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SINGLE-OBJECTIVE SIZING AND TOPOLOGY OPTIMIZATION OF CABLED-TRUSSES USING GENETIC ALGORITHMS

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Summary: *This paper demonstrates the application of genetic algorithms to design cabled-truss structures with minimum weight. These structures can be described as a system of cables and triangular units constructed with straight bars connected at their ends by hinged connections to form a rigid framework. Optimized lightweight structures are determined through a discrete topology and sizing optimization process which is based on ground structure approach and genetic algorithms. Structural static response is computed using nonlinear finite element iterative procedure. Simulations are presented showing comparisons between obtained cabled-trusses with truss benchmarks. In addition, simulation results highlight the potential benefits of using cables for improving truss structural performance.*

Keywords: ***Cabled-trusses, trusses, structural optimization, genetic algorithms, ground structure approach.***

1. Introduction

The reduction of structural mass has been always desired in engineering structures, e.g., bridges, machines, cars, trains, among others (Ahmeti, 2010). Traditionally, the notion of producing a lightweight and high strength structure was often attained through the optimization of basic structural types, e.g., multi-arch structures, frames, trusses, grid shells, etc. In order to improve the trade-offs between mass and rigidity that stems from traditional approaches, modern lightweight approaches often comprise the combination of different structural members. Most of these approaches take into account tensile members since, from a simple strength of materials standpoint, the most efficient material use is considered to be achieved when tensioned members are employed. This consideration takes into account that all the material is equally stressed when the member is uniformly tensioned (Whitman, 2005). In particular, fabrics and/or cables have been used to form structures that radically depart from traditional constructions.

Despite their benefits, modern lightweight structures are not often applicable in mechanical engineering fields where structural members stresses can alternate between tension and compression during operation. In such applications, trusses have proven the most successful

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lightweight structures, once that all its members can resist tension and compression. In this sense, this work investigates the optimization of a novel lightweight structure, namely cabled-truss, which combines truss structures with pretensioned cables.

The present work proposes a optimization framework (OF) for topology and sizing optimization of cabled-trusses. The developed framework is based on ground structure approach, nonlinear finite element analysis and genetic algorithm. Minimal weight design is performed to ground structures with 15 and 66 members in order to investigate the influence of cables in truss structural performance. The results indicate that cabled-trusses have shown a significant improvement in structural mass minimization when compared with trusses.

2. Cabled-trusses

In lightweight structures research, trusses have attracted tremendous interest since their members are arranged to transfer external loads through axial forces rather than bending. If buckling does not occur, the cross sectional area of each truss member is equally stressed, therefore the material is used efficiently and lightweight designs are obtained.

With the fast development of tension structures, tensile elements, such as, cables and membranes, have been combined with truss structures to form efficient hybrid systems (Ando et al., 2000; Bosch, 1990). Among tensile members, cables are the most used because tensile stresses are distributed uniformly over the cross-sectional areas of members. Due to their flexibility, cables have negligible bending stiffness and can develop tension only. Thus, under external loads, a cable will develop the shape that is necessary to support the load by tensile forces alone.

The combination of trusses and cables is often performed in such a manner that cables are extrinsically parts forming sharp angles with the truss structure. Common applications of this approach are found in civil engineering, e.g., cable-stayed towers, cable-stayed truss bridges and roofs (Fig. 1). In these applications, the use of extrinsic positioned cables allows for changing the directions of the main loads and reinforce the truss structure.

Despite of the advantages offered by the extrinsic approach, it has limited applications in engineering fields where the structure must attend rigorous space requirement, e.g., automotive, robotics, aerospace, naval, etc. Usually, in these applications, the structure must be designed within a bounded design space, which is usually fulfilled by the truss structure; consequently, there is not enough space for staying cables.

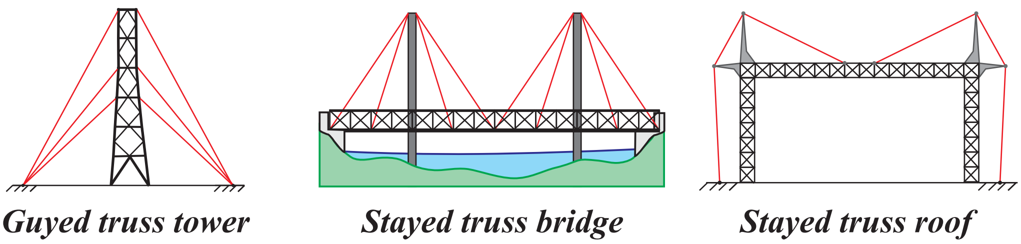

Fig. 1: Extrinsic cables combined truss structures.

Cables are able to withstand tension forces only; besides, they pre-stress the truss system in order to redistribute mass and stresses along the structure. Cabled-trusses are formed by cables and triangular bar formations jointed at their ends by hinged connections to form a rigid framework. Applied loads are assumed to be located at joints only; therefore cabled-truss members either elongate or shorten and material is thereby efficiently employed.

Cabled-trusses, illustrated in Fig. 2, can be classified as a cabled structure and/or a truss-like structure. However, differently from most of the cabled structures, the stability of cable-trusses relies on the triangular bar formations.

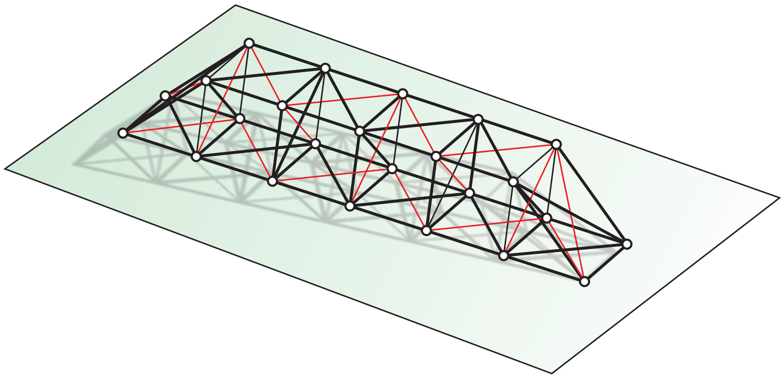

Fig. 2: Cabled-truss structure.

Contrary to most cabled structures, cabled-trusses present a high potential in mechanical applications where forces can rapidly change over time depending on the aimed movement. These applications are significantly different from those from civil engineering, in which the main loads are not likely to change direction. Different to stayed trusses, in cabled-trusses, the cables are positioned within the design space (intrinsic approach). Differently from tensegrity structures, cabled-trusses use pin-joints for interconnecting several bar elements in order to form triangular bar formations.

3. Formulation of the optimization problem

In the sizing optimization of cabled-trusses, cross-sectional areas of members and pres-stress levels in each cable are considered as design variables. To reduce the number of design variables, pre-stress can be applied as an initial strain on cable elements. If the initial strain is equal for all cable elements the pretension forces vary accordingly to the cross-sectional area of cable elements. In addition, to obtain solutions that are applicable in practice, cross-sectional areas can be restricted to take only certain pre-specified discrete values (Deb and Gulati, 2001; Richardson et al., 2012).

In the topology optimization of cabled-trusses, the connectivity of members is determined. In particular, interconnections can be performed by either cable or bar element. Note that, classical optimization methods are not suitable for topology optimization of truss-like structures (Deb and Gulati, 2001; Richardson et al., 2012; Su et al., 2009). The main reason for such inadequacy is lack efficient ways to represent connectivity of members in discrete topology optimization.

It is important to observe that prestressed elements can redistribute stresses along the structure and are dependent on initial strain (sizing) and the length and orientation of the element (topology). Therefore, an efficient way to achieve optimal design of cabled-trusses has to combine both sizing and topology optimization methods.

Structural weight minimization was selected as an optimization problem since it is one of the most commonly studied objective functions in truss topology and sizing optimization. The optimization problem can be explicitly formulated as follows:

$$
\begin{aligned}
&\text{minimize} \quad f(\kappa) = \sum_{j=1}^{m_e} \rho_j . l_j . A_j, \\
&\text{subject to} \\
&\qquad R_1 \equiv \text{Structure is valid}, \\
&\qquad R_2 \equiv \text{Structure is kinematically stable}, \\
&\qquad R_3 \equiv S_j - \sigma_j(\kappa) \geq 0,\ j = 1, 2, ..., m_e, \\
&\qquad R_4 \equiv \delta_w^{max} - \delta_w(\kappa) \geq 0,\ w = 1, 2, ..., n_s \\
&\qquad R_5 \equiv A_i^{min} \leq A_i \leq A_i^{max},\ i = 1, 2, ..., m_a,
\end{aligned}
\tag{1}
$$

where $f(\kappa)$ is the objective function, which indicates the weight of the cabled-truss κ. ρ_j, l_j and A_j are the material density, length and cross-sectional area, respectively, of the *j-th* member. The A_j values are within the pre-defined interval $[A_i^{min}, A_i^{max}]$. m_a indicates the number of possible discrete areas and m_e the number of elements. S_j and $\sigma_j(\kappa)$ are the allowable stress limit and the current stress, respectively, in the *j-th* element. δ_w^{max} and $\delta_w(\kappa)$ are the maximum allowed displacement and the current displacement of the *w-th* node.

In the constraint R_1 from Eq.(1), the validity of the structure is checked. The user specifies the location and the number of the nodes for supports and loads, and in case any one of such is absent on the cabled-truss, a large constant penalty is assigned to the solution.

In the constraint R_2 from Eq.(1), kinematically instable cabled-truss structures are filtered using Grubler's criterion that presented in (Deb and Gulati, 2001; Richardson et al., 2012; Su et al., 2009). If the number of degrees of freedom is higher than 0, then a penalization is applied to the fitness function (2). The penalization is proportional to the number of degrees of freedom (DOF) obtained.

The stress constraint R_3 from Eq.(1) penalizes structures with stresses that surpass the allowable strength of the material for all loading cases. The penalization is proportional to the percentage of stress that exceeded the limits.

The displacement constraint R_4 from Eq.(1) penalizes structures in which any of the nodes deflects more than the allowable limits when loading is applied. In this case, the penalization is proportional to the constraint violation.

In the last constraint from Eq.(1) areas are bounded based on pre-defined limits. This constraint is automatically satisfied because the sizing design variables only assume user defined values.

Similarly to the formulation proposed by (Deb and Gulati, 2001), the fitness of a solution is computed by Eq. (2), which depends on constraint violations.

$$
F(\kappa) = \begin{cases}
10^6 \, . \, {s_p}^{-1} & , \text{ if } R_0 \text{ is violated}, \\
10^8 & , \text{ if } R_1 \text{ is violated}, \\
10^7 . \sqrt{DOF} & , \text{ if } R_2 \text{ is violated}, \\
f(\kappa) = 10^6 . \sum_{j=1}^{m_e} |\langle R_3 \rangle| + 10^6 . \sum_{j=1}^{m_e} |\langle R_4 \rangle| & , \ otherwise,
\end{cases}
\tag{2}
$$

wherein the operator $\langle\rangle$ is the bracket-operator of the penalty term.

The proposed optimization framework uses the ground structure approach because this is the most common in literature and allows the comparison of results to benchmark problems. The ground structure approach takes into account a full structure with all possible member connections among nodes (Deb and Gulati, 2001).

As for trusses, in cabled-truss design the user specifies the nodes that must exist in the feasible design. Such nodes are known as the set of basic nodes and carry loads or support the structure. Other nodes which are part of the structure are known as non-basic nodes. They are defined as such because they can be eliminated during the optimization process. It is important to observe that the optimization of cabled-trusses aims to find the best interconnectivity between nodes and the elements (bar or cable) that should be used in each interconnection. Therefore increasing the number of nodes in the ground structure sharply increases the number of feasible solutions since more structural elements are used to build the cabled-truss system.

The proposed proposed framework uses a fixed length vector to represent the design variables. The first set of numbers stores the topology variables. These are encoded into integer numbers 0 (absent connection), 1 (connection performed by bar element) and 2 (connection performed by cable element). The second set of numbers stores the size variables that are also encoded into integer numbers that varies from 1 to m_{sz}, where m_{sz} is the total number of possible discrete areas that the elements can assume.

4. Proposed optimization framework

The presence of discrete variables in optimization problems has led to the successful application of stochastic search methods. In particular, genetic algorithms (GA) have grown in popularity in the optimization of truss-like structures (Su et al., 2009; Hajela and Lee, 1995; Ohsaki, 1995; Kawamura et al., 2002). GAs are stochastic adaptive methods that can be used for searching and optimization problems (Goldberg, 1989). Compared to traditional optimization methods, such as calculus-based and enumerative strategies, GAs are robust, global and may be applied generally without recourse to domain-specific heuristics. Although performance is affected by these heuristics, GAs operate on a population of potential solutions, applying the principle of survival of the fittest to produce successively better approximations to a solution (Mitchell, 1998; Han and Kim, 2002).

Individuals in a population compete for resources and mates. The most successful individuals, in this case, structures with lower mass, in each competition will produce more offspring than those that performed poorly. Genes from the fittest individuals propagate throughout the population so that two good parents will sometimes produce offspring that are better than either parent. As a consequence, each successive generation will become more suited to their environment.

To sum up, it can be said that GAs aim to use selective breeding of the solutions to produce offspring better than the parents by combining information from the chromosomes (Mitchell, 1998). GAs uses highly customizable genetic operators (selection, cross over and mutation) to perform optimal solution searching.

The GA that was used as basis for the proposed framework was introduced in (Chipperfield and Fleming, 1995). The use of this, which is a well established algorithm, allows for clarifying the effects of modifications to the algorithm. As schematized in Fig. 3, the proposed optimization framework starts with a set of initial inputs, such as the number and length of the chromosomes, the crossover and mutation rates, the number of generations and, in the particular case of this work, the binary representation scheme. Subsequently, an initial uniformly distributed random binary population is generated. The objective function Equation 1 is then evaluated by the NFEM procedure described in the (Finotto and Valášek, 2012) to produce the vector of objective values.

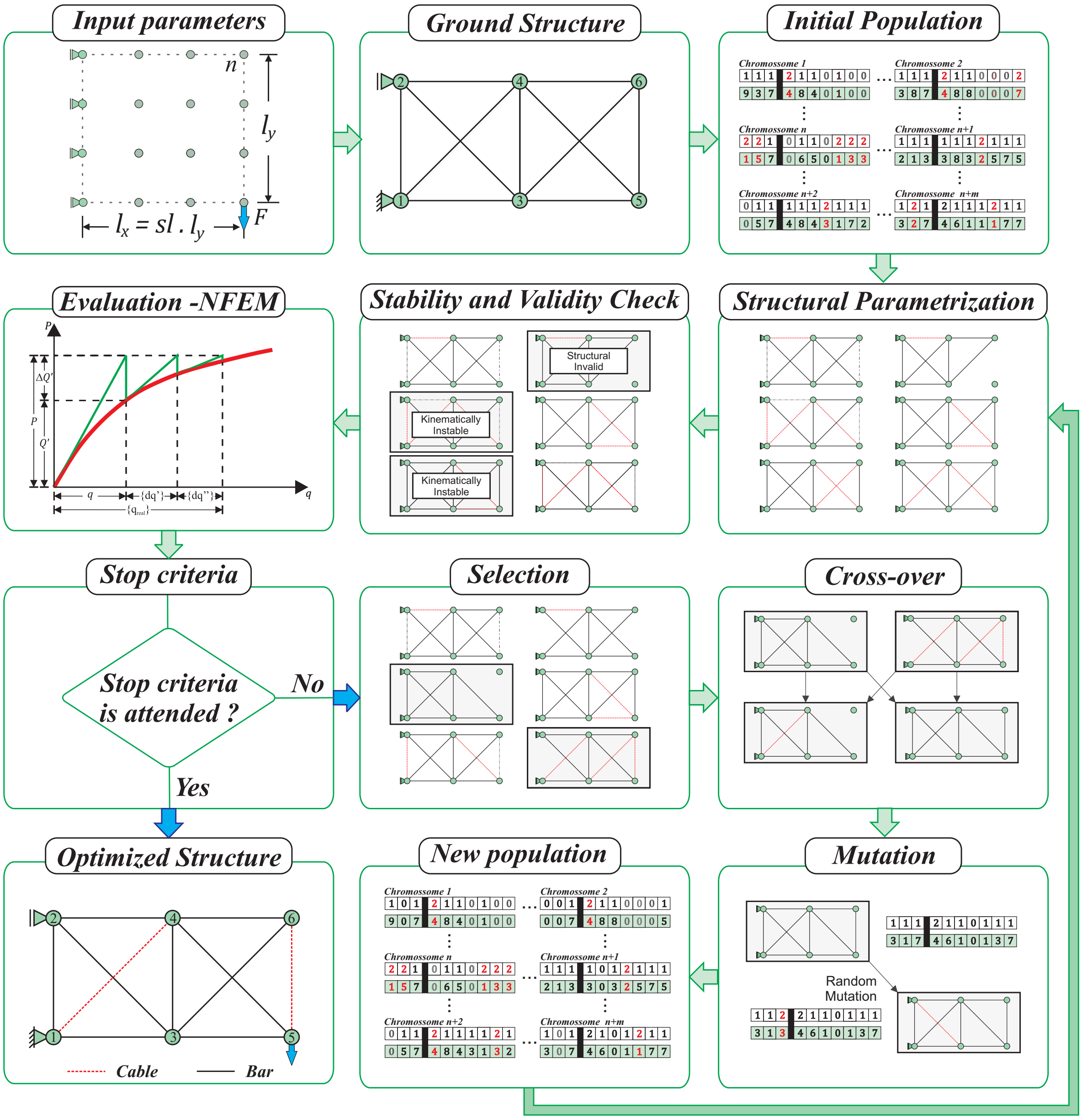

Fig. 3: Cabled-truss optimization based on genetic algorithm.

After the initialization is completed, the proposed optimization framework enters the generational loop that follows. First, a fitness vector is determined using the ranking scheme presented in (Baker, 1987). Next, individuals from the population are selected using the stochastic universal sampling algorithm with a generation gap ($GGAP = 0.8$). The selected individuals are then recombined using single-point crossover with probability of 80%. Further, mutation operator with probability of 1.75% is applied to the offspring, and the values of the objective function are calculated for the new individuals. Finally, the new individuals are re-inserted in the population and the generation counter is incremented. The GA terminates after the maximum number of iterations around the generational loop is reached.

5. Examples

The proposed optimization framework was used as a single-objective method for the optimization cabled-truss structures. Examples comprise ground structures with 15 and 66 elements. Notice that, as suggested in (Deb and Gulati, 2001), the population size in a simulation should be dependent on the discretization level of the ground structure. This consideration is performed otherwise the optimization problem could become multi-modal, i.e., there may exist many different topologies with almost equal overall weight as the discretization level increases.

5.1. GS15: Truss benchmark

In this section, a 15 elements ground structure (GS15) is optimized. Boundary conditions and ground structure of the present analysis are depicted in Fig. 4.

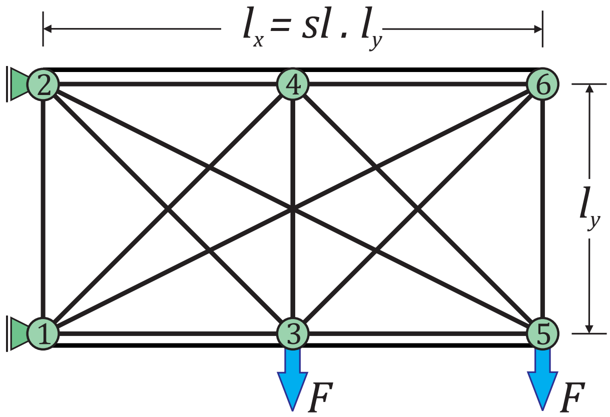

Fig. 4: 15 elements ground structure (GS15).

Sizing and topology optimization were performed concurrently to achieve structural mass optimization. In particular, 15 individual runs were performed. The population size was set as 420 and the maximum number of generations as 500. Geometric and material parameters for the following example is shown in Table 1 .

The optimal truss and cabled-truss topologies for this benchmark are depicted in Fig. 5. The optimized results are presented in Table 2 together with the results obtained by (Deb and Gulati, 2001).

Stresses and displacemets obtained for the optimized solutions using GS15 are presented in Tables 3 and 4, respectively. The numerical error E_{CT} between the results (stresses and displacements) obtained by the proposed optimization framework, implemented in Matlab, and ANSYS is also listed in these tables.

Tab. 1: Geometric and material parameters for GS10 and GS15.

$Parameter$	$Value$
Structure hight (L_y)	9.144 m
Loading Force (F)	448.2 kN
Admissible Areas (A_{ad})	from 645.16 to 19359.00
	with increment $645.16\ mm^2$
Elasticity Modulus (Al)	$6.895\ 10^4 MPa$
Material Density (Al)	$2.768\ kg/m^3$
Tensile Modulus (Carbon Fiber)	$20.685\ 10^4 MPa$
Material Density (Carbon Fiber)	$0.4613\ kg/m^3$
Max. allowable stress σ_j^{max}	$1.724\ 10^4 MPa$
Max. allowable displacement δ_w^{max}	$50.8mm$

Tab. 2: Areas obtained for the optimized solutions using GS15.

	Truss		Cabled-truss
$A_i[mm^2]$	(Deb and Gulati, 2001)	OF	OF
A_0	3367.09	3225.80	3225.80
A_1	13103.19	13548.36	9677.40
A_2	9414.81	9677.40	9032.24
A_3	5014.18	5161.28	5161.28
A_4	18185.12	18064.48	14193.52
A_5	13322.55	12903.20	9032.24
A_6			7096.76
$W_b[kg]$	2146.2403	2160.312	1813.363

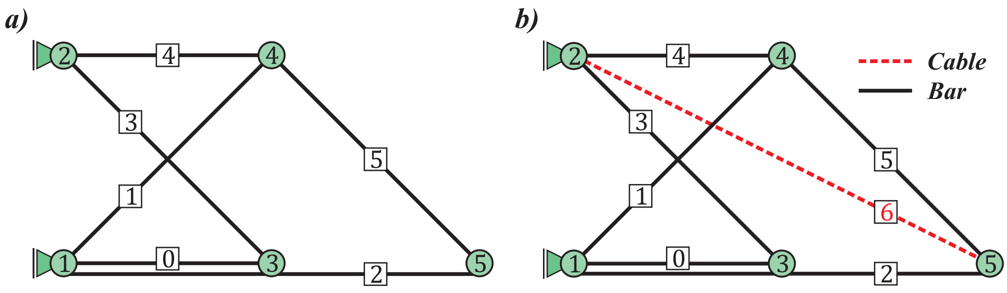

Fig. 5: Optimized structures obtained for GS15: a)Truss and b) Cabled-truss.

Tab. 3: Stresses obtained for the optimized solutions using GS15.

	Truss	Cabled-truss		
$\sigma_i[MPa]$	OF	ANSYS	OF	$E_{CT}[\%]$
σ_0	-137.8900	-137.8900	-137.8940	0.0029
σ_1	-46.4310	-13.9440	-13.9648	0.1490
σ_2	-45.9650	-87.9320	-87.9170	0.0171
σ_3	121.8800	121.8800	121.8823	0.0019
σ_4	49.2480	13.4450	13.4623	0.1285
σ_5	48.7530	14.9400	14.9542	0.0949
σ_6	-	121.1000	121.0583	0.0344

Tab. 4: *Displacements obtained for the optimized solutions using GS15.*

	Truss		Cabled-truss				
$\delta_w[mm]$	OF		ANSYS		OF		
Node	dx	dy	dx	dy	dx	dy	$E_{CT}[\%]$
3	-18.2870	-50.6150	18.2870	50.6150	18.4276	50.6257	0.7901
4	6.5312	-18.8460	-5.0304	15.4643	-5.0075	15.4473	0.5647
5	-12.1920	-50.5000	18.2201	49.8940	18.2590	49.7927	0.0105

6. GS66S: Planar robotic manipulator

In this section, a 66 elements ground structure (GS66S) is optimized. Discrete size and topology optimization are performed for a robotic arm, which is approximated by a horizontal cantilever beam. The planar idealization of the robotic arm and the corresponding boundary conditions and ground structure are depicted in Fig. 6.

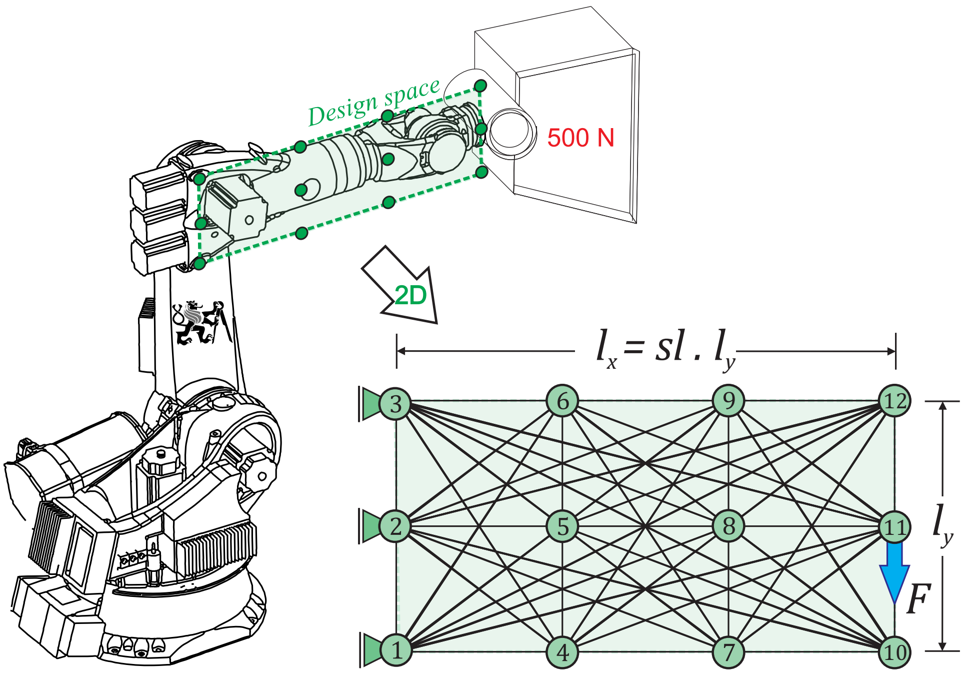

Fig. 6: 66 elements ground structure (GS66S).

An additional constraint was used for maintaining the symmetric vertical structural response. This is because the robotic manipulator is capable of pulling or pushing objects and the same precision is desired in both operations (directions). Geometric for the following example is shown in Tab. 5. Material parameters are considered to be the same as for the previous example. Sizing and topology optimization were performed concurrently to achieve structural mass optimisation. In particular, 15 individual runs were performed. The population size was set as 400 and the maximum number of generations as 500. The optimal truss and cabled-truss topologies are depicted in Fig. 7. The optimized results are presented in Table 6. Displacements obtained for the optimized solutions using GS66S are presented in Tables 7.

Tab. 5: Geometric and material parameters for GS66S.

$Parameter$	$Value$
Structure hight (L_y)	1.0 m
Loading Force (F)	500 N
Slenderness ratio (S_l)	6
Admissible Areas (A_{ad})	$r_b^2\pi\ mm^2$, for $r_b = 2, 4, 6, ..., 40$
Max. allowable stress σ_j^{max}	$1.724\ 10^4 MPa$
Max. allowable displacement δ_w^{max}	$0.2mm$

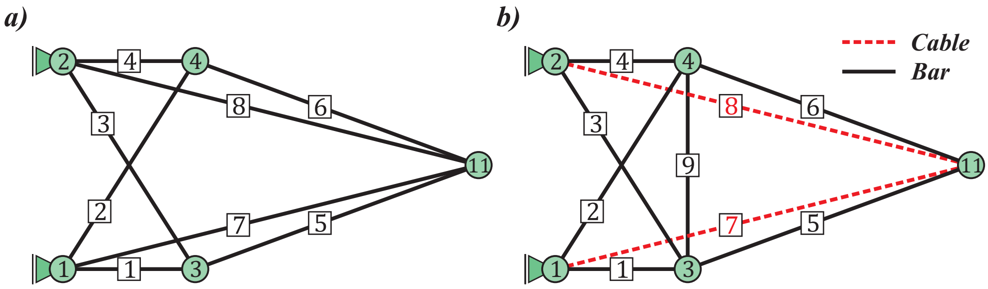

Fig. 7: Optimized structures obtained for GS66S: a)Truss and b) Cabled-truss.

Tab. 6: Areas and stresses obtained for the optimized solutions using GS66S.

$A_i[mm^2]$	Truss	Cabled-Truss	$\sigma_i[MPa]$	Truss	Cabled-truss
A_1	1256.6370	1256.6370	σ_1	-1.2576	-6.5241
A_2	452.3893	615.7522	σ_2	-0.81476	-3.1496
A_3	452.3893	615.7522	σ_3	0.81476	-1.6983
A_4	1256.6370	1256.6370	σ_4	1.2576	-3.4751
A_5	615.7522	804.2477	σ_5	-2.0761	-11.435
A_6	615.7522	804.2477	σ_6	2.0761	-7.5813
A_7	452.3893	50.2655	σ_7	-1.3410	147.18
A_8	452.3893	50.2655	σ_8	1.3410	155.22
A_9	-	201.0619	σ_9	-	9.4611
$W_b[kg]$	8.1073	6.8033			

Tab. 7: Displacements obtained for the optimized solutions using GS66S.

$\delta_w[mm]$	Truss		Cabled-truss	
Node	dx	dy	dx	dy
3	-0.60798E-02	-0.19061E-01	-0.31540E-01	-0.33957E-01
4	0 .60798E-02	-0.19061E-01	-0.16800E-01	-0.65135E-02
11	0.0000	-0.19643	-0.12023	-0.19637

7. Results and discussion

From Table 2 it can be observed that the optimized truss found by the GA is similar to those found in previous research. This indicates that the implemented GA is capable of obtaining reliable results. In addition, further mass reduction was reached when cable and bar elements were combined.

The optimization of cable-trusses requires the use of NFEM procedure, which comprises several evaluations of the same individual before structural equilibrium is reached. The numerical error from Tables 3 and 4 is lower than 0.80%, validating the results from the nonlinear finite element analysis performed by the proposed optimization framework. In addition, these results indicate that the use of cables significantly changed the stress path along the structures. Moreover, from 2 and 6, it can be observed that cable-trusses reached significant weight reduction over trusses.

8. Conclusion

In this work, a single objective optimization framework for discrete topology and sizing optimization of cabled-trusses was presented. The proposed system successfully combined ground structure approach, nonlinear finite element analysis and genetic algorithm. The comparison between optimized trusses and cabled-truss structures shows that optimized cabled-trusses had a significant improvement over trusses in the minimization of the structural mass.

In addition, performing cabled-trusses optimization, the increase of the ground structure discretization led to a sharp increase of the search space. In addition, an increase in the number of evaluations of the FE model was also observed. This is because iterative procedures become part of the optimization problem when cable elements are used. For this reason, the effectiveness of the GA can be compromised since a relatively high number of evaluations may lead to a prohibitive computational cost.

Simulation results indicates that cables can increase truss structural performance. Complementary analysis are recommended to evaluate the influence of cables on trusses under multiple loading cases. In addition, modular designs could be adopted in order to decrease the search space and improve computational cost. As result, such study would allow to optimize more realistic structures.

9. Acknowledgment

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10. References

Ahmeti, F.. Efficiency of lightweight structural forms: The case of treelike structures - a comparative structural analysis. Master's thesis; Vienna University of Technology; Austria; 2010.

Whitman, Z.. Improvement in Energy Absorption Through Use of Bistable Structures. Department of Mechanical Engineering, University of Utah; 2005.

Ando, K., Mitsugi, J., Senbokuya, Y.. Analyses of cable-membrane structure combined with deployable truss. Computers & Structures 2000;74(1):21 – 39.

Bosch, H.R.. Aerodynamic stability of a truss-stiffened cable-stayed bridge. Journal of Wind Engineering and Industrial Aerodynamics 1990;36, Part 2(0):1331 – 1340.

Deb, K., Gulati, S.. Design of truss-structures for minimum weight using genetic algorithms. Finite Elements in Analysis and Design 2001;37(5):447 – 465.

Richardson, J.N., Adriaenssens, S., Bouillard, P., Coelho, R.F.. Multiobjective topology optimization of truss structures with kinematic stability repair. Structural and Multidisciplinary Optimization 2012;46:513–532.

Su, R., Gui, L., Fan, Z.. Topology and sizing optimization of truss structures using adaptive genetic algorithm with node matrix encoding. In: Natural Computation, 2009. ICNC '09. Fifth International Conference on; vol. 4. 2009, p. 485 –491.

Goldberg, D.E.. Genetic Algorithms in Search, Optimization and Machine Learning. 1st ed.; Boston, MA, USA: Addison-Wesley Longman Publishing Co., Inc.; 1989. ISBN 0201157675.

Hajela, P., Lee, E.. Genetic algorithms in truss topological optimization. International Journal of Solids and Structures 1995;32(22):3341 – 3357.

Ohsaki, M.. Genetic algorithm for topology optimization of trusses. Computers & Structures 1995;57(2):219 – 225.

Kawamura, H., Ohmori, H., Kito, N.. Truss topology optimization by a modified genetic algorithm. Structural and Multidisciplinary Optimization 2002;23:467 – 473.

Mitchell, M.. An Introduction to Genetic Algorithms (Complex Adaptive Systems). Third printing ed.; A Bradford Book; 1998. ISBN 0262631857.

Han, K.H., Kim, J.H.. Quantum-inspired evolutionary algorithm for a class of combinatorial optimization. Evolutionary Computation, IEEE Transactions on 2002;6(6):580 – 593.

Chipperfield, A.J., Fleming, P.J.. The matlab genetic algorithm toolbox. In: Applied Control Techniques Using MATLAB, IEE Colloquium on. 1995, p. 101 – 104.

Finotto, V.C., Valášek, M.. Discrete topology optimization of planar cable-truss structures based on genetic algorithms. In: Naprstek, J., Fischer, C., editors. Proceedings Engineering Mechanics 2012. Prague: ITAM AS CR, v.v.i.; 2012, p. 245 – 254.

Baker, J.E.. Reducing bias and inefficiency in the selection algorithm. In: Proceedings of the Second International Conference on Genetic Algorithms on Genetic algorithms and their application. Hillsdale, NJ, USA: L. Erlbaum Associates Inc. ISBN 0-8058-0158-8; 1987, p. 14–21.