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MODEL OF TURBINE BLADES BUNDLES

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Abstract: *A detail analysis of dynamical properties of five-blades-bundle mathematical model was carried out with the aim to investigate how the damping elements made of special rubber inserted into slots between blades heads influence the response curves at different distribution of exciting harmonic forces. Viscous-elastic linear Voigt–Kelvin model was used for modeling the rheological properties of damping elements. Constant values of stiffness and damping parameters were supposed at analysis, but the frequency dependent parameters are mentioned as well. The effect of complicated form of higher modes of vibration on blades bundle amplitude resonance level is shown. The importance of orthogonality of excitation forces distribution to the other eigenmodes of blades bundle is discovered at isolating selected resonance.*

Keywords: ***Free and forced vibrations, eigenmodes, mathematical model, bundle of blades, viscous-elastic damping elements.***

1. Introduction

Rotor blades in turbine are subjected to a high dynamical load produced by pressure fluctuation in the steam or gas stream with the basic frequency that are multiples of the rotor angular frequency and number of stator blades. Because the material damping of blade steel is very low, several damping devises are used for reduction of dangerous stresses.

A very effective way against undesirable vibrations of turbine blades vibrations is introducing additional damping into blade shroud usually by using some type of friction elements. A lot of theoretical, numerical and experimental studies were done in Institute of Thermomechanics ASCR in this field. The reduction of undesirable vibrations of blades is realized by using blade damping heads, which on the experimental model of turbine disk in the laboratories of IT are the heads of blades connected either directly through friction contact or connected by inserted friction elements.

For detail discovering of friction processes and their influence on blades vibrations, the dynamic tests of separated blade couple were performed. Blades were modeled by 1 DOF slightly damped systems connected by a friction element.

In presented paper, the two-blades-model is enlarged on the study of five-blades-model. Effect of various mathematical models of internal material damping described by viscous-elastic linear Voigt–Kelvin model will be investigated. This theoretical study is focused particularly on the elaboration of a theoretical background for analysis of data gained by measurement on the experimental physical model of blades bundles prepared in laboratories IT-ASCR.

2. Vibrations of blades bundle –experimental model

Laboratory measurements of blade bundle will be realized on the experimental set consisting of five models of blades with shroud heads, which were rigidly fastened to a steel plate basement, see Fig. 1. These heads are connected by inserted damping elements made of special rubber. The rubber FKM with a trade mark VITON of hardness 70 ShA was selected as material for the damping elements. This rubber is known for its high resistance against high temperatures (up to T=220^{0}C), resistance against majority of aggressive chemicals, synthetic and mineral oils, sunbeams and ozone. It makes a good presumption for utilization of such a material in hard conditions of a technical praxis

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such as they are in steam turbines. This material similarly as many other polymers has very strong dependence of its mechanical behavior on the temperature and frequency and therefore the mathematical modeling of the dynamic properties of systems containing these elements need a special treatment.

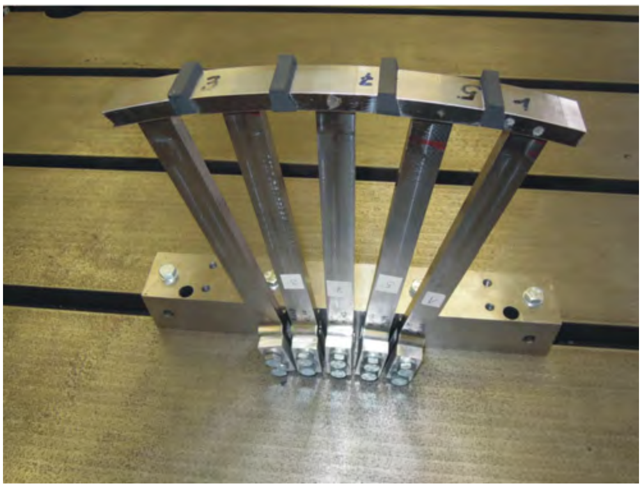

Fig. 1

The dependences of the complex Young modulus $E^* = E_{Re} + iE_{Im}$ of VITON rubber is described by the analytical formula in [1]:

$$E^* = 1049*(1+0.1i) + 1588*(1+0.06i)\,(i\,f\,\alpha T)^{0.64}/(1+0.0063(i\,f\,\alpha T))^{0.64} \qquad (1)$$

where $$\alpha T = 10\wedge(-20(T-4.4)/(134.5+T)\,.$$

Since the VITON elements used in the blade bunch have prismatic-form and their placement in the blade shroud presumes the shear deformation therefore we have used the geometric parameters (base area A = 0.00025m^2, high h = 0.012m) and volume incompressibility assumption for computing the complex shear stiffness K_s^* of these elements in our numerical simulations.

$$K_s^* = E^* h/(3A)\,.$$

For the calculation of dynamic properties of five blades bundle with inserted rubber viscous-elastic damping elements we need linear stiffness k and damping b coefficients. Presentation of these coefficients in dependence on temperature ($T \in (20,\ 120)$ °C) and on frequency ($f \in (100,\ 300)$ Hz) is depicted in Fig. 2 and 3.

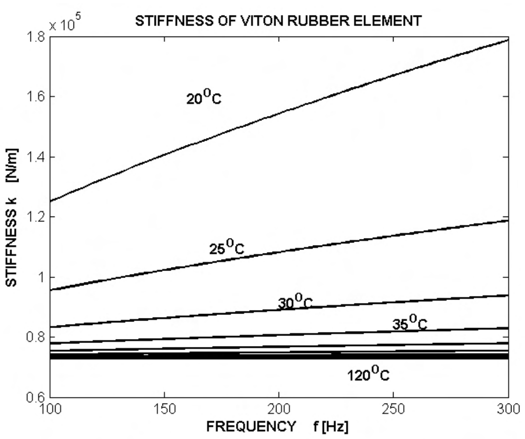

Fig. 2

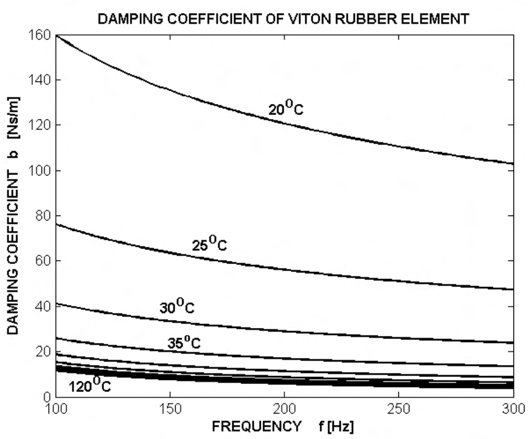

Fig. 3

It is evident that the properties of this viscous-elastic damping element are very variable near to the room temperature. At higher temperature (120 °C) these properties stabilize on the approximately constant stiffness $k \approx 72800$ N/m and damping coefficient b [Ns/m] independent on temperature but variable with frequency f according to the hyperbolic law $b*f \approx$ const., but for near the room temperature, both these rheologic parameters strongly varies with frequency. The linear regression function of rubber element stiffness at 20 °C is

$$k_1 = 102330 + 250f \quad \text{[N/m, Hz]} \tag{2a}$$

and at 50 °C

$$k_1 = 73380 + 2.12f \quad \text{[N/m, Hz].} \tag{2b}$$

3. Simplified mathematical model of five blades bundle

Because the experimental research is usually encumbered with a lot of marginal influences and uncertainties, e.g. by impossibility of measurement of deformation of damping elements, uncertainty in contact forces during operation, etc., the additional analytical and numerical solution of simplified mathematical model with exact parameters is very useful and enables to complete knowledge of dynamic behavior of studied system by a lot of new information. Experimental system in Fig. A can be modeled by a simple five masses system shown in Fig. 1, where the blades are replaced by 1 DOF systems, the eigenfrequencies of which corresponds to the first bending eigenfrequencies of real blades. The eigenfrequencies of these blades are supposed to be much higher than the bending ones (mass m, stiffness k, damping coefficient b) and therefore, in the first approximation, the torsion deformations and vibrations of blades are not taken into account.

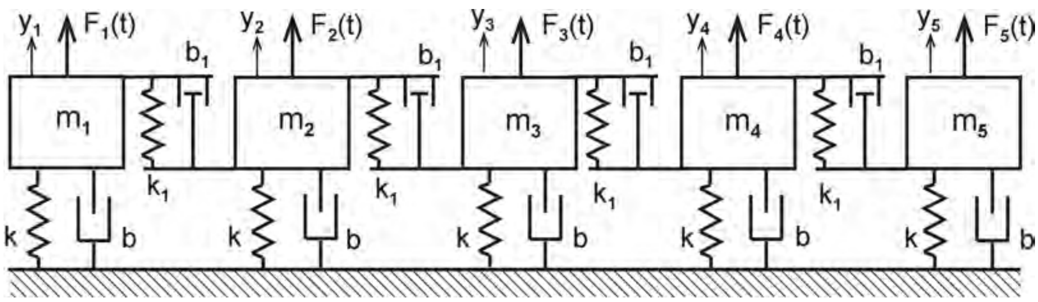

Fig. 4

Damping of all individual separated blades is in Fig. 4 modeled by small viscous damping with very low coefficients b=0.4 Ns/m. According to the lowest bending eigenfrequency of the real blade structure f = 120.88 Hz, the reduced mass m = 0.182 kg, and stiffness k = 105000 N/m are ascertained

The rubber damping elements are on theirs sides loaded by such sufficiently great friction forces in the contact areas with the neighbourhood masses that no slips occur in these connections during operation. The linear Voigt–Kelvin model, describes deformation properties of rubber damping elements. The stiffness k_1 [N/m] and damping coefficient b_1 [Ns/m] must be ascertained from the diagrams in Fig. 2 and 3 or from the equations (1),(2).

Masses m are loaded by the harmonic forces

$$F_i(t) = F_{0i}\cos(\omega t),\ i = 1,..5, \tag{3}$$

where the amplitudes F_{0i} can be of various values. The excitation frequency ω of force $F_{0i}\cos(\omega t)$ varies over the whole eigenfrequencies spectrum of investigated system.

Another type of excitation forces that models excitation from running waves or produced by the periodically distributed pressure of stator blades, contains time or phase delays between neighbour blades forces;

$$F_i(t) = F_{0i}\cos(\omega t + j\Delta\varphi), \quad \text{or} \quad F_i(t) = F_{0i}\cos\omega(t + j\Delta t), \tag{3a}$$

where $\Delta\varphi = \omega\Delta t$, $i = 1,..5$, $j = i-1$.

Differential equations of motion for the first case of excitation by given vector of force amplitudes

$$\boldsymbol{F} = [F_{01}, F_{02}, F_{03}, F_{04}, F_{05}]$$

without phase shifts are:

$$\begin{aligned}
&m\ddot{y}_1 + b\dot{y}_1 + ky_1 + g_1(y_1, y_2) = F_{01}\cos\omega t\\
&m\ddot{y}_2 + b\dot{y}_2 + ky_2 + g_2(y_2, y_3) - g_1(y_1, y_2) = F_{02}\cos\omega t\\
&m\ddot{y}_3 + b\dot{y}_3 + ky_3 + g_3(y_3, y_4) - g_2(y_2, y_3) = F_{03}\cos\omega t\ ,\\
&m\ddot{y}_4 + b\dot{y}_4 + ky_4 + g_4(y_4, y_5) - g_3(y_3, y_4) = F_{04}\cos\omega t\\
&m\ddot{y}_5 + b\dot{y}_5 + ky_5 - g_4(y_4, y_5) = F_{05}\cos\omega t
\end{aligned} \tag{4}$$

where the linkage functions g_i are described by the linear Kelvin Voigt parallel viscous-elastic model

$$g_i(y_i, y_{i+1}) = k_1(y_i - y_{i+1}) + b_1(\dot{y}_i - \dot{y}_{i+1}), \qquad i = 1,\dots,4. \tag{4a}$$

Physical values of the blade parameters in equations (4) are constants: m = 0.182 kg, b = 0.4 Ns/m, k = 105000 N/m, but the rheologic parameters of the inserted damping elements depend on temperature and frequency $k_1(T, f)$ N/m, $b_1(T, f)$ Ns/m, see eqs. (2a), (2b).

4. Free vibrations of blade bundle

First of all, the eigenfrequencies of such 5 DOF system have to be ascertained. Neglecting external and damping forces, the corresponding equations are

$$\begin{aligned}
&m\ddot{y}_1 + ky_1 + k_1(y_1 - y_2) = 0\\
&m\ddot{y}_2 + ky_2 + k_1(y_2 - y_3) - k_1(y_1 - y_2) = 0\\
&m\ddot{y}_3 + ky_3 + k_1(y_3 - y_4) - k_1(y_2 - y_3) = 0\\
&m\ddot{y}_4 + ky_4 + k_1(y_4 - y_5) - k_1(y_3 - y_4) = 0\\
&m\ddot{y}_5 + ky_5 - k_1(y_4 - y_5) = 0
\end{aligned} \tag{5}$$

or in the matrix description

$$M\ddot{Y} + KY = 0\,, \tag{5a}$$

where

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0\\ 0 & m & 0 & 0 & 0\\ 0 & 0 & m & 0 & 0\\ 0 & 0 & 0 & m & 0\\ 0 & 0 & 0 & 0 & m \end{bmatrix}, \quad K = \begin{bmatrix} k+k_1 & -k_1 & 0 & 0 & 0\\ -k_1 & k+2k_1 & -k_1 & 0 & 0\\ 0 & -k_1 & k+2k_1 & -k_1 & 0\\ 0 & 0 & -k_1 & k+2k_1 & -k_1\\ 0 & 0 & 0 & -k_1 & k+k_1 \end{bmatrix}. \tag{5b}$$

Because the stiffness k_1 of inserted elements are not constant, the five blades system eigenfrequencies have to be determined by means of modified programme *eig* in Matlab with consideration to (2). The eigenfrequencies at 20 °C are

$$[\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5] = [120.88,\ 148.41,\ 211.12,\ 279.45,\ 330.82]\ \text{Hz}, \tag{6a}$$

at 50 °C

$$[\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5] = [120.88,\ 136.06,\ 169.48,\ 203.69,\ 227.61]\ \text{Hz} \tag{6b}$$

and at 120 °C

$$[\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5] = [120.88,\ 135.96,\ 169.19,\ 202.88,\ 226.48]\ \text{Hz}. \tag{6c}$$

The graphical illustrations of these spectra are in Fig. 5.

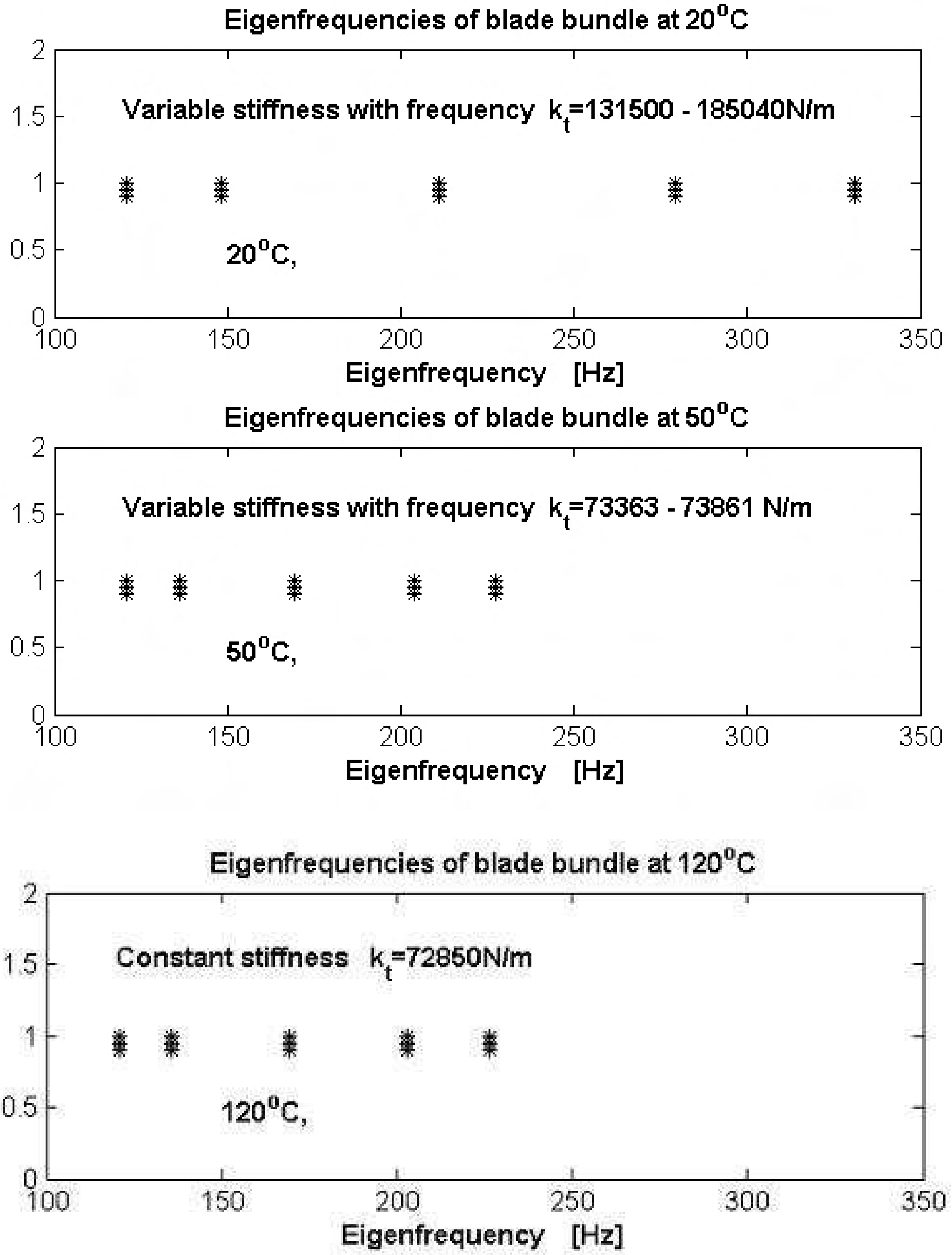

Fig. 5

It is evident that the eigenfrequency spectrum varies mainly in the temperature range under 50 ^{o}C and in range 50 – 120 ^{o}C the influence of temperature is negligible.

The same negligible effect has the temperature also on the forms of eigenmodes, but in this case in the whole investigated temperature area. Corresponding modes of five eigenfrequencies plotted in Fig. 6 are common for all used temperature.

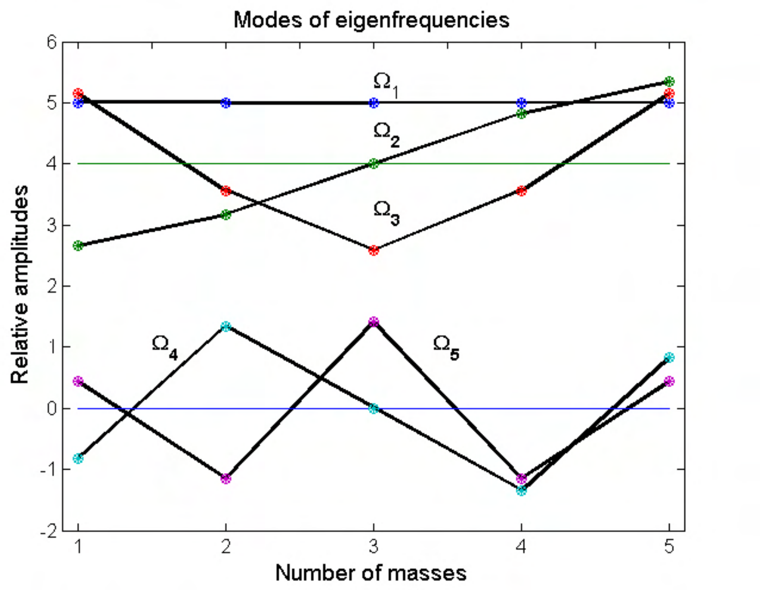

Fig. 6

5. Response on external sweep excitation

The simplest way how to gain the response curves of general multi-degrees dynamical systems with the temperature-variable stiffness and damping characteristics is to calculate the response of mathematical model on the sweeping excitations. However due to the non-stationary excitation, these response curves are a little distorted against the poorly stationary excited responses, but the quickness of solution and the possibility to become great number of alternatives in a short time is very often decisive.

The influence of sweep velocity on the response curves distortion is shown in the next figure Fig. 7, where the passages of a resonance peak by three accelerations 0.15, 0.2, 0.25 rad/s^2 are plotted.

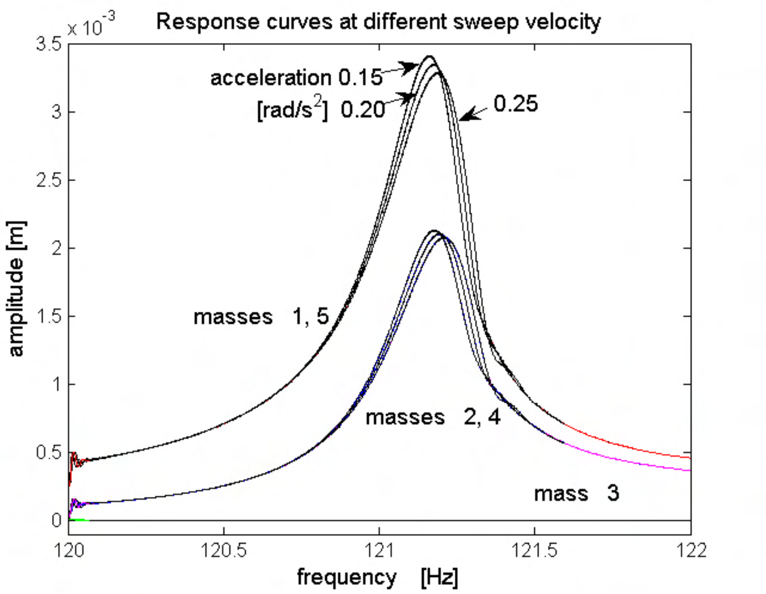

Fig. 7

6. Forced vibrations – linear frequency-dependent damping and stiffness elements

The experimental physical model of five blades bundles, prepared in laboratories IT-ASCR, will be excited by various combinations of external harmonic forces. Therefore also in this anticipatory analytic-numerical model analysis, the responses on various combinations of external harmonic forces must be investigated. In order to keep the same condition in laboratory and at numerical modeling, the temperature 20 °C must be supposed.

The eigenfrequency spectrum is then given by eq. (6a) with Fig. 5 and the stiffness variation (eq. 2a)

$$k_1 = 102330 + 250 f \quad \text{N/m}$$

together with the linear damping coefficient variation $b_1 = 173.97 - 0.2385 f$ Ns/m.

have to be included into the equations (4), (4a) for solution of 5 blades bundle response curves.

This responses in the frequency range $f = \in$ (120, 330) Hz depend also on the form of excitation force vector $\boldsymbol{F} = [F_{01}, F_{02}, F_{03}, F_{04}, F_{05}]$. For the comparison of the influence of forces distribution on response curves, the simplest model of five-blades bundle with constant and smaller rheologic parametres will be investgated in the following chapters. The reason is the fact that the range of eigenfrequencies of blades bundle with inserted VITON rubber elements is very large, the damping very low, so that the resonance peaks in the entire response curve are depicted only by vertical lines. Therefore the five-blades bundle system with the damping elements stiffness $k_1 = 2000$ N/m and linear damping coefficient $b_1 = 2$ Ns/m will be analyzed.

If the force vector consists of only one force F_{01} = 1 N, (i.e. [1, 0, 0, 0, 0]), then the response curves calculated at sweep excitation with angular acceleration $\varepsilon = 0.20\ rad/s^2$ and passing through the frequency range f = 120 – 126 Hz including all eigenfrequencies of the new five-blades bundle system are plotted in Fig. 8.

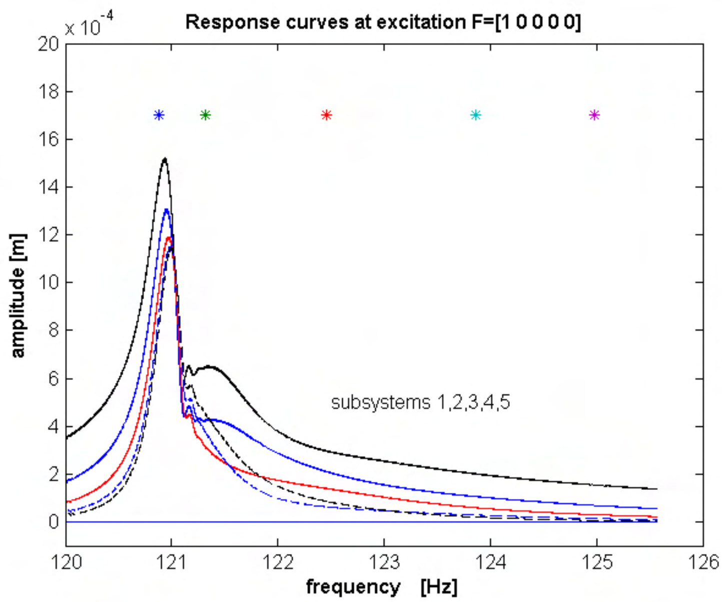

Fig. 8

Response curve in the place of force application is drawn by the highest curve; the curves of other masses are gradually falling at beginning near 120 Hz, but due to the influence of other modes they are crossing in the higher frequency range. Small sterns above the curves indicate the positions of eigenfrequencies. The second resonance causes small increase of response curves near 121.4 Hz. In spite of the theoretical possibility of excitation of higher resonances, the other resonances are not indicated by this one-force action,

Forced oscillations excited by two opposite harmonic forces acting on the corner blades 1 and 5 ($\boldsymbol{F}$= [1, 0, 0, 0, -1]), are axis anti-symmetric and therefore the blades 1 and 5 as well as 2 and 4 have common response curves. They differ only by different phase shifts.

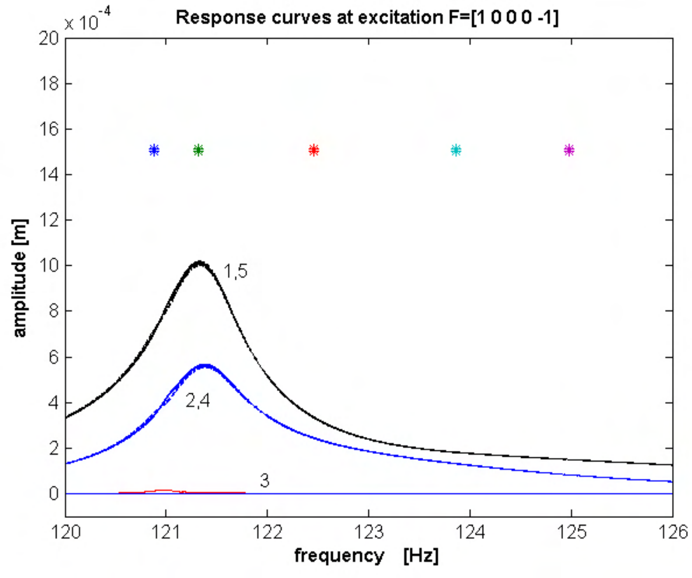

Fig. 9

These curves are shown in Fig. 9, where the response curves of the point 1,5 of force application are again the highest. Responses on the used anti-symmetric excitation with two opposite forces contain only one resonance peak in spite that also the fourth eigenmode is also anti-symmetric and can be theoretically indicated. As we will see further, the more and appropriate forces must be used for its accentuation.

By means of three forces distributed according the force vector ***F***= [1, 0, -1, 0, 1] and corresponding to the mode of the third eigenfrequency Ω_3, the third resonance should be excited. However, result of numerical solution plotted in Fig. 10 shows that the highest resonance peak has the first resonance and the third one is only indicated by the small increase at f = 122.46 Hz. Using modified force vector ***F***= [1, 0, -2, 0, 1], having the balanced force components, the first resonance peak is suppressed and the third one is moderately amplified as shown in Fig. 11. The great damping $b_1 = 2$ Ns/m is evidently very high for this mode and causes the flat form of third resonance peak.

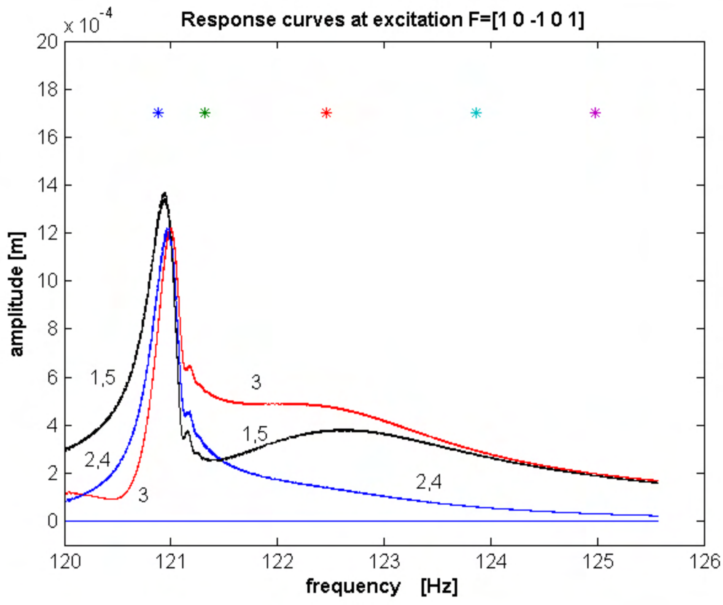

Fig. 10

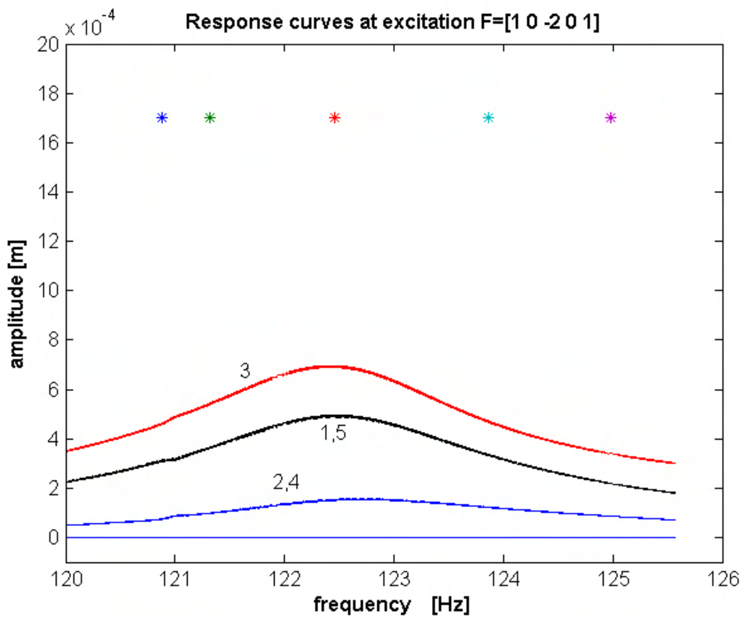

Fig. 11

The higher modes are more complicated as seen from Fig. 6 and therefore the higher damping due to the greater deformations of viscous-elastic elements inserted into slots among bladed heads must be expected. The reasonable fourth and fifth resonance peaks even at the appropriate force vector ***F*** containing more than three acting forces can be expected only at systems with lower damping coefficient b_1 of inserted elements.

Force vector ***F***=[1, –1, 0, 1, –1] excites the flat forth resonance peak at 123.86 Hz, but it is connected with the rise of the comparatively sharp resonance peak at 121.32 Hz as shown in Fig. 12. The reduction of damping coefficient b_1 on the lower value does not help, as both peaks

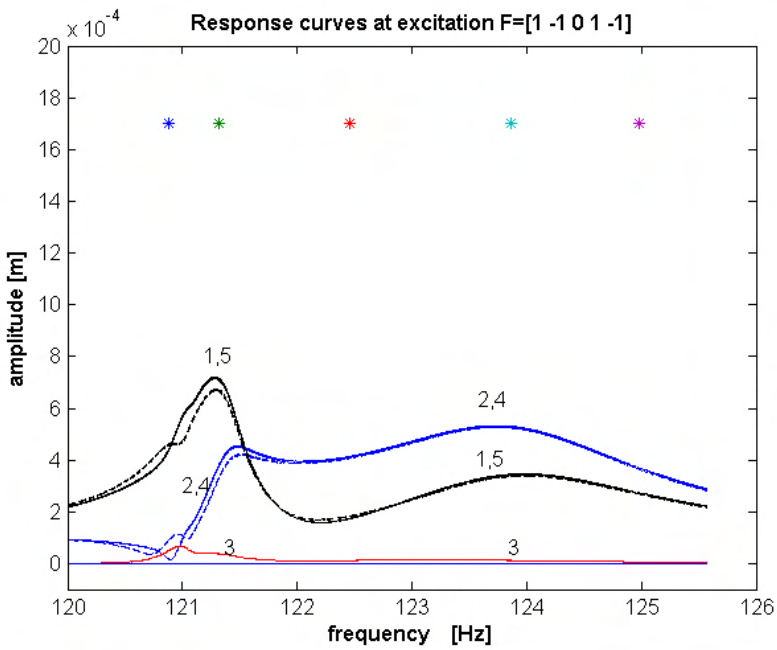

Fig. 12

proportionaly increase. Suppression of second resonance peak can be reached by transformation of force vector ***F*** into orthogonal form to the second eigenmode. Response curves of 5-blades bundle excited by the modified force vector ***F***=[1, –1.625, 0, 1.625, –1] show only one resonance peak corresponding to the forth eigenfrequency – Fig. 13.

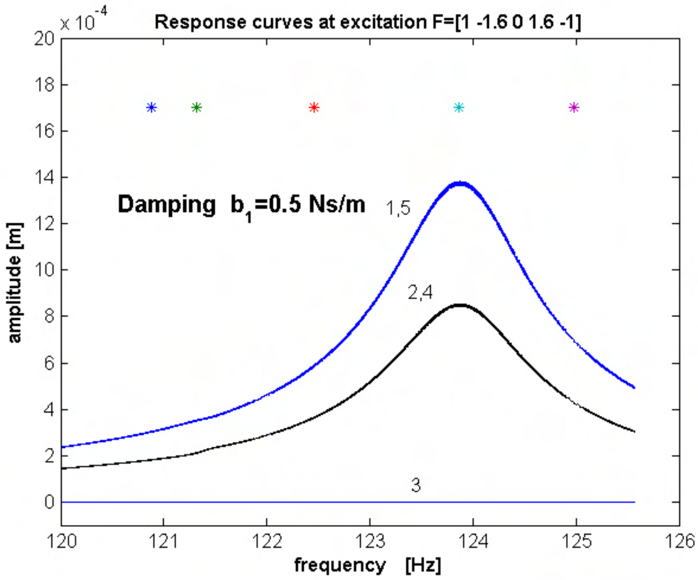

Fig. 13

The 5-blades bundle has 5 eigenfrequencies, from which three $\Omega_1, \Omega_3, \Omega_5$ have axis symmetric modes. If we wish to separate the fifth eigenmode, the excitation force vector must be orthogonal to the first and third eigenmodes. Response curves of such system excited by $\boldsymbol{F}$ = [1, –2.49, 2.474, -2.49, 1] are plotted in Fig. 14. The middle, third mass has the highest resonance amplitude, amplitudes of the second and forth masses are lower and both end masses 1,5 vibrate with minimal amplitudes. The small distortions near the thirst eigenfrequency $\Omega_3 = 122.46\ Hz$ show that the applied force vector is not quite correct orthogonalized to the other modes.

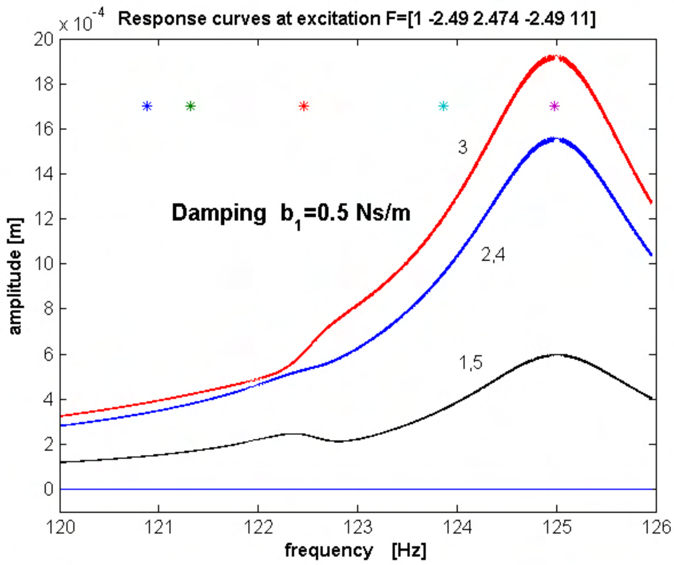

Fig. 14

7. Conclusions

- A mathematical model of five blades bundle, connected in the heads slots by damping elements made of special rubber, has been developed and applied for the ascertaining of dynamic behavior of blades bundle at different external excitation.

- Properties of rubber FKM with a trade mark VITON of hardness 70 ShA, which was selected as material for the damping elements for the prepared experimental research in laboratory IT AVCR, were discussed particularly in view of the dependence on frequency of oscillations.
- Detail analysis of frequency and temperature effect on this material showed that the eigenfrequency spectrum varies mainly in the temperature range under 50 ^{o}C and in the range 50 – 120 ^{o}C the influence of temperature is negligible. The variability of stiffness and damping properties on frequency is similar.
- Effect of the temperature and frequency on the forms of eigenmodes in the whole investigated area is negligible.
- The analysis of influence of different force distribution on the response curves of blades bundle was realized at the lower stiffness k_1 = 2000 N/m.
- It was proved that due to the more complicated forms at higher eigenmodes the mode's damping increases with frequency even at the constant material viscous damping coefficient.
- The application of orthogonality of excitation forces distribution to the other eigenmodes of blades bundle is necessary for analysis and isolation of selected resonance.

Acknowledgement

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