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# COMPARISON OF MODAL PARAMETER ESTIMATION TECHNIQUES FOR EXPERIMENTAL MODAL ANALYSIS

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**Abstract:** This paper deals with comparison of three commonly used modal parameter estimation methods (Pick Picking, Least Square Complex Exponential and Eigensystem Realization Algorithm) in order to judge about their accuracy and limitation. There have been invented a great amount of modal parameter estimation techniques in the field of experimental modal analysis. However, there is not universal method which would be suitable for every possible structure and/or with acceptable accuracy. As a test structure, 15 degrees of freedom plane with given mass, damping and stiffness was chosen. The foundations of discussed methods are described, their implementation on the test structure is shown and finally, natural frequencies and damping obtained by analytical approach are compared with those estimated using mentioned techniques. In addition, mode shapes differences are illustrated by means of Modal Assurance Criterion. Based on these comparisons, an accuracy and limitation of the methods is summarized in the conclusion.

## Keywords: Experimental Modal Analysis, Modal Parameter Estimation, Modal Assurance Criterion.

# 1. Introduction

Experimental modal analysis is used for the fast determination of the modal properties, determination of the structure vibration behavior or verification of finite element model and its correlation. There can be found a lot of methods that are used in an experimental modal analysis for determination of the modal properties (natural frequencies  $f_r$ , damping  $b_r$  and mode shapes) from Frequency Response Function (FRF). None of those methods is perfect and none of them is suitable for all cases with guarantee of accuracy (Avitabile, 2001). The aim of this paper is to compare three commonly used modal parameter estimation techniques in order to judge their accuracy on the test structure, which is represented by simulated 15 degrees of freedom plane.

# 2. Modal Parameter Estimation Methods

According to operation domain, the modal parameter estimation methods can be divided into timedomain and frequency-domain methods. There can be found methods, which work on single degrees of freedom (SDOF) systems only, and also more complex algorithms which work on multi degrees of freedom (MDOF) systems. As the representative of frequency-domain method, Least Square Complex Exponential (LSCE) was chosen. Eigensystem Realization Algorithm (ERA) is from group of the timedomain methods and Pick Picking (PP) method stands for classical, SDOF system method.

# 2.1. Pick Picking method

This method is sometimes referred also as peak-amplitude method (Ewins, 1986). It is one of the simplest modal parameter estimation methods. Pick Picking method belongs to SDOF methods group and from this fact yields its great limitation. It works well just for structures with well-separated (uncoupled) modes and in addition, for structures with a "good" damping. For heavily damped systems the response at a resonance is influenced by more than one mode. On the other hand the accurate measurements at resonance are difficult to obtain for light-damped structures. The detailed description of the Peak Picking method can be found in Ewins (1986).

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# 2.2. Least Square Complex Exponential method

Least Square Complex Exponential (LSCE) method is representative of the frequency-domain methods. It operates on MDOF systems directly. This means that it is able to find all modes in frequency range of interest at once. Basically, this method fits a theoretical expression for frequency-domain transfer function on the measured data. The process itself is very sophisticated and has been optimized into time efficient and robust algorithm (Verbover, 2002; Cauberghe, 2004). Results of this method are natural frequencies, damping and, in this case, complex mode shapes. The only problem here is an unknown order of the system. This deficit is removed by using of a stabilization chart (Cauberghe et al., 2005).

## 2.3. Eigensystem realization algorithm

The Eigensystem realization algorithm (ERA) is the typical time-domain method, which was published by Juang and Pappa (1985). All details, together with mathematical proofs, can be found in this publication. The core of this method lies in finding state-space matrices of the system from a time-domain data. In case of the experimental modal analysis, an impulse response function (IRF) can be used instead of the time-domain data. The IRF is computed from FRF by means of inverse Fourier transform (Ewins, 1986). Again, the problem is to determinate the system order, so similar stabilization chart has to be used as in case of LSCE.

## 3. Application of the Methods on the Test Structure

## 3.1. Description of the test structure

The simulated plane with 15 DOF was used as a test structure. The rectangular plane (Fig. 1) has fixed supports on two sides and free edges on another two sides. Properties of the plane (mass and stiffness) were chosen and proportional viscous damping model was used for determination of the damping matrix. The exact modal properties of the plane were computed and will be used for comparisons. The theoretical frequency response functions (Fig. 1) were calculated by direct inverse method (Ewins, 1986). Seven close coupled modes of vibration can be found in frequency range 270 Hz – 370 Hz.



Fig. 1: Simulated test structure (left: Geometry, right: Frequency Response Function).

#### 3.2. Setting of used modal parameter estimation techniques

Every modal parameter estimation technique requires an influence of analysts (Avitabile, 2001). It needs to be judged about system order or choose an individual peak in the FRF. When we deal with MDOF systems identification techniques we have to define the system order. In general, this is not easy task since we don't know the number of modes which are included in measured frequency range. Stabilization charts are constructed in order to remove this little disadvantage. It is convenient to mark a stable frequencies and damping in the stabilization charts. The stable, in this content, means that the frequencies and damping change just in small range of their values (usually 1% for frequency and 2% for damping) with different system order (Cauberghe et al., 2005).

For the test structure, we were able to identify all 15 peaks in FRF using manual peak picking. The stabilization charts of ERA and LSCE for our test structure can be seen in Fig. 2. ERA products cleaner stabilization diagram, however, the setting of ERA for successful identification (Caicedo, 2011) is more

complicated than setting of LSCE. In every case, both methods were able to identify all 15 modes with required accuracy.



Fig. 2: Stabilization charts (left: LSCE, right: ERA).

#### 3.3. Comparison of natural frequencies and damping

The comparison of natural frequencies and damping is made by graphical method that is described in Ewins (1986) and the results are plotted in Fig. 3.



Fig. 3: The comparison of natural frequencies and damping.

It can be seen from these graphs that estimated natural frequencies are almost independent on the used estimation method. On the other hand, the damping, which is estimated by PP method, varies from analytical, LSCE and ERA values. The most differences can be found in the close coupled modes where the values of estimated damping are different of up to 300% from analytically calculated damping.

# **3.4.** Comparison of mode shapes

Naturally, it would be possible to compare mode shapes from different method by overlap them into one graphs. But picture like this would be confusing and comparison would be uneasy (Ewins, 1986). Therefore, numerical methods have been developed for comparison of the mode shapes. One of the most used parameter is a Modal Assurance Criterion (MAC). The MAC is scalar value designed for two mode shapes from different sets (e.g. predicted vs. measured). It can be used for real and even for complex mode shapes. Modal Assurance Criterion indicates the degree of correlation between two mode shapes (Ewins, 1986). If the mode shapes are similar (or identical) the MAC values is equal or close to one, different mode shapes produce zero or almost zero MAC value. The MAC values from different combinations of mode shapes can be written into matrix and subsequently plotted. Comparison of the mode shapes obtained by PP and LSCE with analytical mode shapes is made in Fig. 4.



Fig. 4: The comparison of mode shapes (left: Analytical vs. PP, right: Analytical vs. LSCE).

From Fig. 4 can be observed that mode shapes produced by PP are very inaccurate in area of coupled modes. Mode shapes produced by LSCE are highly similar with analytical mode shapes.

#### 4. Conclusion

The paper presented the comparison of modal parameter estimation methods. Three commonly used methods were chosen – Pick Picking, Least Square Complex Exponential and Eigensystem Realization Algorithm. The assessment about the quality of methods was based on comparison of modal properties. All frequencies estimated by these methods corresponded with analytical very well. However, the damping was different in the case of Pick Picking method. From the comparison of mode shapes by means of Modal Assurance Criterion it could be found the same trend i.e. the mode shapes estimated by Pick Picking method were not determined very well.

From these comparisons clearly state that Pick Picking method fails in case of the close coupled modes of vibration. Least Square Complex Exponential method and Eigensystem Realization Algorithm show very good performance in area of close coupled mode shapes, but the estimation itself requires more computing time and has higher demands on the quality of the measured FRFs.

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