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OPTIMIZATION OF DYNAMICALLY LOADED CIVIL ENGINEERING STRUCTURES

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Abstract: *Contribution presents a tool for general optimization on the macro level of civil engineering structures that are subjected to dynamic phenomena. These phenomena are represented mainly by harmonic loads. Optimization process is based on communication between new script and widespread program for structural analysis including FEM solver. Communication is accomplished using Extensible Markup Language (XML). Very robust optimization method Simulated Annealing is used due to the requirement of a general optimization of dynamic problems. In the last part of contribution two optimizations of simple structures of different materials (reinforced concrete, steel) are performed. Long-term task of this work is to put academic knowledge in the field of optimization into common practice of structural analysis.*

Keywords: ***Optimization, civil engineering structures, dynamics, simulated annealing, software.***

1. Introduction

A lot of research work has already been done on the academic field on the topic of optimizing and structure dynamics, see the following subsection. Unfortunately, this knowledge often cannot be successfully put into practice. Several aspects are to blame. One of them is the absence of supporting software that would quickly and effectively help to engineers with the optimization process. This process leads to optimal design and economical structure. Such software has already found its use in mechanical engineering.

Nowadays, optimization process in practice is perceived only as an iterative computation of one or two steps that fulfills standard requirements. Thus design of optimal shapes and dimensions of structures remains only on designer’s knowledge and experiences. Another aspect related to above mentioned problem is the complexity of optimization functions in civil engineering and many restrictive conditions, which are in codes. These are the reasons why to deal with the optimization of structures and make support tools for designers.

1.1.Previous work

Large number of groups each with different main material could face dynamic effects. For every single group we could consider different objective functions. In general, it is better to avoid multi-criteria tasks, namely those where two or more objective functions are optimized. Therefore there is an effort to convert above mentioned problems to task of only one objective function. For structures of homogeneous material usually total weight is optimized. Among these materials we can consider steel.

Heterogeneous materials such as concrete or reinforced concrete consist of various components with different properties. Thus total weight of structure could not be considered as objective function. Variable that takes into account non-homogeneousness could be total costs composed of unit costs multiplied by appropriate amounts. Another advantage of this attitude is that total costs can also contain costs of transport, errection, manufacturing, storage etc. An example of such function for concrete structures is shown in equation 1 (Sarma & Adeli, 1998)**.**

$$C_o = C_m + C_{FB} + C_{TR} + C_S + C_{CD} + C_E \tag{1}$$

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Where C_m are cost of materials (concrete, reinforcement, formwork, fibers etc.), C_{FB}, C_{TR}, C_S, C_{CD} and C_E are costs of fabrication, transport, substructures, cladding and errection, respectively.

Due to the nature of optimization tasks in civil engineering (including dynamics) where discreet variables, discontinuous functions and high nonlinearity have to be taken into account, robust genetic, evolutionary or special optimization methods are used. Comparing methods for optimization of great structures subjected to dynamic loads is one of issues in (Chahande & Arora, 1994).

One of the most cited works in connection with optimization methods for dynamic tasks is (Xie & Steven, 1996), where ESO – Evolutionary Structural Optimization method has been published. Modification of the same method that is used in this paper (Simulated Annealing) was applied for optimization in (Pantelides & Tzan, 2000).

Optimization with dynamic phenomena is widespread topic especially in last decade. Let us mention some of papers on which presented work is linked to. Robust optimization of dynamic response of structures was presented in (Chen et al., 2011). One of the most common techniques to cope with dynamic loads is to transfer them to static loads. This process was used inter alia in (Park et al., 2005) or in (Yi, 2011) where authors are dealing with contact conditions. Eigen frequencies and Eigen mode shapes play an important role in topology optimization of structures in (Maeda, 2006).

2. Optimization process

Process of optimization as such could be divided into two sections according to the program, which is used for that particular part of the process. In the first part software for structural analysis is used for creating model of optimized structure. Than parameters have to be implemented to the model. These parameters will be updated according to optimization method commands and hence will be a change of objective function and constraints. In this section it is necessary to make XML file(s) as well. This file ensures communication between new optimization script and the program with FEM solver.

The second part of the process is provided by new script. Main tasks of the script are to select and upload desiderative inputs and outputs from XML file for objective function definition and constraint designation. Optimization will be preceded by Simulated annealing method. Setting of this method is also part of the script. Last but not least, script could be able to save results and evaluate gained optima using post processing. For better clearness and understanding, above mentioned process is shown below in few steps.

- **Static model**

Structure modeling in program for structural analysis

- **Parameterization of model**

Definition and implementation of parameters to the model and assignment of these parameters to individual entities. With these parameters model can be changed and controlled based on partial optimization algorithm outputs.

- **XML files**

Creating of XML files. With these files it is possible to transfer data between script and model. It is recommended to make two XML files. One is with inputs (parameters) and the other with outputs (weight of structure, internal forces, unit checks etc.).

- **Settings**

Objective function, constraints and parameter boundary values have to be set as well as optimization method.

- **Course of optimization**

After the evaluation of output variables from static software the optimization algorithm initiates the change of appropriate parameters. After that model is recalculated, results or output variables are exported back to the script to be assessed again. This cycle is repeated until acceptable optimum is found, Fig. 1.

- **Result management**

Application saves current results during optimization. Thus it is possible to analyze optimization process and user has ability to choose other optima according to his (new) priorities. From the practical point of view, it is definitely useful to investigate the dependence of the parameters on the objective function.

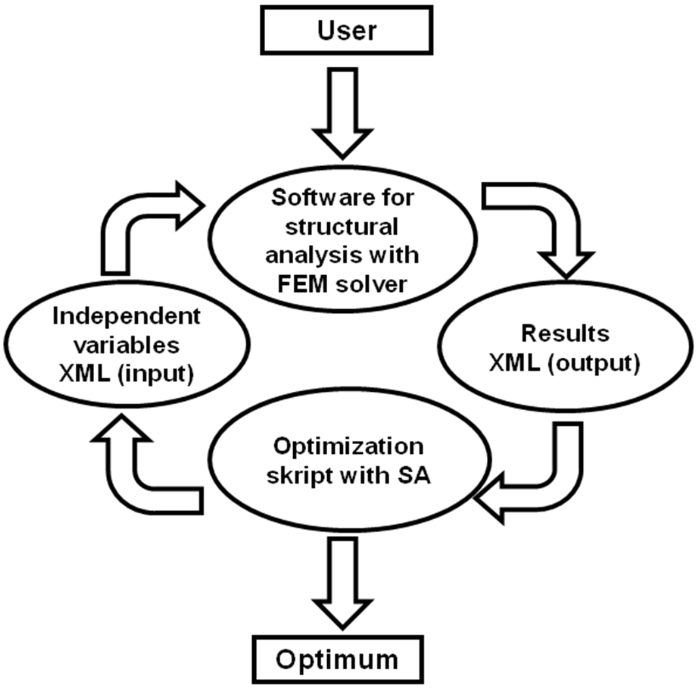

User
Software for structural analysis with FEM solver
Independent variables XML (input)
Results XML (output)
Optimization skript with SA
Optimum

Fig. 1: Optimization schema.

3. Optimization script

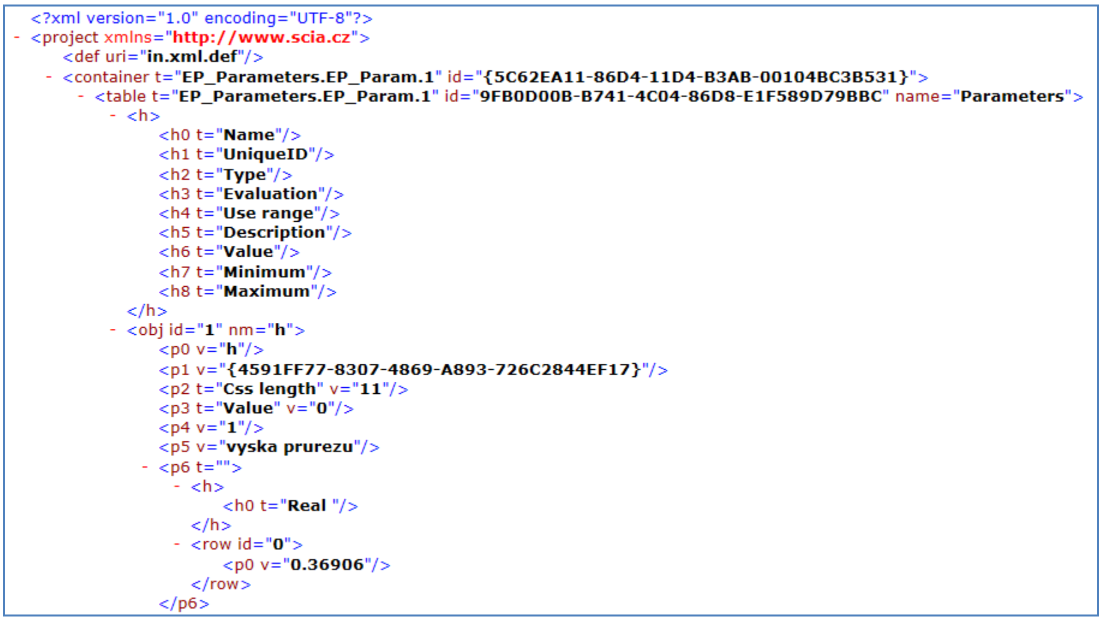
To create optimization script, programming language and numerical computing environment MATLAB was chosen. In fact script has to face three main tasks:

- Upload, update and save XML files
- Ensure execution of recalculation in software for structural analysis
- Include the optimization algorithm

3.1. XML files operations

Content of the input XML (IN) consists of table with parameters implemented to static structure model. Piece of input document in markup language generated from model is shown in Fig. 2. Given that the table can be variously modified depending on the type and number of parameters or the version of static program, so this part of script is “made to measure” XML file.

In fact, script has to found position (line), where parameter or its bounds are located. After finding the line, script searches parameter value and change it according to the command from the optimization algorithm. Considering the shown example, second row with index *p0* need to be located to update parameter and after that the value of the attribute *v* is changed. In the sample it was therefore the value of 0.36906 (Note: The parameter unit is [m] and the parameter value is based on consideration of continuous value distribution for optimization process).



```
  <?xml version="1.0" encoding="UTF-8"?>
- <project xmlns="http://www.scia.cz">
      <def uri="in.xml.def"/>
    - <container t="EP_Parameters.EP_Param.1" id="{5C62EA11-86D4-11D4-B3AB-00104BC3B531}">
        - <table t="EP_Parameters.EP_Param.1" id="9FB0D00B-B741-4C04-86D8-E1F589D79BBC" name="Parameters">
            - <h>
                  <h0 t="Name"/>
                  <h1 t="UniqueID"/>
                  <h2 t="Type"/>
                  <h3 t="Evaluation"/>
                  <h4 t="Use range"/>
                  <h5 t="Description"/>
                  <h6 t="Value"/>
                  <h7 t="Minimum"/>
                  <h8 t="Maximum"/>
              </h>
            - <obj id="1" nm="h">
                  <p0 v="h"/>
                  <p1 v="{4591FF77-8307-4869-A893-726C2844EF17}"/>
                  <p2 t="Css length" v="11"/>
                  <p3 t="Value" v="0"/>
                  <p4 v="1"/>
                  <p5 v="vyska prurezu"/>
                - <p6 t="">
                    - <h>
                          <h0 t="Real "/>
                      </h>
                    - <row id="0">
                          <p0 v="0.36906"/>
                      </row>
                  </p6>
```

Fig. 2: Input XML document.

The situation is similar with the output (OUT) XML file. Document structure is the same, but the script searches for objective or other functions such as code limits, unit checks etc. Part of the script for searching and loading variables from generated XML files could have the following form:

```
Doc = xmlread ('in.xml');
elements = Doc.getElementsByTagName('p0');
parCon = [];
parValues = [];
for i=1:parNum
    ConMin = str2double(elements.item(4*i-2).getAttribute('v'));
    ConMax = str2double(elements.item(4*i-1).getAttribute('v'));
    parCon = [parCon; ConMin ConMax];
    PAR = str2double(elements.item(4*i-3).getAttribute('v'));
    parValues = [parValues PAR];
end
```

The first line of the script loads the file *in.xml* into variable *Doc*. Searching elements containing crucial character *p0* follows. Then attribute value for chosen elements is determined. In this case, into variables *ConMin* and *ConMax* are gradually saved parameter limits and in variable *PAR* current parameter values are kept. Script for searching and loading output parameters is resembled. But there is a change of source file.

3.2. Recalculation

To avoid complicated programming of FEM principles for calculation and design of structures, among users-designers widespread software Scia Engineer is utilized. It is necessary to update model after changing the input parameters. Execution of appropriate type of calculation follows. Thereafter output XML file is updated. All these operations are accomplished by a single command with several attributes. The command has following form:

```
Dos (' "ESA_XML.exe" CALCULATIONTYPE "ProjectFile" ["XMLUpdateFile"] [switches]')
```

Each argument of the command will be explained. The first part of command *"ESA_XML.exe"* shows full path to the boot XML file. The second one *CALCULATIONTYPE* specifies the calculation type. In this paper batch analysis will be used. This allows us to run gradually modal analysis and linear calculation. Argument *"ProjectFile"* determinates full path to file with model. Next two items are partly optional. Attribute *"XMLUpdateFile"* includes full path to the XML file that contains parameters and allows model updating. The last part of command *[switches]*serves for selecting additional operations such as saving results in a new or existing output XML file or exporting results into different data formats.

3.3. Optimization algorithm

There are mentioned in chapter Previous work at the beginning of the contribution some of optimization methods or algorithms that provide us ability to solve structure optimization including dynamic factors. Our goal is to create application that will be useful for general optimization. This requires using robust optimization method suitable for various types of optimization problems.

One of such methods is Simulated annealing. This method is based on physical process of metal annealing. At high temperature, the particles are arranged randomly in the material matrix and the slow cooling leads to arrangement of particles in equilibrium, which corresponds to the minimum energy state. In optimization this value represents the global minimum. Simulated annealing was discovered in the middle of eighties independently by two authors (Kirkpatrick et al., 1983, Černý, 1985).

Principally particle in Simulated annealing represents one possible solution with function value. This solution is modified by random transformation from neighborhood of actual solution. Original solution is replaced with some probability. If there is a better value of new solution then this will always substitute the original one. But if new solution has worse value, it still could replace the old one thanks to the probability. Acceptance probability of worse solution is reduced during optimization according to temperature, which is one of method parameters. With this acceptance process when we can get out of local extremes the method is so robust. Algorithm could have following form (Lepš, 2000):

```
$T = T_{max}$, Create P, rate P
while (not stopping condition) {
    $count = succ = 0$
    while ($count < count_{max}$ & $succ < succ_{max}$) {
        $count = count + 1$
        Change P with operator O, the result is N
        $p = \exp((F(N) - F(P))/T)$
        if (random number u[0,1] < p) {
            $succ = succ + 1$
            P = N
        $\}_{if}\}_{while}$
    reduce $T$
$\}_{while}$
```

Algorithm has few parameters that regulate the method, T_{max}, T_{min}, $count_{max}$, $succ_{max}$ or $iter_{max}$. First two variables comprise maximal and minimal temperature value. During cooling process temperature is multiplied by T_{multi} in accordance with traditional formula:

$$T = T_{multi} \cdot T_i \tag{2}$$

But the multiplier can be determinate by various ways. Examples in next chapter are optimized with multiplier defined both with numeric value (0.99) and formula below.

$$T_{multi} = \left(\frac{T_{min}}{T_{max}}\right)^{\frac{succ_{max}}{iter_{max}}} \tag{3}$$

Method parameter $count_{max}$ specifies the maximum number of iterations at given temperature level and $succ_{max}$ number of successful iterations at given temperature level. A recommendation for the ratio of these values is that $succ_{max}$ is tenth of $count_{max}$. Parameter $iter_{max}$ determinates maximum number of all iterations.

3.4. Script and algorithm verification

Settings of some Simulated annealing parameters often requires try/error method. An example is settings of the initial temperature T_{max}. This should be set to a value that ensures approximately fifty percentage acceptation of newly created solutions. It is also good to verify the optimization algorithm on a simple example and compare results with other methods for the problem certainly successful. Therefore to verify proposed code optimization of support positions of statically indeterminate continuous beam follows.

The goal of the optimization was to determine position of two supports to minimize stress caused by bending moment. Objective function represents the maximum absolute value of the minimum and maximum bending moment on the beam. Beam is made of concrete and loaded only with self weight. Example schema is shown in Fig. 3.

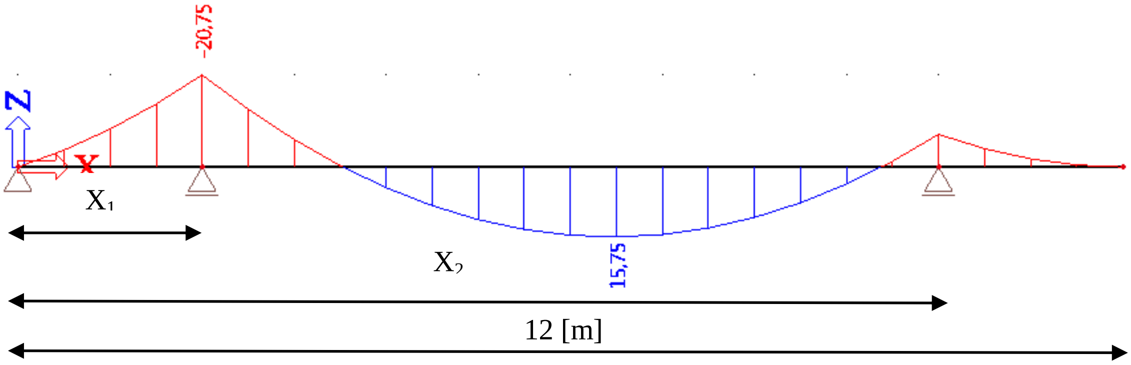


Fig. 3: Scheme of the optimization task

There were two parameters (distance from the beginning of the beam). Range for the first parameter was from 1.0 to 7.0 meters and for the second one 7.0 to 12.0. Distributions of values within these intervals were considered continuous. There was no constrains for objective function.

Settings for Simulated annealing was according to next table. Stop condition was reaching maximum number of iterations.

Tab. 1: Simulated annealing settings

parameter	T_{max}	T_{min}	$count_{max}$	$succ_{max}$	$iter_{max}$
value	*5000*	*50*	*100*	*10*	*5000*

Simulated annealing in script was compared with Nelder-Mead optimization method that is suitable for continuous problems and Modified simulated optimization. Both of new mentioned methods are parts of the Engineer Optimization Toolbox (EOT) used for optimization of static problems (Šedlbauer et al., 2012). Results (optima) together with description of optimization courses are summarized in following table.

Tab. 2: Results of optimization for verification

	X1 [mm]	***X2 [mm]***	***optimum [Nm]***	***opt. time [s]***	***average sol. time [s]***	***recalculation number***
EOT – N-M	*4742*	*9813*	*8799*	*752*	*4,6*	*165*
EOT – MSA	*4750*	*9825*	*8852*	*7788*	*6,2*	*1250*
script - SA	*4692*	*9814*	*8806*	*26124*	*5,2*	*5000*

It is obvious that proposed script is quite functional. Simulated annealing achieved similar optimum as already proven attitudes. Little disadvantage for this example could be longer total optimization time. This phenomenon was expected due to the nature of method and its settings. Graphic optimization result is shown in Fig. 4. It should be noted that script was verified on more examples. This example was chosen for its clarity and simplicity.

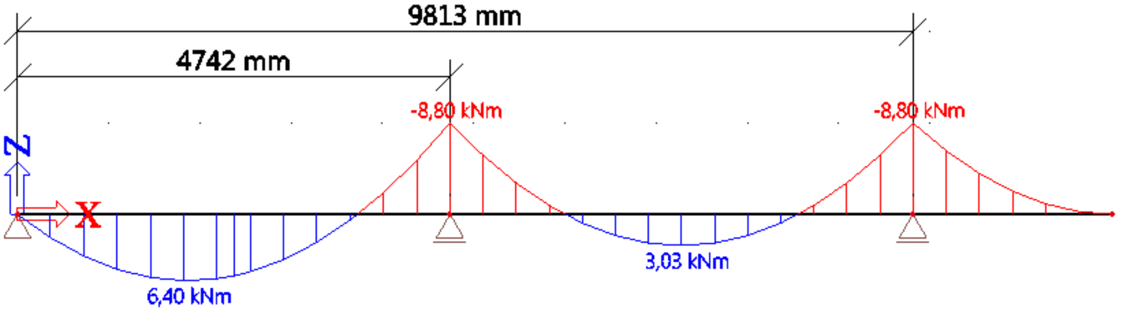


Fig. 4: Graphic result of optimization for verification

4. Optimization dynamically loaded structures

In this section will be presented and solved two optimization problems. The first one is single cantilever column with head exposed to two different harmonic loads. This problem represents optimization of statically determinate reinforced concrete structure. The second one is statically indeterminate steel frame also with harmonic forces.

4.1. Reinforced concrete column

First optimized structure is column made of reinforced concrete. Column is exposed to four load cases: 1) Self weight, 2) Dead load, 3) Dynamic X, 4) Dynamic Y. On Fig. 5 there is static schema and gradually sorted load cases except self weight.

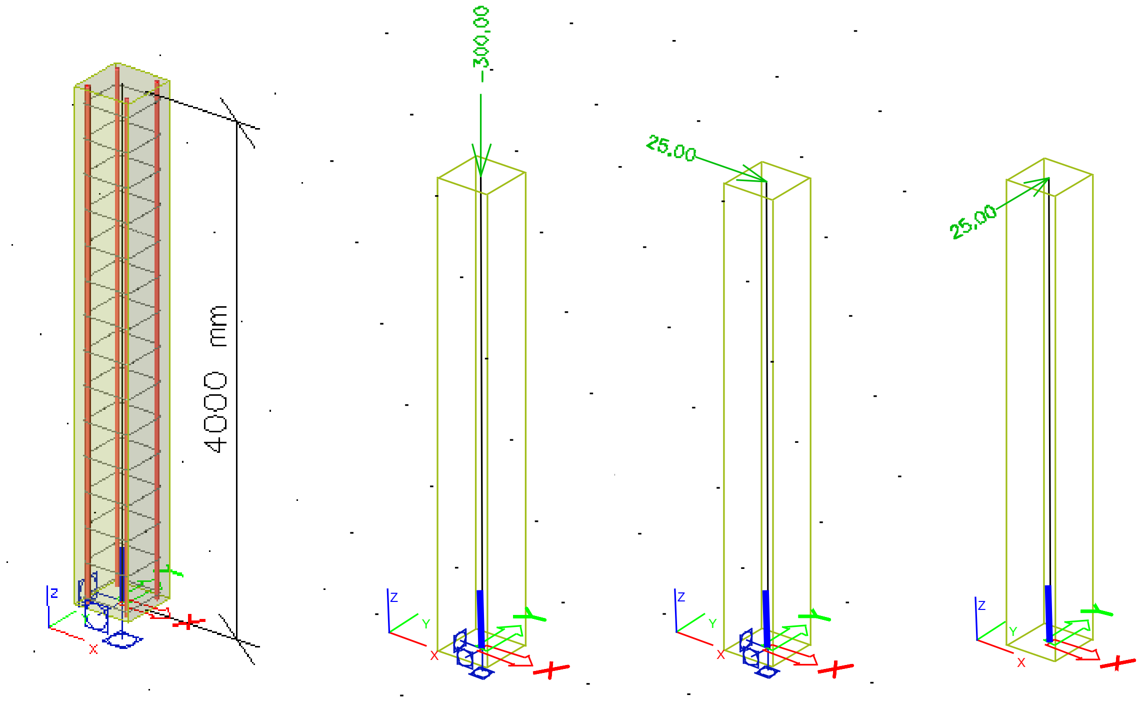
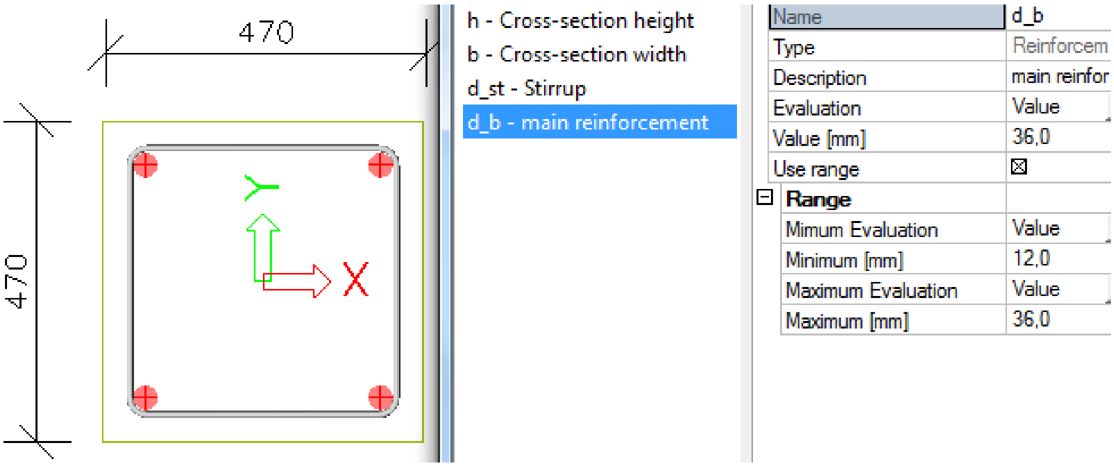


Fig. 5: Static schema and loads on column

Harmonic force in the third load case has frequency 1.0 s^{-1} and amplitude 25*kN*. Parameters of harmonic force in Y direction are: frequency 20 s^{-1} and amplitude 25*kN*. Frequency of forces is not randomly chosen. These frequencies are close to the first two Eigen frequencies of original structure. This concludes to unacceptable state. So the goal is to design column cross-section with reinforcement to meet Eurocode requirements at minimum costs. Eurocode requirements include Ultimate limit state and Serviceability limit state both with maximal unit check = 1.

Main model materials are concrete C25/30 and steel bars B500B. Column cross-section is rectangular and concrete bars are situated in section corners. There are four parameters implemented into model, two for section and two represents reinforcement diameters, Fig. 6 and Tab.3.



h - Cross-section height
b - Cross-section width
d_st - Stirrup
d_b - main reinforcement

Name	d_b
Type	Reinforcem
Description	main reinfor
Evaluation	Value
Value [mm]	36,0
Use range	☒
Range	
Mimum Evaluation	Value
Minimum [mm]	12,0
Maximum Evaluation	Value
Maximum [mm]	36,0

Fig. 6: Parameters for column optimization

Tab. 3: Parameters and limits for column optimization

parameter	***h [mm]***	***b [mm]***	***d_{st} [mm]***	***d_b [mm]***
initial value	*470*	*470*	*36*	*10*
minimum	*250*	*250*	*12*	*6*
maximum	*500*	*500*	*36*	*12*

Settings of optimization method Simulated annealing was a little different from settings for verification, see Tab. 4.

Tab. 4: Simulated annealing settings for column optimization

parameter	***T_{max}***	***T_{min}***	***$count_{max}$***	***$succ_{max}$***	***$iter_{max}$***
value	*2000*	*20*	*100*	*10*	*2000*

Objective function is composed of costs of main materials (concrete, steel bars) and formwork, formula 4. Unit costs were defined as average of Czech market costs.

$$C = UC_{con} \cdot V_{con} + UC_{steel} \cdot m_{steel} + UC_{form} \cdot A_{form} \tag{4}$$

Where C are total costs, UC_{con} = cost of 1m^3 of concrete = 1700 CZK, V_{con} = volume of concrete, UC_{steel} = cost of 1kg of reinforcement = 17 CZK, m_{steel} = mass of steel bars, UC_{form} = hire for 1m^2 of formwork = 100 CZK/for 10 days, A_{form} = area of formwork**.**

4.1.1. Results for reinforced concrete column

As has already been said first performed recalculation during optimization is modal analysis. Based on these results that are out of harmonic load frequency (± 0.5 s^{-1}), linear analysis and assessment is carried out. Therefore following graphs and tables are according to results that satisfy above constraints.

Found optimum of given problem in comparison with initial solution is approximately 18 percent cheaper. Optimum parameter values are listed in Tab. 5. Due to settings of operator (no.6 in Simulated annealing algorithm) that ensures practical changes of parameters (*mm* or *dm*), we got reasonable optimum value in terms of practice. It is obvious that all parameters play role both in determining objective function and in fulfillment of code conditions. None of them had reached its boundary value.

Tab. 5: Comparison of initial and optimal parameter values

parameter	***h [mm]***	***b [mm]***	***d_{st} [mm]***	***d_b [mm]***	***Costs [CZK]***
initial	*470*	*470*	*36*	*10*	*4555*
optimal	*460*	*400*	*32*	*8*	*3802*

Data from previous table clearly demonstrates the importance of dynamic loads. Due to ratio of Eigen frequencies and force frequencies column is stressed more in one direction, which explains final rectangular cross-section. Cost ratios of components are: concrete: steel: formwork = 3:5:2.

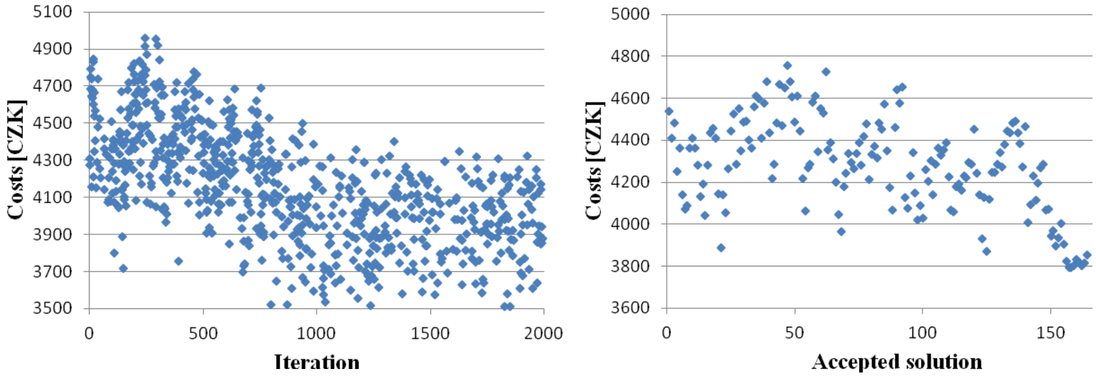


Fig. 7: Tested solutions and accepted solutions during optimization

Fig. 7 shows graphs describing optimization with Simulated annealing. The first chart displays all tested solutions. As we can see in later part of optimization process method examines farther solutions to avoid local extremes. The second graph shows convergence of adopted solutions to final optimum. Figures also indicates margin in optimization method settings where there would be more iterations and changes of temperature limits.

4.2. Steel frame

Another example is optimization of steel frame structure that is shown on Fig. 8. Frame is supported by hinges, which make this structure statically indeterminate. This makes the task considerably more difficult that in the first case. Redistribution of internal forces depends on stiffness of components and therefore more of their dimensions. In this optimization example objective function consists of total structure mass, since steel is homogeneous material from macro point of view.

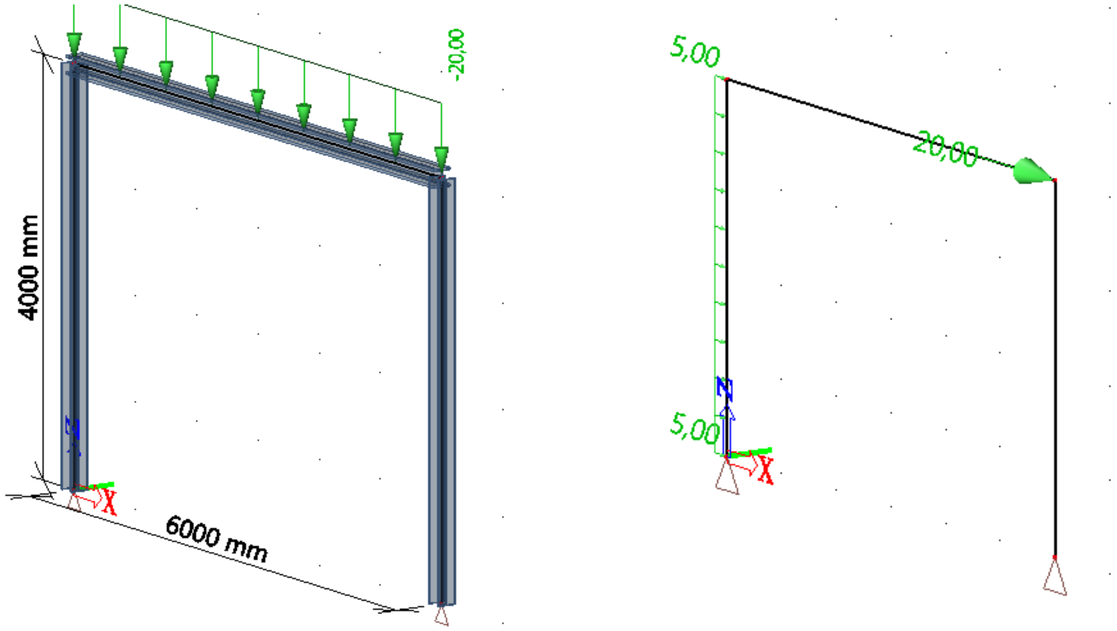


Fig. 8: Static schema and loads on frame

Frame is loaded with automatically generated self weight, live load (20kN/m), wind (5kN/m) and dynamic load including harmonic force with frequency *20s*$^{-1}$ and amplitude 20 kN, Fig. 8.

Structure consists of two different cross-sections. One is for columns and other is for vertical beam. Cross-sections are considered as welded I profiles. Each of these cross-sections is defined by four parameters – cross-section height, width of flanges, thickness of flanges and web thickness. Initial values of these parameters as well as their limits for optimization process are shown in Tab. 6.

Tab. 6: Parameters and limits for frame optimization

BEAM

parameter	***H_1 [mm]***	***B_1 [mm]***	***T_{f1} [mm]***	***T_{w1} [mm]***
ini. value	*200*	*120*	*15*	*10*
minimum	*120*	*80*	*10*	*8*
maximum	*250*	*170*	*20*	*20*

COLUMNS

parametr	***H_2 [mm]***	***B_2 [mm]***	***T_{f2} [mm]***	***T_{w2} [mm]***
ini. value	*200*	*160*	*20*	*15*
minimum	*150*	*100*	*15*	*12*
maximum	*300*	*250*	*30*	*25*

There is again an effort to design structures in accordance with European standards – Eurocodes. Extreme values for unit checks were set for illustration to 0.9 for ULS and 1.0 for SLS. Settings of optimization method Simulated annealing are similar to previous example. One of long-term goals of our work is to provide optimization duration up over night and make full use of computers out of office hours. Settings shown in Tab. 4 ensures above mentioned duration for this type of optimization tasks.

4.1.1. Results for steel frame

Final optimum was reached at time of 29160*s*. This time slightly exceeds required "night" time of eight hours. It is worth noting that final weight of structures differs from the original by about ten percent, Tab. 7-8**.** Great savings are mainly a matter of beam. Not higher saving is due to relatively reasonable previous design and due the fact that smaller number of iterations was selected.

Tab. 7: Initial solution for steel frame

	H [mm]	***B [mm]***	***T_f [mm]***	***T_w [mm]***	***m [kg]***
beam	*200*	*120*	*15*	*10*	*967*
columns	*200*	*160*	*20*	*15*	

Tab. 8: Final optimum for steel frame

	H [mm]	***B [mm]***	***T_f [mm]***	***T_w [mm]***	***m [kg]***
beam	*135*	*105*	*12*	*16*	*882*
columns	*193*	*195*	*15*	*14*	

Results of each optimization should be soberly commented and evaluated. There is always a collision of mathematical and real word. This is confirmed by the results of frame optimization. It is hard to imagine that beam with bending moment has the web thickness greater than flange thickness. Therefore it is desirable to save interim results and do the post-processing. Or it is possible to add at the beginning of optimization other constraints or conditions that would consider parameters ratio. Considering above mentioned user-selected optimum could have following form.

Tab. 9: Optimum selected by user

	***H* [mm]**	***B* [mm]**	***T_f* [mm]**	***T_w* [mm]**	***m* [kg]**
beam	*240*	*90*	*12*	*9*	*915*
columns	*200*	*150*	*18*	*18*	

Let us present other options that offer results of optimization by presented script. It is often required in addition to parameters values to determine relation to the objective function. This can be designated as post-processed sensitivity analysis. Graphs showing dependence of parameters on objective function are on Fig. 9. For example (as shown in the top two graphs for beam) flange thickness parameter is decisive for the objective function. This relation is expected and among other indicates correctness of proposed script.

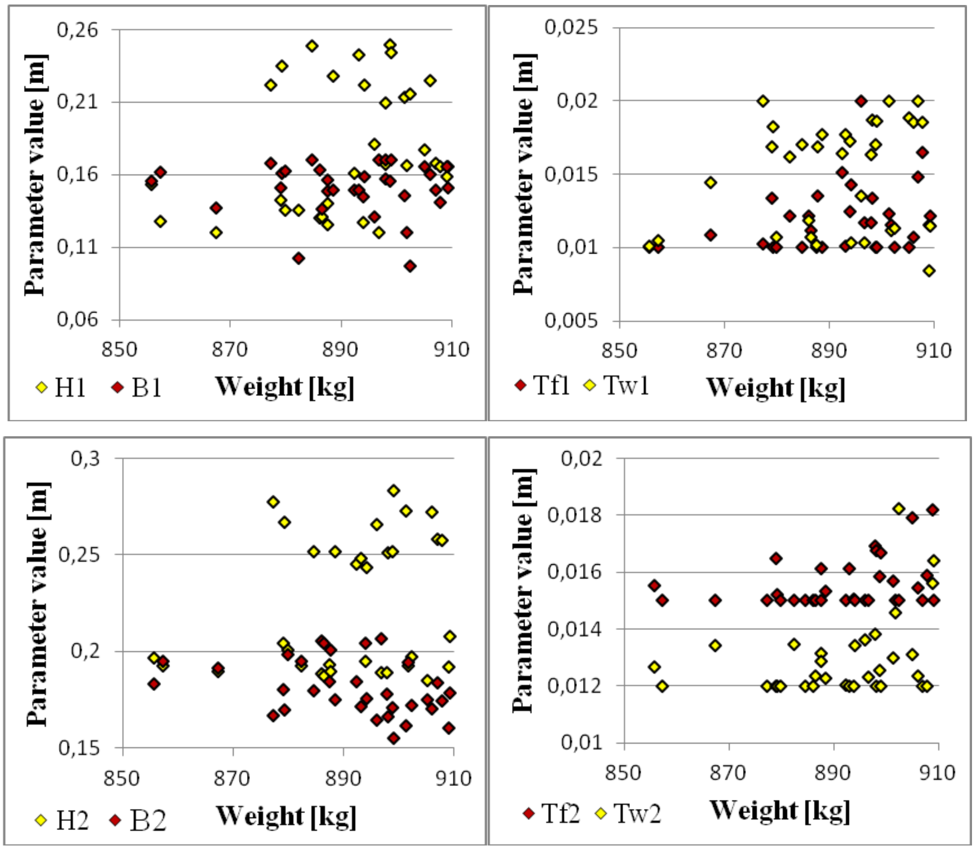


Fig. 9: Dependence of parameters on objective function for best solution

5. Conclusion

Possible solution of optimization problems with dynamic phenomena (especially harmonic forces) was introduced. Optimization is based on cooperation between widely extended static software including FEM solver and new script in program Matlab. Main tasks of new script are to get data from software for structural analysis, change parameters in model according to the optimization algorithm and ensure recalculation. An important role in this process play XML files, which are mediators of communication between new script and software for structural analysis.

Simulated annealing was used as an optimization algorithm located in the body of the script. For this method exist mathematical proofs of convergence and to its robustness is suitable for wide range of difficult tasks involving a large number of parameters or discrete functions. However, there is a disadvantage – time consumption.

Above mentioned optimization process was used for optimization of two examples involving harmonic loads. The first was optimization of cantilever column made of reinforced concrete. Second one was optimization of hinged steel frame. There were around fifteen percent savings of main materials in both examples compared to initial design.

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