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NUMERICAL SIMULATION OF TRANSONIC FLOW OF CONDENSING STEAM

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Abstract: *The paper is aimed at a modeling of transonic flow of steam with pressure and temperature range corresponding to conditions in steam turbines. The flow model is based on the model published in (Šťastný and Šejna, 1995). A possibility of droplet size spectra reconstruction are discussed. Numerical results are compared to experimental data for nozzle flow.*

Keywords: ***Wet steam, nucleation, droplet size distribution, condensation***

1. Introduction

Steam during expansion usually goes from dry to wet region. This expansion is very fast in steam turbines, therefore the condensation of initially dry steam starts later, when the steam temperature drops sufficiently below the saturation temperature. The state of steam, when the temperature is below the saturation temperature and before the start of condensation is called metastable state (steam is not in the thermodynamic equilibrium). The mass exchange between the gas and the liquid phase has different mechanisms. One of them is nucleation, which means nearly instant creation of tiny droplets dispersed in gas phase. New droplet forms either from a cluster of water molecules (homogeneous nucleation) or it is created on some impurity particle (heterogenous nucleation). Existing droplets grow or evaporate according to the state of surrounding vapor. Another important phenomena is a film condensation, which is not in the focus of present paper. The flow of mixture composed of vapor and droplets is in reality a very complex two-phase flow. Droplets appearing due to homogenous nucleation have sub-micron size and one can consider, that they are convected by the vapor. Condensation phenomena should not be neglected in the simulations of flow in turbines, since condensation reduces thermal efficiency of turbine, it changes significantly shock wave structure in the transonic flow field and it can initiate pressure pulsations.

Research of non-equilibrium condensation is demanding, experiments as well as numerical simulations use certain level of simplifications. Experiments are often based on indirect methods like light scattering method. Numerical simulations usually use simplified equation of state for the vapor and they approximate, sometimes very roughly, the droplet size spectra. Despite these facts simulations together with experiments provide helpful insight in condensing flow phenomena and help to improve steam turbines design.

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Numerical simulations of condensing steam flow started three decades ago, the first simulations (Bakhtar and Tochai Mohammadi, 1980), (Bakhtar and Alubaidy, 1984), (Moheban and Young, 1984) or (Young, 1992) were based on the solution of flow field in the Eulerian frame (fixed grid) and condensation was simulated as one-dimensional problem along streamlines in the Lagrangian frame, where the streamlines were obtained from the velocity field from the Eulerian frame. The main disadvantage was the need to recompute streamlines in each iteration of time-marching algorithm. Recent works are based mainly on full Eulerian approach, i.e. all transport equations for the mass, momentum and energy of the mixture and the supplementary transport equations for the parameters of liquid phase are solved on a fixed grid. Some authors approximate wet steam by a monodispersed mixture e.g. (Heiler, 1999), (Dykas, 2001), (Dykas et al, 2003), (Gerber and Kermani, 2004) or (Sun et al, 2007), another authors, e.g. (Šejna and Lain, 1994), (Mousavi et al, 2006) or (John et al, 2007), use models based on the method of moment originally published in (Hill, 1966), which is able to recognize the polydispersity of mixture.

2. Flow model

Presented flow model is based on the model published in (Šejna and Lain, 1994). The velocity of droplets is considered equal to the velocity of vapor. The model is based on the transport equations for mass, momentum and energy of the mixture, mass fraction of liquid phase and moments for liquid phase. Mass exchange between vapor and liquid is modeled by homogeneous nucleation and droplet growth. Let us denote the three moments according to (Hill, 1966) as

$$Q_0 = N, \qquad Q_1 = \sum_{i=1}^{N} r_i, \qquad Q_2 = \sum_{i=1}^{N} r_i,^2 \tag{1}$$

where N gives the total number of droplets in unit mass and r_i is the radius of $i-th$ droplet. The average droplet radius is taken as $r_{20} = \sqrt{Q_2/Q_0}$ according to (Šejna and Lain, 1994). When the wetness χ is below chosen minimum χ_{min} we set $r_{20} = 0$ to avoid numerical errors. The system of all transport equations for the case of one-dimensional flow with variable cross-sectional area $A(x)$ reads

$$\frac{\partial}{\partial t}\big(A(x)\mathbf{W}\big) + \frac{\partial}{\partial x}\big(A(x)\mathbf{F}(\mathbf{W})\big) = \mathbf{P}(\mathbf{W}) + A(x)\mathbf{Q}(\mathbf{W}), \tag{2}$$

where

$$\mathbf{W} = \begin{bmatrix} \rho \\ \rho u \\ e \\ \rho\chi \\ \rho Q_2 \\ \rho Q_1 \\ \rho Q_0 \end{bmatrix}, \ \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ (e+p)u \\ \rho\chi u \\ \rho Q_2 u \\ \rho Q_1 u \\ \rho Q_0 u \end{bmatrix}, \ \mathbf{P} = \begin{bmatrix} 0 \\ pA'(x) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{Q} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{4}{3}\pi r_c^3 J\rho_l + 4\pi\rho\mathcal{M}_2\rho_l \\ r_c^2 J + 2\rho\mathcal{M}_1 \\ r_c J + \rho\mathcal{M}_0 \\ J \end{bmatrix},$$

$$\mathcal{M}_n = \int_0^\infty r^n N(r)\dot{r}(r)dr.$$

The symbol ρ denotes the mixture density, u the mixture velocity, e the total energy of mixture per unit volume, p the pressure of the mixture, χ the mass fraction of liquid phase (wetness).

The symbol J is used for the number of new droplets due to homogenous nucleation in unit volume per one second. The radius of these droplets is equal to the critical radius r_c. The variable ρ_l denotes the density of liquid phase. The integral $\int_{r_1}^{r_2} N(r)dr$ equals to the number of droplets with the radius $r \in < r_1, r_2 >$ in the unit mass of mixture. The function $\dot{r}(r)$ describes the droplet growth speed. The system (2) is closed by additional equations (equation of state, material properties), for detail see Appendix.. The equation for pressure under the perfect gas assumption reads

$$p = \frac{(\gamma - 1)(1 - \chi)}{1 + \chi(\gamma - 1)} \left[e - \frac{1}{2}\rho u^2 + \rho\chi L \right], \tag{3}$$

where the pressure p is considered the same for both vapor and liquid and L denotes the latent heat of condensation/evaporation. The specific heat ratio is taken as a function of temperature $\gamma(T) = c_p(T)/(c_p(T) - R_v)$, where R_v denotes the gas constant for vapor, $c_p(T)$ the specific heat at constant pressure and T the temperature of vapor. The system (2) turns into the common single-phase flow model if $\chi = Q_0 = Q_1 = Q_2 = 0$. The original model considers the droplet growth speed, which depends on average radius $\overline{r}$, i.e

$$\mathcal{M}_n = \int_0^\infty r^n N(r)\dot{r}(r_{20})dr = \dot{r}(r_{20}) \int_0^\infty r^n N(r)dr = \dot{r}(r_{20})Q_i. \tag{4}$$

This model is further denoted as AVG-P model. The original model has been modified in order to model droplet spectra by apriori given distribution represented by moments Q_i. The model, which is further denoted as DSDF-P model, considers the log-normal distribution defined by the moments Q_0, Q_1 and Q_2 according to (John et al, 2007)

$$N(r) = Q_0 \frac{1}{r \ln(\sigma_g)\sqrt{2\pi}} \exp\left(-\frac{\ln^2(r/\overline{r_g})}{2\ln^2(\sigma_g)} \right), \tag{5}$$

where

$$\overline{r_g} = \frac{r_{20}}{\exp(0.5\ln^2(\sigma_g))}, \qquad \sigma_g = \exp\sqrt{\ln(c_v^2 + 1)},$$
$$r_{20} = \sqrt{\frac{Q_2}{Q_0}}, \qquad c_v = \sqrt{\frac{Q_0 Q_2}{Q_1^2} - 1}. \tag{6}$$

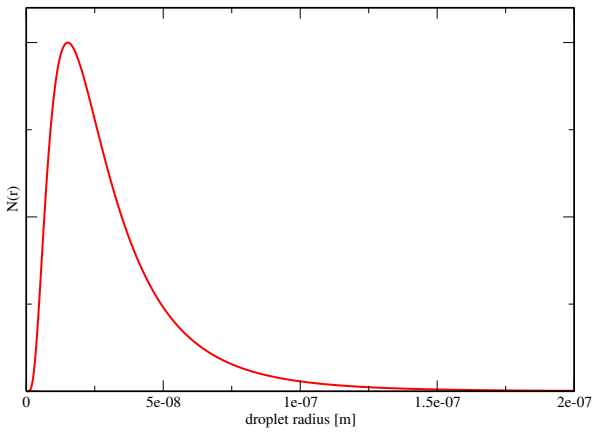

Figure 1: Example of log-normal droplet size distribution.

3. Numerical solution

Consider a convergent-divergent nozzle, see the Fig. 2.

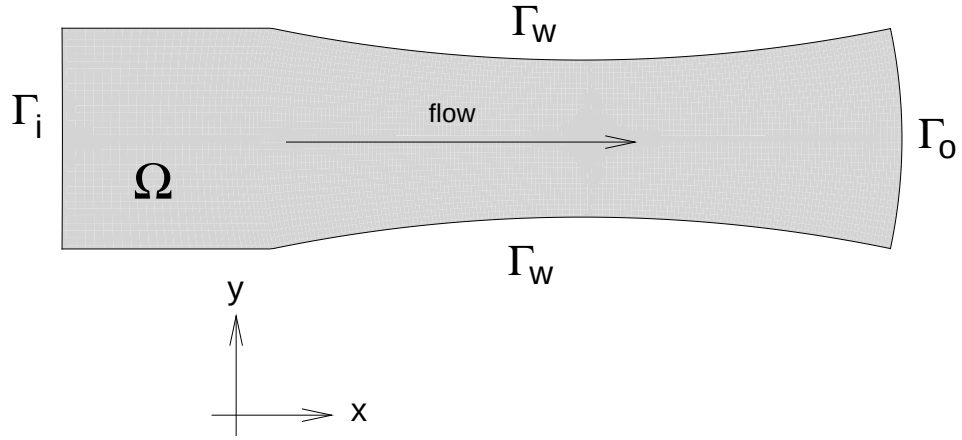

Figure 2: Solution domain - nozzle.

We consider subsonic flow at the inlet boundary Γ_i . Therefore six parameters have to be set according to the 1D theory of characteristics of linearized problem. We prescribe a constant values of total temperature T_0, total pressure p_0, wetness χ and moments Q_0, Q_1 and Q_2. The flow is supersonic at the outlet boundary Γ_o, therefore according to the 1D theory of characteristics for linearized problem we must not prescribe any condition.

The numerical method has to cover very different time scales of convection, nucleation and droplet growth. Numerical method is therefore based on the splitting method of (Strang, 1968) in order to deal with different time scales of convection and condensation. The splitting method also allows to use different time integration for individual phenomena. The splitting method means the successive solution of following three problems

$$\frac{\partial}{\partial t}\mathbf{W}^{*} = \mathbf{Q}(\mathbf{W}^{*}) \tag{7}$$

$$\frac{\partial}{\partial t}\big(A(x)\mathbf{W}^{**}\big) = -\frac{\partial}{\partial x}\big(A(x)\mathbf{F}(\mathbf{W}^{**})\big) + \mathbf{P}(\mathbf{W}^{**}) \tag{8}$$

$$\frac{\partial}{\partial t}\mathbf{W}^{***} = \mathbf{Q}(\mathbf{W}^{***}), \tag{9}$$

where (7) is solved with initial data $\mathbf{W}^{*}(t) = \mathbf{W}(t)$, the (8) with initial data $\mathbf{W}^{**}(t) = \mathbf{W}^{*}(t + \Delta t/2)$ and (9) with initial data $\mathbf{W}^{***}(t) = \mathbf{W}^{**}(t + \Delta t)$. The value $\mathbf{W}^{***}(t + \Delta t/2)$ approximates the solution $\mathbf{W}(t + \Delta t)$ of the original system (2). The single step of the Lax-Wendroff finite volume method is applied for the solution of (8) and several steps of the Runge-Kutta method is used to solve (7) and (9).

4. Numerical results

We consider the geometry of the Barschdorff nozzle (Barschdorff, 1971), see the Fig. 2. The domain has been discretized using 200 cells with the grid spacing equal to $1.4 \cdot 10^{-3}$ in the convergent-divergent part. The inlet total pressure is $78390\ Pa$, the inlet total temperature is

373.1 K and the steam does not contain the liquid phase at the inlet (i.e. $Q_0 = Q_1 = Q_2 = \chi = 0$). Following Figures 4 - 6 show the graphs of pressure, temperature, wetness and average radius r_{20} along the nozzle axis for both AVG-P flow model (droplet growth computed using the average r_{20} only) and DSDF-P flow model (droplet growth computed using log-normal distribution of droplet size). The Figure 7 shows how many droplets appears due to homogenous nucleation in each grid point and the Figure 8 shows the size (critical radius) of these new droplets. It is clear, that nucleation takes place in a small region. Nucleation almost cancels the thermodynamic non-equilibrium, see the difference between wetness and the equilibrium wetness in the Figure 5. Nucleation stops around $x = 0.04$ and the condensation follows more-less the equilibrium case from that point. The nucleation releases a non-negligible amount of the latent heat in a relatively small region, it yields the so called nucleation shock, see the pressure rise around $x = 0.03$ in the Figure 3. Both models yield nearly the same results for pressure, temperature and wetness distributions. However they differ significantly in the composition of the droplet spectra. The DSDF-P model gives smaller total amount of droplets, see the Figure 7 or Q_0 at the nozzle outlet in the Table 1. The DSDF-P model also yields slightly higher wetness, see the Table 1, so the DSDF-P model predicts 'bigger droplets'. The graph of average radius r_{20} in the Figure 6 can be somehow confusing, since the DSDF-P model assumes bigger range of droplet sizes and the average radius r_{20} is not a good parameter to compare DSDF-P and AVG-P models from the point of the average droplet size, see comparison of different droplet size averages in the table 2, where r_{10} is the linear average radius, r_{20} is the surface average radius, r_{30} is the volume average radius and the r_{32} is the Sauter mean radius.

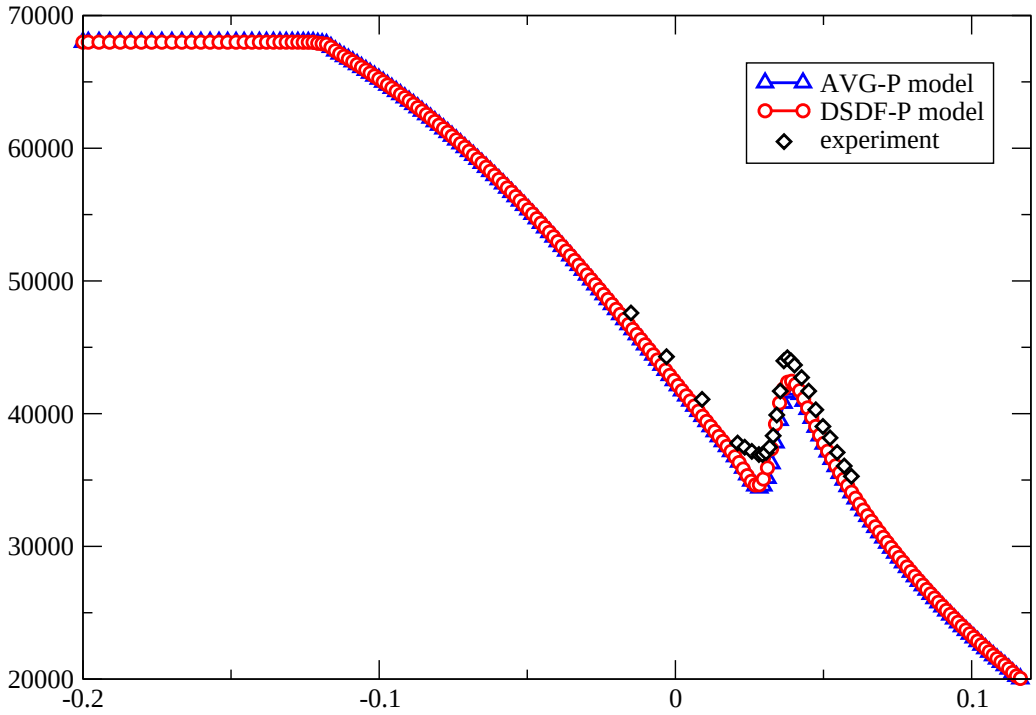

Figure 3: Pressure along nozzle axis, experimental data from (Barschdorff, 1971).

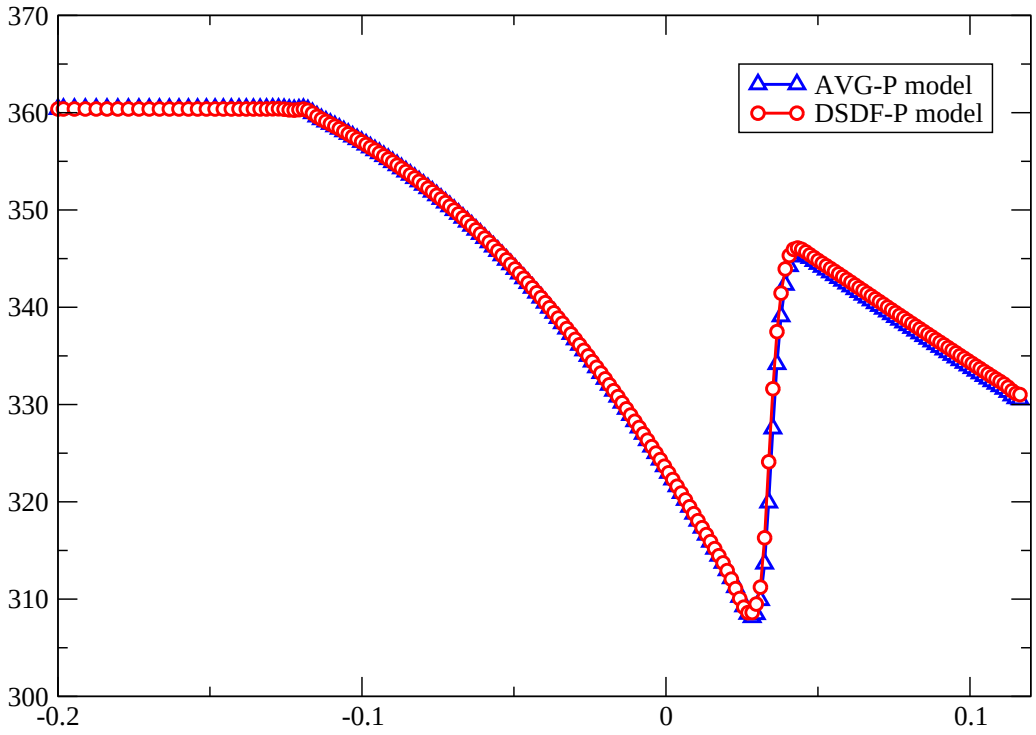

Figure 4: Temperature along nozzle axis.

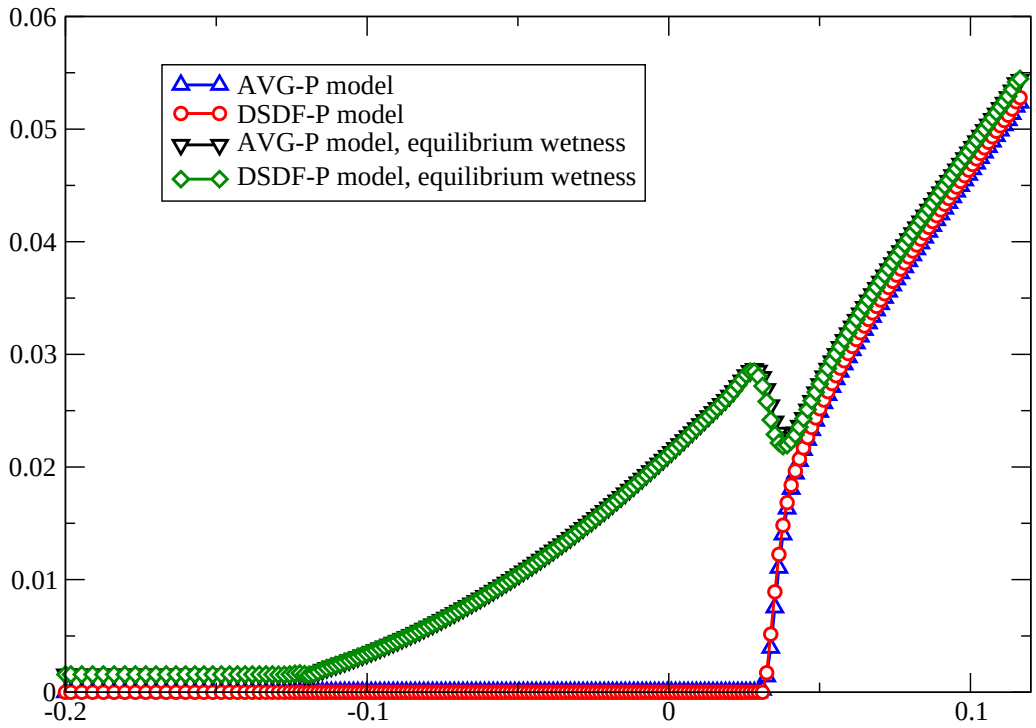

Figure 5: Wetness along nozzle axis.

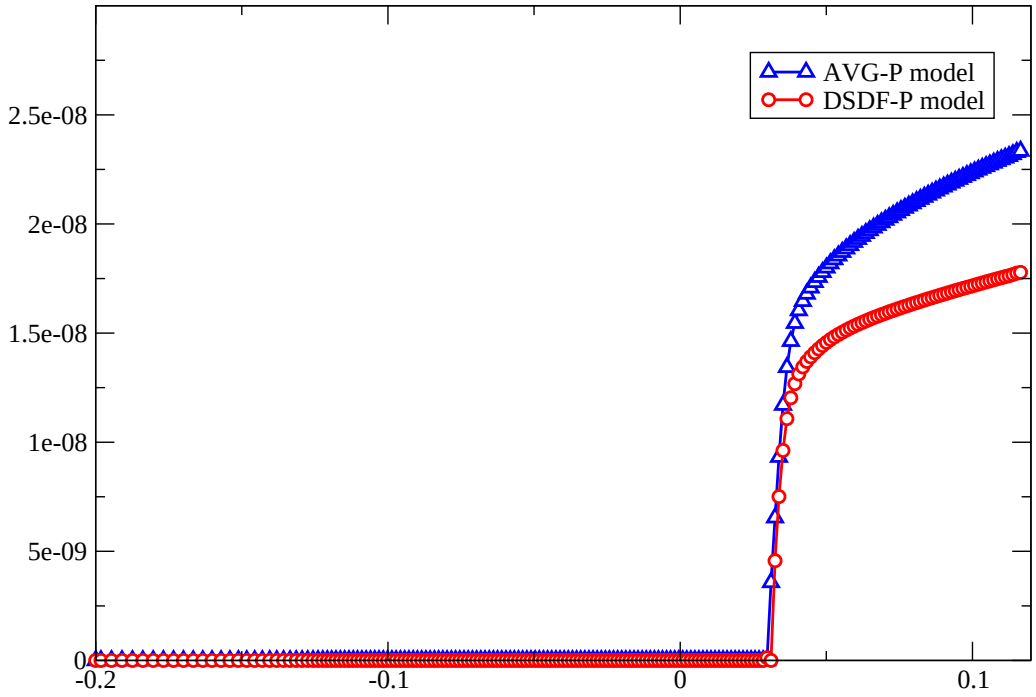

Figure 6: Average radius $r_{20} = \sqrt{Q_2/Q_0}$ along nozzle axis.

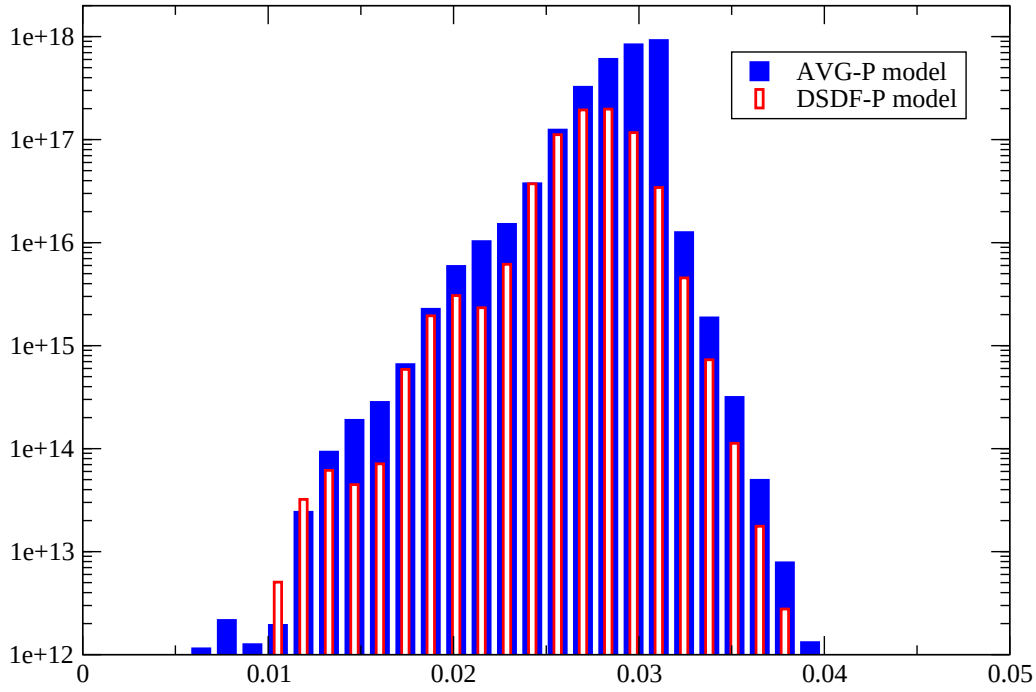

Figure 7: Increments of number of droplets $Q_0(x_i) - Q_0(x_{i-1})$ within the nucleation zone in grid points x_i.

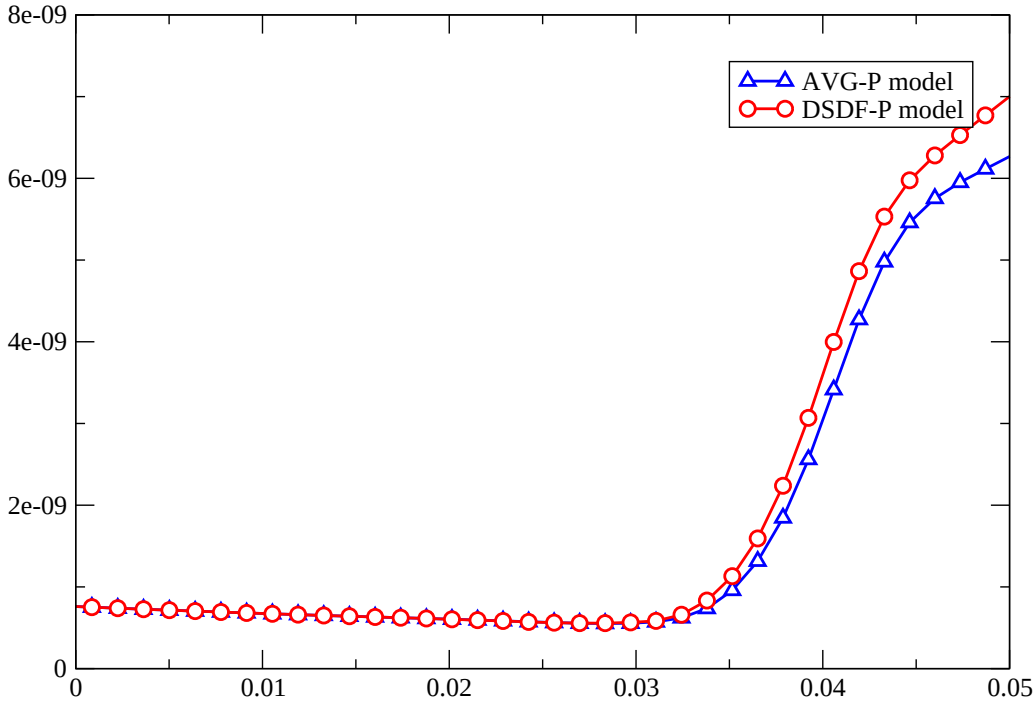

Figure 8: Critical radius r_c within the nucleation zone.

Table 1. parameters of liquid phase at the nozzle outlet

	Q_0	Q_1	Q_2	χ
AVG-P model	$9.45 \cdot 10^{17}$	$2.16 \cdot 10^{10}$	515.28	0.0523
DSDF-P model	$7.12 \cdot 10^{17}$	$0.72 \cdot 10^{10}$	225.16	0.0527

Table 2. different average radii of droplets at the nozzle outlet

	$r_{10} = Q_1/Q_0$	$r_{20} = \sqrt{Q_2/Q_0}$	$r_{30} = \sqrt[3]{Q_3/Q_0}$	$r_{32} = r_{30}^3/r_{20}^2$
AVG-P model	$2.28 \cdot 10^{-8}$	$2.33 \cdot 10^{-8}$	$2.36 \cdot 10^{-8}$	$2.42 \cdot 10^{-8}$
DSDF-P model	$1.01 \cdot 10^{-8}$	$1.77 \cdot 10^{-8}$	$2.61 \cdot 10^{-8}$	$5.67 \cdot 10^{-8}$

5. Conclusions

Presented results show, that the model DSDF-P could be a good option for more detailed modeling of droplet size spectra. The results also show, that one has to be very careful when computing the average size of droplet. The original model AVG-P has the average size of droplets nearly independent of definition, it means that AVG-P model yields nearly mono-dispersed mixture. The average size of droplet for the DSDF-P model strongly depends on the used definition.

6. Acknowledgment

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8. Appendix

Homogenous nucleation rate is computed using a formula of (Becker and Döring, 1935)

$$J = \sqrt{\frac{2\sigma}{\pi m_v^3}} \cdot \frac{\rho_v^2}{\rho_l} \cdot \exp\left(-\beta \cdot \frac{4\pi r_c^2 \sigma}{3 k_B T}\right), \tag{10}$$

where $\sigma(T)$ is the surface tension of water and β is correction parameter from (Petr and Kolovratník, 2001)

$$\beta = 1.328 p_{cor}^{0.3}, \quad p_{cor} = p_{saturated}(s_{01}) 10^{-5}, \tag{11}$$

where s_{01} is the total inlet entropy and p_{cor} is considered in $[bar]$. The critical radius is computed according to

$$r_c = \frac{2\sigma}{L\rho_l \ln(T_s/T)}, \tag{12}$$

where $L(T)$ is the latent heat of condensation/evaporation and $\rho_l(T)$ denotes the density of water. The saturation temperature T_s is evaluated using IAPWS-IF97 function. The droplet growth speed is given by the Gyarmathy's formula

$$\dot{r}(r) = \frac{\lambda_v(T_s - T)}{L\rho_l(1 + 3.18 \cdot Kn)} \cdot \frac{r - r_c}{r^2}, \qquad Kn = \frac{\eta_v \cdot \sqrt{2\pi R_v T}}{4rp}, \tag{13}$$

where vapor thermal conductivity λ_v and vapor viscosity η_v are functions of temperature and r is the droplet radius.