

Svratka, Czech Republic, 12 – 15 May 2014

## SIMULATION ASSESSMENT OF SUPPRESSION OF MACHINE TOOL VIBRATIONS

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**Abstract:** The paper presents an approach for finding the locally optimal parameters of a mass which connected to a machine tool suppresses vibrations during machining process. The proposed approach is mainly based on simulation modeling of mechatronic systems with flexible bodies and consequent reduction of the obtained model order. Obtained parameters can be structurally interpreted and included in the model e.g. as a damper or an absorber. The complex model containing the machine tool model with the connected mass then makes possible to analyze the influence of the connected mass on the dynamic compliance of the machine tool.

# Keywords: Machine tool, Dynamic compliance, Flexible bodies, Simulation assessment, Mechatronic systems.

### 1. Introduction

The influence of a connected mass on the dynamic compliance of the whole system provides quite important information for a designer of a machine tool. The importance generally arises from consequent possibility of the revision and correction of the whole engineering design of the machine according to the obtained information.

The mentioned information was often based only on the experience of the design engineers. The proposed concept is on the contrary based on the simulation modeling of mechatronic systems which utilizes different simulation engines (Brezina et al., 2011). The main advantage of the simulation modeling is then possibility of fast modifications of the model and its low cost compared to the experiments with the real machine.

However, finding the suitable locally optimal parameters of the connected mass, in order to tune the dynamic compliance of the system, might be quite complicated because its integration influences all of machine axes thus improvement of the behavior for one axis can lead to the worsening in the other axis. It should be also taken into account that the reliable model of the machine should contain flexible bodies which is leading to the dramatic increase of the model order.

The paper presents an approach to obtaining of the locally optimal parameters of the mass which is flexibly connected to the tool holder. The parameters of the mass are designed in a way to reduce the tool vibrations during the machining. The simulation modeling of the mechatronic systems concept is utilized for that purpose. The method based on the canonical form of the obtained LTI model of the system for the model order reduction is presented as well.

The modeled machine tool (Fig. 1) by TOSHULIN, a.s. producer was modeled in axes X, Y, Z at the center of gravity of the tool holder for the excitation frequency range 0 - 280 Hz. The asked correspondence with the real machine was up to 10% at both time and frequency domain.

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#### 2. Formulation of the Task

The connected mass A is described by mechanical parameter weight  $m_A$ . It is connected to an appointed place (the mass center of a tool holder) by an ideal damper with damping  $b_A$  and by an ideal spring with stiffness  $k_A$  (Fig. 2). The mass is moving in  $\varphi_A$ ,  $\theta_A$  direction (Fig. 2).  $\varphi_A$ ,  $\theta_A$  represent geometric parameters.





*Fig. 1: Analyzed machine tool from TOSHULIN (Brezina et al., 2013).* 

Fig. 2: Scheme of the integration of the mass A to the system (Brezina et al., 2013).

The dynamic compliance of the machine with the integrated mass A is described by following equations

$$(s^{2}\mathbf{m} + s\mathbf{b} + \mathbf{k})\mathbf{q} = \mathbf{Q} - s^{2}m_{A}\mathbf{q} + s^{2}(\mathbf{h}m_{A})q_{A} (s^{2}m_{A} + sb_{A} + k_{A})q_{A} = Q_{A} + s^{2}(\mathbf{h}m_{A})^{T}\mathbf{q}$$
(1)

where **m**, **b**, **k** mean one by one matrix of mass, matrix of damping, matrix of stiffness and  $\mathbf{h} = [\cos(\varphi_A)\cos(\theta_A) \cos(\varphi_A)\sin(\theta_A) - \sin(\varphi_A)]^T$ .

The improvement of the dynamic compliance of the machine tool with the connected mass A is formulated as a minimization of the objective function  $g(\mathbf{p})$ :

$$\mathbf{p} = \begin{bmatrix} m_A & b_A & k_A & \varphi_A & \theta_A \end{bmatrix}$$
$$\Delta_{i,j} \left( \mathbf{p} \right) = \frac{\max_{\omega} \left| \boldsymbol{\alpha}_{i,j}^A \left( \boldsymbol{\omega}, \mathbf{p} \right) \right|}{\max_{\omega} \left| \boldsymbol{\alpha}_{i,j} \left( \boldsymbol{\omega}, \mathbf{p} \right) \right|} - 1 , \qquad (2)$$
$$g \left( \mathbf{p} \right) = \max_{i,j} \Delta_{i,j} \left( \mathbf{p} \right)$$
$$\mathbf{p}_{out} = \arg \min g \left( \mathbf{p} \right)$$

where i, j = X, Y, Z,  $\max_{\omega} | \boldsymbol{\alpha}_{i,j}^{A}(\omega, \mathbf{p}) |$  is a dominant amplitude – maximum of the amplitude characteristics of the dynamic compliance of the machine with the integrated mass *A* and  $\max_{\omega} | \boldsymbol{\alpha}_{i,j}(\omega, \mathbf{p}) |$  has the same meaning but the machine is considered without the mass *A*. There are considered following limitations:

$$m_{A}, b_{A}, k_{A}, \varphi_{A}, \theta_{A} > (m_{A})_{min}, (b_{A})_{min}, (k_{A})_{min}, 0, 0$$
  

$$m_{A}, b_{A}, k_{A}, \varphi_{A}, \theta_{A} \le (m_{A})_{max}, (b_{A})_{max}, (k_{A})_{max}, 360, 360.$$
(3)  

$$\max_{A} \Delta_{i,i}(\mathbf{p}) < 0$$

The first two conditions represent limitations of the magnitude of the parameters values and the last one is introducing requirement for the improvement of the dynamic compliance in the particular axis. The minimization of the objective function guarantees the least possible degradation of the dynamic compliance between the different axes. It is necessary to obtain the low order model of the dynamic compliance to implement the minimization process first of all.

#### 3. Obtaining of the Low Order Model via CAD, FEM and MBS Models

To efficiently minimize the target function, a credible model of as low order as possible has to be used. The low order model is created according to the simulation modeling of mechatronic systems concept. The first step is to identify rigid and flexible parts in the CAD model. There are consequently created FEM models of the flexible parts (order around  $10^5 - 10^6$ ) and modal analysis is performed. The high order is reduced by utilization of the Craig-Bampton method (Craig & Bampton, 1968) for the given frequency range. The reduced models (the order around  $10^2 - 10^3$ ) are integrated to the multi-body machine model together with rigid bodies models. The order of the complete machine remains  $10^2 - 10^3$ . The final state LTI model  $\alpha = (A, B, C, D = 0)$  is obtained by the linearization of the reduced model around the selected operation point. Let's note that L-image of  $\alpha$  is for the zero initial conditions described as

$$s\mathbf{x}(s) = \mathbf{A}\mathbf{x}(s) + \mathbf{B}\mathbf{Q}(s)$$
  
$$\mathbf{q}(s) = \mathbf{C}\mathbf{x}(s) + \mathbf{D}\mathbf{Q}(s),$$
 (4)

where  $\mathbf{x}(s)$ ,  $\mathbf{Q}(s)$  and  $\mathbf{q}(s)$  mean L-images of the vector of (inner) states, input vector of exciting force actions and output vector of corresponding displacements. The dynamic compliance of the machine tool or dynamic compliance only between axis *j* and *i* could be expressed simply as  $\mathbf{q}(s) = \mathbf{\alpha}(s)\mathbf{Q}(s)$ , or  $q_i(s) = \mathbf{\alpha}_{i,j}(s)Q_j(s)$ . The obtained model is of the 428<sup>th</sup> order. The asked correspondence with the real machine was according to measurements achieved.

#### 4. Further Reduction of the LTI Model

There are many methods of reduction of the LTI order using similarity transformations based on the controllability and observability matrix (e.g. Antoulas & Sorensen, 2001; Moore, 1981) which, however, often disturb the original physical structure of the model. Therefore, the further reduction of the LTI model is performed by transforming the matrix **A** into canonical modal form  $\mathbf{A} = diag(\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_n)$ . The matrices  $\mathbf{A}_k$  are 2/2 type and a couple of single eigenvalues of LTI  $\left(\delta \pm i\sqrt{\omega_0^2 - \delta^2}\right)_k$  or  $\left(\delta \mp \sqrt{\delta^2 - \omega_0^2}\right)_k$  is projected as

$$\mathbf{A}_{k} = \begin{bmatrix} \delta & \sqrt{\omega_{0}^{2} - \delta^{2}} \\ -\sqrt{\omega_{0}^{2} - \delta^{2}} & \delta \end{bmatrix}_{k}, \text{ or } \mathbf{A}_{k} = \begin{bmatrix} \delta - \sqrt{\delta^{2} - \omega_{0}^{2}} \\ \delta + \sqrt{\delta^{2} - \omega_{0}^{2}} \end{bmatrix}_{k}.$$
 (5)

The symbol  $\delta$  or expression  $\delta \mp \sqrt{\delta^2 - \omega_0^2}$  means damping factors and  $\omega_0$  is the natural frequency of the couple. The application of such LTI form has two principal consequences: the original model can be decomposed into a set of partial models  $\boldsymbol{a}_k \equiv (\mathbf{A}_k, \mathbf{B}_k, \mathbf{C}_k, \mathbf{D}_k = \mathbf{0})$  of the second order, for which applies

$$\mathbf{q}(s) = \left(\sum_{k} \alpha_{k}(s)\right) \mathbf{Q}(s) = \sum_{k} q_{k}(s)$$
(6)

The modeled dynamic compliance a(s) is according to (6) a sum of partial dynamic compliances  $a_k(s)$ . This can be utilized for the reduction of the LTI model a(s) which respects the structure of the original model of the mechanical system. It is possible to perform the reduction of a(s) by simple elimination of partial models  $a_k(s)$  with no significant contribution to the original model.

The order 428 of LTI model was reduced to just 80 with relative accuracy downgrade 7%. The minimization according to (2) and (3) is consequently performed for the reduced LTI. The found parameters are then used as initial parameters for the minimization on original LTI (order 428) because of the inaccuracy suppress.

The examples of the locally optimal solutions for the mass A (Fig. 3) are presented in (Brezina et al., 2013). There was observed significant suppression of the original dominant amplitudes (Fig. 3 - left).

Simultaneous worsening in some of the other axis also appeared (Fig. 3 - right) but outside the working range of the machine.



Fig. 3: The integration of the mass A with the locally optimal parameters, left - The amplitude characteristics of the dynamic compliance of the machine in the Z axis (the best integration influence), right - The amplitude characteristics of the dynamic compliance of the machine between the X and Y axis (the worst integration influence) (Brezina et al., 2013).

#### 5. Conclusions

The proposed approach based on the modeling of mechatronic systems presents possible efficient way to the reduction of the obtained LTI model which contains also flexible bodies. The reduced model order reaches approximately 20% of the original one. The observed difference between the resonant frequencies of both models was typically up to 10%.

There was also presented an objective function for finding of the locally optimal parameters of the connected mass *A* which was integrated to the model in order to reduce the dominant amplitudes of the system. Initial locally optimal parameters are due to the decreasing of computational time found for the reduced model of the system. Final optimal parameters are then searched for the original non-reduced model. There was observed important decreasing of dominant amplitudes after the implementation of the mass *A* with the locally optimal parameters. Increasing of amplitudes in some other axes was observed outside the working range of the machine thus without an important impact on the machine function. Let's note that mass *A* can be practically implemented as a damper or an absorber.

#### Acknowledgement

This work is supported by project NETME Centre PLUS. The results of this project NETME CENTRE PLUS (LO1202) were co-funded by the Ministry of Education, Youth and Sports within the support programme "National Sustainability Programme I".

#### References

- Antoulas, A. C. & Sorensen, D. (2001) Approximation of Large-Scale Dynamical Systems: an Overview. Int. J. Applied Math. Computer Science, 11, 5, pp. 1093-1121.
- Brezina, T.; Brezina, L.; Marek, J.; Hadas, Z. & Vetiska, J. (2013) Simulation Assessment of Suspension of Tool Vibrations during Machining, in: Proc. 10th International Conference Mechatronics 2013, Springer, London, pp. 273-280.
- Brezina, T., Vetiska, J., Hadas, Z. & Brezina, L. (2011) Simulation, Modelling and Control of Mechatronic Systems with Flexible Parts, in: Proc. 9th Conf. Mechatronics 2011, Springer Verlag, Berlin, pp. 569-578.
- Craig, R. R. & Bampton, M., C. (1968) Coupling of substructures for dynamics analyses. AIAA Journal, 6, 7, pp. 1313-1319.
- Moore, B.C. (1981) Principal Component Analysis in Linear Systems: Controllability, Observability and Model Reduction. IEEE Trans. on Automatic Control, AC-26, 1, pp. 17-31.