plastic behavior of duraluminum.



a) *b)*

Fig. 5: Comparison of estimated critical values of applied tensile stress $\sigma_{appl,crit}$ and experimental data taken from Goméz (2003).

5. Summary

The critical value of the applied stress necessary for the crack initiation from sharp V-notch was estimated in the paper. Tensile specimens with V-notches were modeled by numerical simulations and two different materials were considered: PMMA and duraluminum. Influence of the V-notch opening angle *α* and the V-notch depth *a* was studied. A criterion based on average value of the tangential stress in front of the sharp concentrator was applied. Estimated values of the critical applied stress were compared with experimental data taken from the literature and good agreement was found for brittle PMMA. The paper represents an example of the use of the generalized form of LELM in the case of general singular stress concentrators.

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References

Knésl, Z. (1991) A criterion of V-notch stability. International Journal of Fracture, 48, pp. 79-83.

Knésl, Z. (1993) The application of the strain energy density concept to the determination of a crack propagation direction initiated at a sharp notch tip. Acta Technica ČSAC, 38, pp. 221-234.

Seweryn, A. (1994) Brittle fracture criterion for structures with sharp notches. Engineering Fracture Mechanics, 47, 5, pp. 673-681.

Seweryn, A., Lukaszewicz, A. (2002) Verification of Brittle fracture criteria for elements with V-shaped notches. Engineering Fracture Mechanics, 69, pp. 1487-151.

Goméz, F. J., Elices, M. (2003) Fracture of components with V-shaped notches. Engineering Fracture Mechanics, 70, 14, pp. 1913-1927.

Zouhar, M., Náhlík, L., Hutař, P., Ševčík, M., Knésl, Z. (2012) The effect of critical distance in stability condition for the crack at the interface between two materials. Engineering Mechanics, 19, 2/3, pp. 155-164.

Yosibash, Z. (2010) Singularities in elliptic boundary value problems and elasticity and their connection with failure initiation. Springer Science Business Media.