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VIBRATIONS OF BLADES BUNCHES

L. Půst*, L. Pešek *

Abstract: *Paper describes a mathematical model for analysing the dynamic characteristics of five-blades bunch with different damping connections among individual blades. The connections of blades by means of special rubber elements either with the fixed contact with neighbouring blades or with one-sided slip is presented. Systems with dry friction connections among blades by means of direct contacts or by inserted dry friction elements are described as well. As an example of analysis results, response curves of five-blades-bunch excited on first blade at different dry friction force in the first bending resonance is shown. Elaborated analysis create the basic theoretical background for evaluation of measurement on experimental bladed disk set in the laboratory of Institute of Thermomechanics of ASCR and it is also applied for evaluation of effectiveness of methods for suppression of forced vibration of blades.*

Keywords: Damping, Dry friction, Five-blades-bunch, Harmonic excitation, Response curve.

1. Introduction

Introduction of additional damping elements or friction connection into blades' shroud can very effectively damps undesirable vibrations of turbine blades *vibrations* (Pesek & Pust, 2011a; Pesek & Pust, 2011b; Treyde, 1995). Many theoretical, numerical and experimental studies were done in Institute of Thermomechanics ASCR in this field.

The previous investigation of dynamic properties of two-blades-model realized in the last years in our Institute in cooperation with WCU Plzen and connected either by a rubber element, or by direct friction contact or by inserted friction element showed very positive results. Majority of mathematical models of these two-blades-systems are strongly nonlinear (Awrejcewicz, 2009). In the presented paper, the simple basic two-blades-model is enlarged on the study of five-blades-bunch equiped again by means of different types of damping connections.

Due to the more complicated spectral properties of the five-blades bunch in comparison to two-blades system, a more complicated response behaviour depending also on large variability of external force-excitation can be expected. Content of this article is only the first part of voluminous research.

2. Mathematical Models of Five-Blades-Bunch.

Some laboratory experiments have open this investigation, however, as the experimental research is usually encumbered with a lot of marginal influences, the preliminary analytic-numerical solution of mathematical model has to been done. This initial theoretical phase of research is very useful as it enables comparatively easy to complete knowledge of dynamic behaviour of studied system and optimise experimental procedures.

Several types of mathematical models of five-blades-bunches have been investigated. Five-blades-bunch can be modelled by a simple five masses system shown in Fig. 1, where the blades are replaced by 1 DOF systems, the eigenfrequencies of which correspond to the first bending eigenfrequencies of real blades. The torsion eigenfrequencies of these blades are supposed to be much higher than the bending ones (mass m, stiffness k, damping coefficient b) and therefore, in the first approximation, the torsion deformations and torsion vibrations of blades are not taken into account.

* Ing. Ladislav Půst, DSc., Ing. Ludek Pešek, CSc.: Institute of Thermomechanics AS CR, v.v.i., Dolejškova 5, 18200 Prague, pust@it.cas.cz, pesek@it.cas.cz,

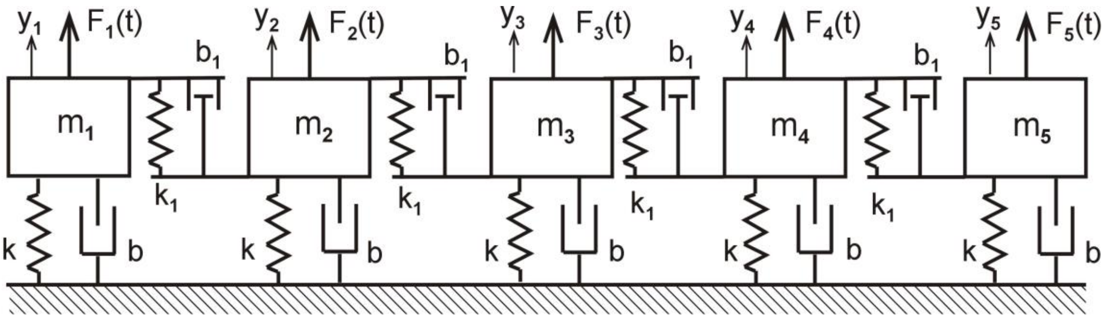


Fig. 1: Computational model of five-blades-bunch.

The connections of blades are in Fig. 1 realized by special rubber elements (Jones, 2001), which are on theirs sides loaded by such sufficiently great friction forces in the contact areas with the neighbourhood masses that no slips occur in these connections during operation. The linear Voigt–Kelvin model describes deformation properties of rubber damping elements.

If the friction forces in the contact surfaces between rubber element and the neighbourhood masses are low, than a slip can occur at one side of rubber element, Connection elements among blades become nonlinear The rubber element is completely fixed connected only to one model of blade, but it touches the other blade by the dry friction contact.

In such case, the dynamic computational model of the connection between two blades masses is shown in Fig. 2. One half of element's mass $m_e/2$ is fully connected with mass m_i, the second $m_e/2$ is connected with blade's mass m_{i+1} only by the sliding contact with the dry friction force F_t. Spring and damping element modelling the deformation properties of rubber link both half-element-masses together. Motion $y_{ei}(t)$ of the second half-element-mass $m_e/2$ is determined by the friction force and viscous elastic rubber forces and must be described by the individual differential equation.

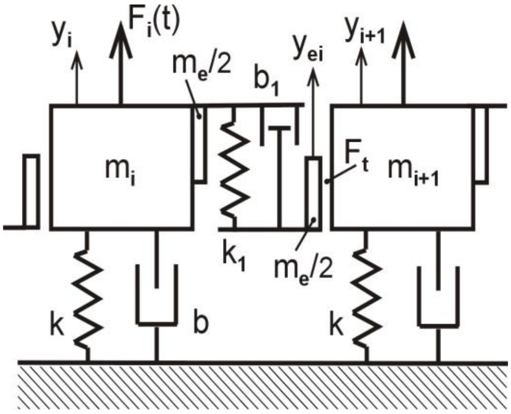


Fig. 2: Model of rubber damping element with one-side slip.

Motion of such a whole system, in which the same slipping properties according to Fig. 2 is supposed, is described by nine nonlinear differential equations; five equations are for masses m_i, $(i = 1 – 5)$ and four ones for damping elements' masses $m_{ej} = m_e/2$, $(j = 1 – 4)$.

The simplest damping connections among the blades heads are realized by *direct friction contacts*, where dry friction forces can be realized by direct contacts, where Coulomb law (see Fig. 3a) describes the classical dry friction forces: This law is described by two equations

$$F_t = fF_N \operatorname{sgn}(v) = fF_N v/|v| = fF_N (H(v) - H(-v)) \qquad \text{for } v \neq 0, \qquad (1)$$

$$F_t \in \langle -fF_N, fF_N \rangle \qquad \text{for } v = 0,$$

where f is coefficient of dry friction, F_N normal force, v relative velocity, F_t friction force, $H(v)$ Heaviside function: $H = 1$ for $v > 0$ and $H = 0$ for $v < 0$.

Many of fiction couples made of various materials have friction coefficient f in motion ($v \neq 0$) different from the friction coefficient f_s without motion ($v = 0$, see Fig. 3b). Mathematical model of such a connection is similar to eqs. (1), where in the second equation the slip coefficient f must be replaced by friction coefficient in still-stand stick coefficient f_s.

The tangential elastic micro-deformations in friction contact during vibration with small amplitudes strongly influence the damping characteristics. According to requited exactness it can be modelled by application many types of mathematical models (sometimes very complicated). The simple mathematical model is the modified Coulomb dry friction, graphically represented in Fig. 3c. A similar but continuous model uses function atan. It is shown in Fig. 3d.

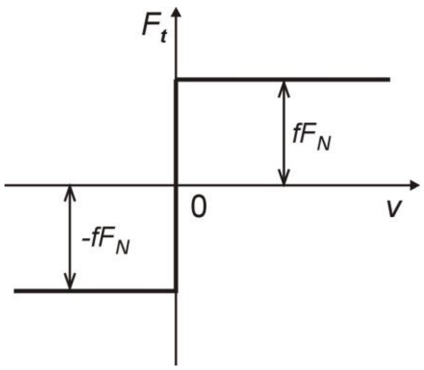


Fig. 3a: Coulomb dry friction characteristic.

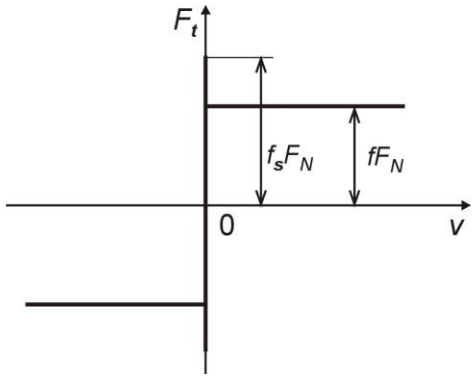


Fig. 3b: Different coefficients $f_s > f$.

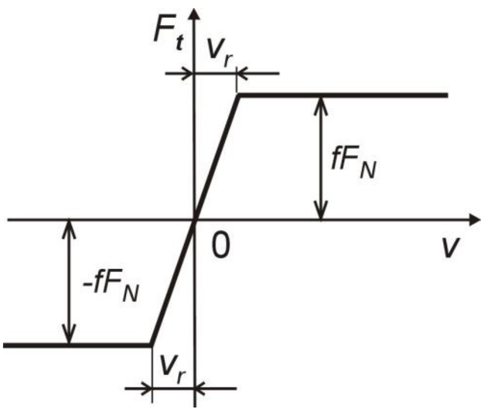


Fig.3c: Modified Coulomb dry friction characteristic.

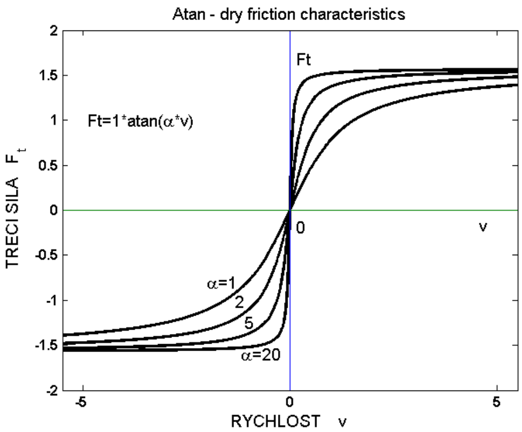


Fig. 3d: Atan dry friction characteristic.

The another type of five-blades-bunch, which have increased number of DOF, is a system with damping connections among the blades heads done by means of inserted stiff dry friction damping elements with masses m_e. These damping elements are held in the usually trapezoidal slot among heads by means of centrifugal force and saved against axial pushing out by means of stops (Fig. 4 left) or by weak springs (Fig. 4 right). Mathematical model in both cases contains eight strongly nonlinear functions in nine differential motion equations.

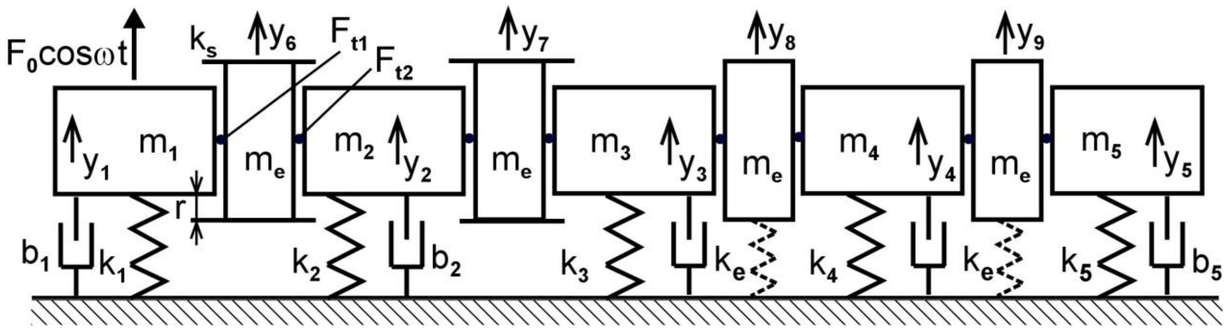


Fig. 4: Five-blades-bunch with inserted dry friction elements.

The harmonic excitation forces $F_{0i}\cos(\omega t)$ acting on blades, modelled here by masses m_i, can be generally of different magnitudes and also different phases [Pesek& Pust , 2011a, Pesek& Pust , 2011b]. Sometimes we need to take into account also the coupling of blades due to the compliance of turbine disk. This coupling can be modelled by linear springs which link neighbouring masses. Effects of different forms of excitation force vector $\boldsymbol{F} = [F_{01}, F_{02}, F_{03}, F_{04}, F_{05}]$, where $F_i(t) = F_{0i}cos(\omega t)$, i =1,..5 on the vibration of five-blades-bunches with different friction connections are analysed in the paper.

3. Free Vibration of Five-Blades-Bunches

The bunch of blades connected by viscous-elastic rubber elements ere investigated both theoretically and experimentally and results of this work were presented on the last conference Engineering Mechanics 2013 and published in (Pust & Pesek, 2013a; Pust & Pesek, 2013b). It has been proved there, that due to the more complicated forms at higher eigenmodes the modes' damping increases with frequency even at the constant material viscous damping coefficient. The application of orthogonality of excitation forces distribution to the other eigenmodes of blades bundle was necessary for analysis and isolation of selected resonance.

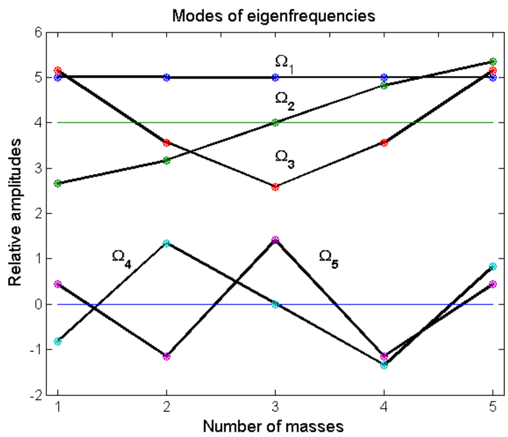


Fig. 5: Eigenmodes of five-blades-bunch.

The first step at the beginning of investigation of nonlinear dry friction systems (Treyde, 1995) of blades bunches was the ascertaining of free vibration's modes (Fig. 5) and theirs exploitation for isolation of selected resonances.

The influence of friction connections in the contact surfaces between blades' heads was investigated as well. It was shown that that in the case of excitation only by one force (e.g. by means of force vector $\boldsymbol{F} = [1, 0, 0, 0, 0]$) the decrease of friction forces in contact surfaces increases the resonance amplitudes of excited mass and decreases amplitudes of other, non-excited masses.

Only one type of excitation force vector excitation only on the first mass was used in this paper. The application of this simple excitation force vector with only one force has an important advantage resulting from the property that it enables to excite all five eigenmodes belonging to eigenfrequencies $\Omega_1,....,\Omega_5$, as this force vector can be decomposed into orthogonal components proportional to all modes.

4. Example of Response Curves of Five-Blades-Bunch

Let us show the influence of different coefficient of dry friction $f = F_t / F_0$ on the response curves of five-blades-bunch excited by a harmonic force F_0 acting on the first mass in the narrow frequency range *freq* = ∈ (120.5, 121.5) Hz containing the first resonance. Amplitude of excitation force is F_0 = 10 N. Response curves of five masses system connected by simple Coulomb dry friction forces (see Fig. 3a) in the contacts among masses are shown in Fig. 6. If the friction forces among blades are very low (F_t = 2 N) then the first, excited mass 1 vibrates with considerable higher amplitude than other four masses, which are fixed connected together due to the friction forces. However, in the resonance zone the increase of amplitudes causes separation of mass 2 from the rest of coupled masses 3, 4, 5. All four masses connect together again in the over-resonance zone (Fig. 5a).

Twice higher friction (F_t = 4 N, Fig. 5b) connects masses nearer. Further increase of friction forces to F_t = 5.5 N (Fig. 5c) decreases again resonance peak of excited mass 1 and moderately increases amplitudes of the remaining masses. The greatest friction force F_t = 6.5 N (Fig. 5d) results in merging of all five response curves into one curve with exception of small part at 121 Hz, where small splitting of mass 1 from masses 2,3,4,5 is recorded.

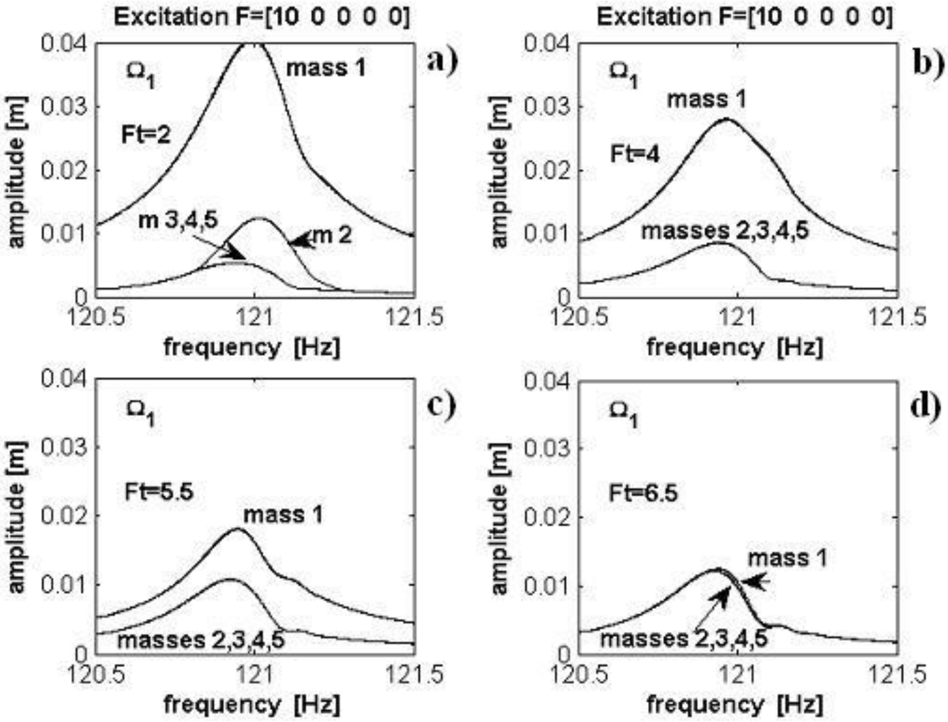
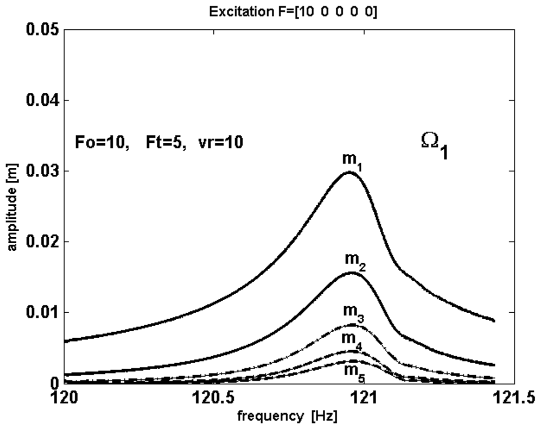


Fig. 6: Response curves of five-blades-bunch excited on first blade at different dry friction force F_t=2, 4, 5.5, 6.5 N.

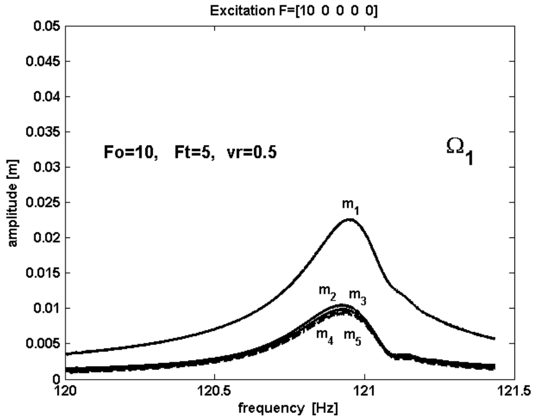
4.1. Modified Coulomb Characteristic

Modified Coulomb characteristic, in which the sharp-edged jump of friction forces at zero velocity is eliminated and replaced by continuous passage (Fig. 3c), is defined .by two parameters: Threshold velocity v_r and friction force F_t.

The influence of threshold velocity changes considerable the course of response curves bunch in comparison with the previous case. At sufficiently smooth passage of friction forces ($v_{r,}$ = 10, Fig. 7a) all masses vibrate by different amplitudes as shown in Fig. 7a. If the threshold velocity limits to zero (e.g. $v_{r,}$ = 0.5), the type of response curves resembles to the Coulomb friction response compare Fig. 7b with Fig. 6.



a)



b)

Fig. 7: Response curves of system with modified Coulomb friction characteristics.

5. Conclusion

Analysis of dynamic behaviour of numerical models of five-blades-bundle with different types of damping elements is presented by means of numerical simulation. The main attention was given to the response curves of individual blades.

There are lot of variations of excitation force type and of the dry friction characteristics modelling at vibration contact motion. Only one type of excitation force vector with harmonic force acting on the first mass has been used in this article. Five-blades-bundle with two types of dry friction connections were investigated**.**

Increasing friction forces among blades decreases maximal resonance peak and causes coherent response curves set.

Response curves of five-blades-bundle with modified Coulomb connections and decreasing threshold velocity v_r get similar to the responses of system containing the exact Coulomb friction.

It was shown that the application of damping connections in blades shroud is advantageous for suppressing of resonance peaks.

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