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ELASTIC CRITICAL BUCKLING STRESS OF THE WEB GIRDER SUBJECTED TO TRANSVERSE FORCE

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Abstract: *Beam subjected to a single concentrated load or transverse uniformly distributed partial load (relative loading length $0 \leqslant c/a \leqslant 1$). Buckling coefficients $k_{\sigma,\,b}$ (Tab.2) for a long rectangular plate $(4 \leqslant \alpha \leqslant 40)$ stiffened at longitudinal edges by flanges with different relative normal flange rigidity $(0 \leqslant \delta \leqslant 3)$ and subjected to a symmetric transverse load only at one longitudinal edge. Comparison of the buckling coefficients k_σ computed by various authors for plate without flanges (Tab.1). Conclusions made from a parametrical study, part of which is presented in the paper. Program PLII (2001) was used in the parametrical study.*

Keywords: ***Buckling coefficient, critical stress, patch loading, parametrical study, comparisons.***

1. Introduction

Already in the older editions of the modern codes DIN 18800 (1989), ENV 1993-1-5 (1997) the rules for the resistance of a web to the patch loading (*Fig.1*) use the same format as the other buckling rules. In the design procedure it is necessary to calculate:

- the yield resistance in the form of the stress f_y DIN 18800 (1989), or the force F_y ENV 1993-1-5,
- the elastic buckling stress σ_{cr} DIN 18800 (1989), or the buckling force F_{cr} ENV 1993-1-5 (1997),
- the relative slenderness $\lambda = \sqrt{f_y / f_{cr}}$ DIN 18800 (1989), or $\lambda = \sqrt{F_y / F_{cr}}$ ENV 1993-1-5 (1997),
- the reduction factor $\kappa = f(\lambda)$ DIN 18800 (1989), or $\chi = f(\lambda)$ ENV 1993-1-5 (1997),
- the resistance of web to patch loading $\sigma_R = \kappa f_y$ DIN 18800 (1989), or $F_R = \chi F_y$ ENV 1993-1-5.

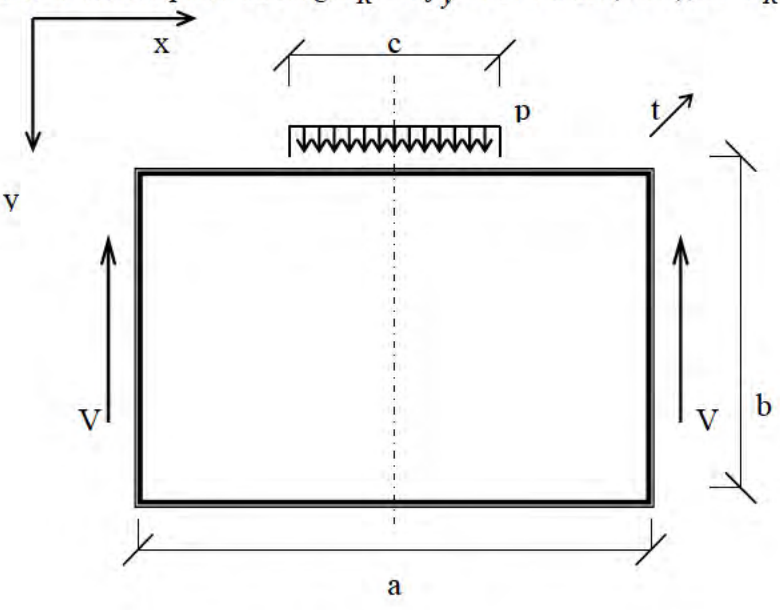

Fig. 1: Notation

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Despite of the formal similarity in the design procedures, there are important differences in the details between the both codes DIN 18800 (1989) and ENV 1993-1-5 (1997).
One of the most important step in a design procedure is calculating of the elastic buckling stress σ_{cr} .

2. Elastic buckling stress

The elastic buckling force is written as

$$F_{cr} = \sigma_{cr} c t \tag{1}$$

where

c is the length over which the applied transverse force is distributed,

t the thickness of the plate,

$\sigma_{cr} = k_\sigma \sigma_E$ the critical stress and the Euler critical stress are as follows

$$\sigma_E = \frac{\pi^2 E}{12\left(1-\nu^2\right)\left(\frac{b}{t}\right)^2}, \quad \sigma_{E,\text{steel}} = \frac{189\,800}{\left(\frac{b}{t}\right)^2}\left[\text{N/mm}^2\right], \quad \sigma_{\text{aluminium}} = \frac{\sigma_{E,\text{steel}}}{3} = \frac{63\,267}{\left(\frac{b}{t}\right)^2}\left[\text{N/mm}^2\right] \tag{2}$$

b is the breadth of the plate or depth of the web,

b/t the slenderness of the plate or the web,

k_σ the buckling coefficient,

E Young's modulus of elasticity (210 GPa for steel, 70 GPa for aluminium alloys),

ν Poisson's ratio in elastic stage (0,3 for steel and aluminium alloys).

The buckling coefficient k_σ depends generally on the

- type of the action (also on the relative loading length $\beta = c/a$ in the case of transverse action),
- boundary conditions (also on the flange rigidities, e.g. on the relative normal rigidity of the flange $\delta = A_f/(bt)$, where A_f is the area of the flange cross-section),
- longitudinal and/or transverse stiffeners locations and their rigidities,
- shape of the plate (e.g. on the aspect ratio of the plate $\alpha = a/b$, in the case of rectangular plate, where a is the length of the investigated plate – spacing of the transverse stiffeners),

The numerical values of the buckling coefficient k_σ may vary a lot and therefore sometimes for the purpose of diagrams the more convenient forms of the buckling coefficients $k_{\sigma,a}$ and $k_{\sigma,b}$ are used. For instance Petersen (1993), von Berg (1989) and Ravinger (1979) use instead of the above defined buckling coefficient k_σ the buckling coefficient $k_{\sigma,a}$:

$$F_{cr} = \sigma_{cr} c t = k_\sigma \sigma_E c t = (k_\sigma \frac{c}{a}) \sigma_E a t = k_{\sigma,a} \sigma_E a t \tag{3}$$

Kutzelnigg (1982) and Protte (1994) use instead of the coefficient k_σ the buckling coefficient $k_{\sigma,b}$:

$$F_{cr} = \sigma_{cr} c t = k_\sigma \sigma_E c t = (k_\sigma \frac{c}{b}) \sigma_E b t = k_{\sigma,b} \sigma_E b t \tag{4}$$

The following formulae are valid

$$k_{\sigma,a} = k_\sigma \frac{c}{a} = k_{\sigma,b} \frac{b}{a} = \frac{k_{\sigma,b}}{\alpha} = k_\sigma \beta \tag{5}$$

$$k_{\sigma,b} = k_\sigma \frac{c}{b} = k_{\sigma,a} \frac{a}{b} = k_{\sigma,a} \alpha = k_\sigma \alpha \beta \tag{6}$$

There are many publications, where the values of the buckling coefficients may be found. Some of them are mentioned in *Tab.1* together with the ranges of dimensionless parameters for which the values of buckling coefficient were calculated.

Tab.1: The way of calculation of the buckling coefficient values according to various authors

	Petersen (1993)	von Berg (1989)	Ravinger (1979)	Kutzelnigg (1982)	Protte (1994)
Type of k_σ	$k_{\sigma,a}$			$k_{\sigma,b}$	
$\alpha = a/b$	0.3 ÷ 35	0.7 ÷ (≥10)	1 ÷ 3	0.5 ÷ 5	2 ÷ 10
$\beta = c/a$	0 ÷ 1	0 ÷ 1	0; 1	0 ÷ 1	0 ÷ 1
$\delta = A_f/(bt)$	0	0	0	0	0; 0.1; 0.3; 0.5; 1
Remarks	simply supported plate (s.s. plate)	simply supported plate, $k_\sigma(\alpha \geqslant 10) = k_\sigma(\alpha = 10)$	4 various boundary conditions incl. s.s. plate	s.s. plate, 0 or 1 or 2 longitudinal stiffeners	simply supported plate

The numerical values of the buckling coefficient $k_{\sigma,b}$ computed by the program PLII for the patch loading, simply supported rectangular plate with no flanges $(\delta = 0)$ and no stiffeners are given in *Tab. 2*. They are compared with the results of the authors from *Tab. 1*. The comparison of the results given in *Tab. 2* leads to the following conclusions:

- for the small aspect ratios ($\alpha < 4$) the differences among the values of all authors are negligible ($\leqslant 10$ %),
- the agreement between PLII's and Protte's results is excellent in the whole *Tab.2*,
- the greatest difference between Protte's and Kutzelnigg's results in Tab. 2 is 60 % for the case $\alpha = 5$, $\beta = 1$. The reason why the results of Protte and Kutzelnigg differ was explained in Ravinger (1979, p. 34, paragraph 4.1). The reason is, that Kutzelnigg took into account only the influence of the vertical normal stresses σ_y. The influences of the stresses σ_x and τ were neglected in his buckling coefficient calculations. The differences are the greater the greater is aspect ratio α, because with increasing α, the influence of so called beam stresses σ_x on the value of buckling coefficient increases,
- the buckling coefficients for the very long plates ($\alpha > 10$), a case which may be important in the design of the cran runway girders without intermittent transverse stiffeners, can be found only in Petersen (1993) and Berg (1989). Ravinger (1979, p.421), reported problems in finding the minimum value of buckling coefficients for the cases with aspect ratios $\alpha > 3$. Petersen's and von Berg's results are based, as it is mentioned in Petersen (1993) and Berg (1989) on the older Protte's publications. They are each other in good agreement except the cases $\beta = 0$, $\alpha \geqslant 8$. They differ a lot from PLII's and Protte's results for $\alpha \geqslant 5$. The difference is the greater the greater is the aspect ratio α. For the case $\alpha = 5$ the Petersen's and von Berg's results does not differ a lot from Kutzelnigg's ones, which are, as it is explained above, not correct. It is therefore believed that Petersen's and von Berg's results for the long plates ($\alpha > 5$) are not correct too and they are, comparing with PLII's and Protte's results, on the unsafe side.

The influence of the relative normal rigidity of the flanges $\delta = A_f/(bt)$ on the buckling coefficient was investigated by Protte (1994). The results of program PLII (2001) are in excellent agreement with Protte's ones also in this case. The part of the large parametrical study is shown in the *Tab. 3*. From the results it may be concluded:

- the influence of the flange normal rigidity δ on the buckling coefficient is negligible in the range $\alpha \leqslant 4$. Maximum difference is $< 25\%$ for the $\beta = 0.005$, $\alpha = 4$,
- for the cases with $\alpha \leqslant 4$ and any δ, we can use the values of buckling coefficients computed for $\delta = 0$, being slightly on the safe side,
- for the longer plates ($\alpha > 4$) is the influence of the δ on the increasing of the buckling coefficient the greater the greater is aspect ratio α.

Program PLII is able to take into account also influence of the torsional rigidity of the flange on the buckling coefficient, which has greater effect on the increasing of the buckling coefficient than normal flange rigidity. The various boundary conditions may be taken into account too.

Tab. 2: Comparison of the values of the buckling coefficients $k_{\sigma,\,b} = f(\alpha, \beta)$ *.*

Patch loading, simply supported rectangular plate, no flanges $(\delta = 0)$ *, no stiffeners.*

$\alpha = a/b$	Author	$\beta = c/a$						
		0	0.2	0.4	0.6	0.7	0.8	1
1	PLII (2001) C	3.32	3.42	3.74	4.24	4.55	4.91	5.67
	Kutzelnigg (1982) D	3.00	3.30	3.60	4.30	4.70	5.20	6.25
	Petersen (1993) D	3.24	3.45	3.76	4.26	4.54	5.04	6.09
	von Berg (1989) T	3.20	3.40	3.70	4.20	-	5.00	6.20
	Ravinger (1979) D	3.50	3.55	3.79	4.25	4.55	4.90	5.70
2	PLII (2001) C	2.37	2.54	2.91	3.46	3.79	4.17	5.09
	Protte (1994) D	2.35	2.50	2.90	3.45	3.80	4.20	5.08
	Kutzelnigg (1982) D	2.26	2.54	2.9	3.65	4.10	4.50	5.45
	Petersen (1993) D	2.30	2.52	2.82	3.31	3.66	4.14	5.21
	von Berg (1989) T	2.36	2.60	2.90	3.40	-	4.40	5.10
	Ravinger (1979) D	2.40	2.60	3.00	3.64	4.00	4.42	5.28
3	PLII (2001) C	2.23	2.54	3.21	4.05	4.52	5.02	6.16
	Protte (1994) D	2.21	2.51	3.24	4.05	4.50	5.05	6.13
	Kutzelnigg (1982) D	2.20	2.64	3.44	4.40	5.00	5.48	6.68
	Petersen (1993) D	2.14	2.61	3.18	3.96	4.47	5.05	6.39
	von Berg (1989) T	2.25	2.40	2.94	3.90	-	5.07	6.30
	Ravinger (1979) D	2.40	2.70	3.27	4.05	4.50	5.07	6.30
4	PLII (2001) C	2.10	2.58	3.58	4.83	5.47	6.02	7.30
	Protte (1994) D	2.07	2.55	3.55	4.80	5.46	6.00	7.26
	Kutzelnigg (1982) D	2.30	2.90	4.00	5.40	6.00	6.76	8.36
	Petersen (1993) D	2.04	2.87	3.72	4.85	5.51	6.22	7.51
	von Berg (1989) T	2.20	2.48	3.40	4.60	-	6.20	7.56
5	PLII (2001) C	1.98	2.61	3.70	4.38	4.76	5.18	6.23
	Protte (1994) D	1.94	2.60	3.75	4.45	4.76	5.20	6.22
	Kutzelnigg (1982) D	2.39	3.22	4.65	6.40	7.20	8.10	9.95
	Petersen (1993) D	2.00	3.31	4.44	5.91	6.71	8.56	9.83
	von Berg (1989) T	2.35	2.70	4.00	5.40	-	7.40	9.15
8	PLII (2001) C	1.62	2.17	2.50	2.88	3.10	3.37	4.04
	Protte (1994) D	1.58	2.20	2.50	2.90	3.13	3.40	4.10
	Petersen (1993) D	2.00	4.46	6.54	8.81	10.16	11.48	14.28
	von Berg (1989) T	3.20	3.68	5.44	7.92	-	10.56	13.92
10	PLII (2001) C	1.42	1.78	2.02	2.32	2.50	2.71	3.25
	Protte (1994) D	1.38	1.80	2.05	2.35	2.55	2.75	3.30
	Petersen (1993) D	2.00	5.19	7.90	10.65	12.65	13.82	17.41
	von Berg (1989) T	3.60	4.20	6.20	9.50	-	12.70	17.00
20	PLII (2001) C	0.81	0.90	1.01	1.14	1.23	1.34	1.60
	Petersen (1993) D	2.00	8.49	14.41	20.17	23.86	27.18	33.67
	von Berg (1989) T	7.20	8.40	12.40	19.00	-	25.40	34.00
30	PLII (2001) C	0.54	0.59	0.63	0.76	0.82	0.89	1.06
	Petersen (1993) D	2.00	11.04	20.55	30.25	35.49	40.60	51.15
	von Berg (1989) T	10.80	12.60	18.60	28.50	-	38.10	51.00
40	PLII (2001) C	0.39	0.45	0.50	0.54	0.58	0.62	0.76
	Petersen (1993) D	2.00	14.11	26.49	41.19	46.92	53.00	66.79
	von Berg (1989) T	14.40	16.80	24.80	38.00	-	50.80	68.00

Numerical values were: C – computed, D – taken from a diagram, T – taken from a table.

Tab. 3: Buckling coefficients $k_{\sigma,b} = f(\alpha, \beta, \delta)$ computed by the program PLII (2001). Patch loading with relative loading length β, simply supported rectangular plate having aspect ratio α, with flanges having only relative normal rigidity δ, no stiffeners.

$\alpha = a/b$	$\delta = A_f/(bt)$	$\beta = c/a$						
		0.005	0.01	0.05	0.1	0.2	0.3	0.4
4	0	2.098	2.104	2.149	2.246	2.577	3.036	3.581
	0.3	2.497	2.504	2.550	2.644	2.966	3.411	3.935
	0.5	2.552	2.559	2.605	2.697	3.013	3.452	3.971
	1.0	2.607	2.612	2.656	2.744	3.056	3.484	3.996
5	0	1.972	1.979	2.037	2.170	2.610	3.194	3.700
	0.3	2.460	2.470	2.531	2.673	3.139	3.762	4.479
	0.5	2.532	2.540	2.604	2.742	3.201	3.817	4.526
	1.0	2.599	2.610	2.670	2.800	3.250	3.860	4.560
	1.5	2.629	2.636	2.693	2.825	3.270	3.873	4.571
	2.0	2.648	2.655	2.709	2.839	3.280	3.880	4.575
	3.0	2.683	2.688	2.738	2.863	3.295	3.888	4.581
8	0	1.606	1.615	1.697	1.884	2.172	2.338	2.501
	0.3	2.329	2.345	2.473	2.781	3.736	4.931	5.950
	0.5	2.451	2.467	2.598	2.908	3.865	5.050	6.298
	1.0	2.566	2.584	2.712	3.016	3.960	5.136	6.384
	1.5	2.612	2.626	2.750	3.053	3.990	5.156	6.406
	2.0	2.641	2.654	2.774	3.072	4.003	5.166	6.416
	3.0	2.683	2.695	2.807	3.098	4.018	5.176	6.426
10	0	1.427	1.427	1.496	1.647	1.781	1.896	2.021
	0.3	2.288	2.288	2.437	2.863	4.143	4.743	5.176
	0.5	2.449	2.449	2.604	3.046	4.368	5.871	6.826
	1.0	2.576	2.600	2.750	3.190	4.510	6.050	7.590
	1.5	2.651	2.651	2.804	3.242	4.546	6.084	7.647
	2.0	2.682	2.682	2.832	3.266	4.563	6.099	7.670
	3.0	2.725	2.725	2.870	3.294	4.579	6.112	7.689
20	0	0.812	0.812	0.835	0.856	0.902	0.954	1.013
	0.3	1.826	1.826	2.078	2.254	2.434	2.600	2.774
	0.5	2.162	2.162	2.574	3.074	3.386	3.646	3.906
	1.0	2.468	2.508	3.020	4.300	5.500	6.060	6.560
	1.5	2.634	2.634	3.166	4.462	7.074	7.890	8.562
	2.0	2.700	2.700	3.232	4.526	7.532	8.780	9.362
	3.0	2.772	2.772	3.296	4.578	7.676	9.488	9.988
30	0	0.545	0.545	0.553	0.565	0.595	0.629	0.667
	0.3	1.402	1.402	1.478	1.534	1.631	1.732	1.842
	0.5	1.832	1.833	2.053	2.150	2.305	2.454	2.614
	1.0	2.339	2.400	3.210	3.570	3.903	4.200	4.500
	1.5	2.619	2.620	3.629	4.827	5.430	5.889	6.330
	2.0	2.726	2.728	3.771	5.799	6.771	7.434	8.070
	3.0	2.834	2.836	3.885	6.132	8.460	9.102	9.612
40	0	0.408	0.408	0.414	0.423	0.445	0.470	0.500
	0.3	1.092	1.096	1.125	1.160	1.229	1.302	1.383
	0.5	1.487	1.500	1.578	1.638	1.744	1.850	1.968
	1.0	2.135	2.200	2.632	2.788	3.000	3.204	3.444
	1.5	2.527	2.436	3.558	3.867	4.224	4.524	4.832
	2.0	2.590	2.692	4.264	4.856	5.392	5.812	6.228
	3.0	2.748	2.854	4.512	6.552	7.384	8.040	8.696

Lagerqvist (1995, p.122) made an attempt to create approximate formula for $k_{FL} = k_{\sigma,b}$, which is valid only in the narrow intervals $(1 \le \alpha \le 4;\ \ 0 \le \alpha\beta \le 1)$ comparing with our *Tab. 2*:

$$k_{FL} = k_{\sigma,b} = 2 + \frac{1.2}{\alpha^2} + \alpha^2\beta^2(0.5 + \frac{2}{\alpha^2}) \qquad (7)$$

There is a print error in the similar formula in Lagerqvist (1995, p.41, formula (2.107)), where the term „2.1α+…“ should be replaced by the term „2.1+…“.

The buckling coefficient k_F in ENV 1993-1-5 (1997), which corresponds to $k_{\sigma,b}$, was determined on the basis of the results from FE analysis using ANSYS, which took into account the influences of all stiffnesses as well as the length of the transverse load applied on the flanges. The more accurate values k_F, which were calculated for more complex boundary conditions and were calibrated with numerous experiments, may be found in Lagerqvist (1995). These expressions were simplified for ENV 1993-1-5 (1997) (for EN 1993-1-5 (2006) and EN 1999-1-1 (2007) too) to formula

$$k_F = 6 + \frac{2}{\alpha^2} \qquad (8)$$

The values computed according to the formula (8) do not differ a lot from the values of buckling coefficient computed for the plate with upper edge fixed in loaded flange when the load is applied on the short relative length β (compare them for instance with the results for $\beta = 0$ in Ravinger (1979, p. 410).

As usually, of course, one cannot mix the parts of design procedures taken from the different codes. This rule is valid also for this topic and the codes DIN 18800 (1989) and ENV 1993-1-5 (1997). In the latter code the design procedure using the formula (8) was calibrated with the experiments in (Lagerqvist 1995); see also (Johansson, Maquoi, Sedlacek 2001).

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