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EFFECT OF SOFTENING FUNCTION TYPE IN THE DOUBLE-*K* FRACTURE MODEL FOR THE EVALUATION OF FRACTURE TESTS ON CONCRETE SPECIMENS WITH AND WITHOUT POLYPROPYLENE FIBRES

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Abstract: *Cement-based composites are traditionally a commonly used material in civil engineering structures. The basic representative of this type of material is concrete, a quasi-brittle composite in which crack resistance can be achieved by the addition of fibres. The double-K fracture model can be used to calculate the fracture-mechanical parameter values of structural concrete with and without polypropylene fibres. This model combines the concept of cohesive forces acting on the crack length with a criterion based on the stress intensity factor, using a 'softening function' to determine the cohesive part of fracture toughness. In this paper, authors determine the effect of the type of this softening function on the evaluation of fracture tests performed on sets of concrete specimens with and without polypropylene fibres.*

Keywords: ***Double-K fracture model, softening function, concrete, polypropylene fibre, fracture test.***

1. Introduction

Concrete, a so-called quasi-brittle material, is a commonly used building material. Its range of applications can be extended using various additives, e.g. polypropylene fibres. Even relatively small volume quantities of these fibres in concrete mixture (1–3 %) can affect the resistance of the composite to crack propagation.

In the study of properties of existing or newly developed cement-based composites the fracture parameters (fracture toughness, fracture energy, tensile strength etc.) have to be quantified. The determination of these parameters is based on standardized fracture experiments on specimens with stress concentrators (typically the three-point bending test, performed on notched beams, or the wedge-splitting testing of compact notched specimens). Subsequently, the results of these experiments in the form of diagrams showing load–deflection or load versus crack mouth opening displacement are evaluated by direct or indirect methods using one of the many fracture models.

In this paper, the double-*K* fracture model (from the pilot papers Reinhardt & Xu, 1999; Xu & Reinhardt, 1999a,b,c and further works up until e.g. the summarizing book Kumar & Barai, 2011) is used. In principle, this model combines the concept of cohesive forces acting on the faces of the fictitious (effective) crack increment with a criterion based on the stress intensity factor. This model can determine the critical crack tip opening displacement and the fracture toughness and is capable of describing different levels of crack propagation: an initiation part, which corresponds to the beginning of stable crack growth (at the level where the stress intensity factor, K_{Ic}^{ini}, is reached), and a part featuring unstable crack propagation (after the unstable fracture toughness, K_{Ic}^{un}, has been reached).

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An evaluation of three-point bending tests using the double-*K* fracture model is presented in this paper, with a principal focus on the effect of softening function type in this model for concrete with and without polypropylene fibres.

2. Fracture testing of concrete specimens

2.1. Material

Fresh concrete mixture was prepared from heavy-weight aggregates of 0–4 mm and 4–8 mm fractions, CEM I – 42.5 R cement, fly-ash, plasticizer, water and stabilizer. The water and stabilizer were dosed by volume, the remaining components by weight. Four mixtures were made: *OB_REF*, *OB_FF*19, *OB_FF*38 and *OB_FF*54. The reference mixture (*OB_REF*) was made without fibres, while the mixtures *OB_FF*19, *OB_FF*38 and *OB_FF*54 included FORTA FERRO polypropylene fibres of 19 mm, 38 mm and 54 mm in length, respectively. Details regarding the composition of the fresh concrete mixtures can be found in the paper Havlíková et al. (2012).

2.2. Concrete specimens, three-point bending tests

Three-point bending tests were performed on a total of twelve beams (comprising three specimens fabricated from each concrete mix) with a central edge notch to obtain the data described below. The nominal dimensions of the specimens were 100×100×400 mm, the depth of the central edge notch was about 1/3 of the depth of the specimen, and the loaded span was equal to 300 mm. A notch was cut before testing. Specimen age was 28 days.

The geometry of a specimen used in the three-point bending tests is shown in Fig. 1, where *D* is specimen depth, *B* is specimen width, *L* is specimen length, *S* is span; a_0 is the initial notch length. The output of the performed measurements was a set of load versus crack mouth opening displacement (*P–CMOD*) diagrams. An example showing data from one specimen from each concrete mix is shown in Fig. 1, where curves depict the typical ductile response of fibre reinforced concretes (*OB_FF).*

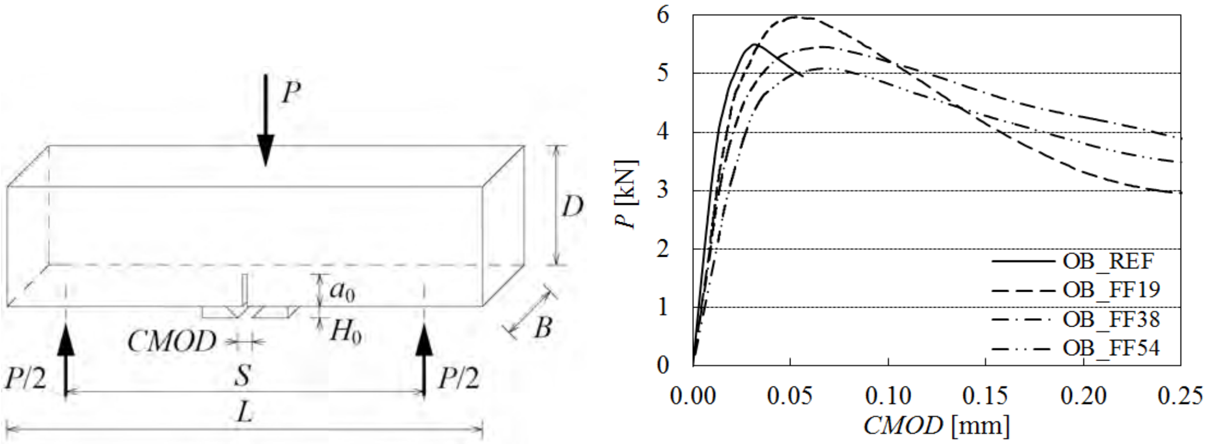

Fig. 1: Three-point bending fracture test geometry (left) and selected P–CMOD diagrams.

3. Application of the double-*K* fracture model

The measured *P–CMOD* diagrams are used to determine the fracture parameters of the double-*K* model. The unstable fracture toughness K_{Ic}^{un} is numerically determined first, followed by the cohesive fracture toughness K_{Ic}^{c}. When both of these values are known, the following formula can be used to calculate the initiation fracture toughness K_{Ic}^{ini}:

$$K_{Ic}^{ini} = K_{Ic}^{un} - K_{Ic}^{c}. \tag{1}$$

Details regarding the calculation of both unstable and cohesive fracture toughness can be found e.g. in Xu et al. (2003), and/or Zhang & Xu (2011).

When calculating the cohesive part of fracture toughness $K_{\mathrm{I}c}^{c}$ it is necessary to accept the assumption of the distribution of the cohesive stress σ along the fictitious crack. Generally, the relation between this cohesive stress σ and the fictitious crack opening displacement w is termed the cohesive stress function $\sigma(w)$. The cohesive stress $\sigma(CTOD_c)$ at the tip of the initial notch length a_0 at the critical state can be obtained from the softening curve. Four types of softening curve are used in the following text and calculations: linear, bilinear, and two exponential variants by Reinhardt (exp_R) and Karihaloo (exp_K).

If a linear softening curve is used, the value $\sigma(CTOD_c)$ can be calculated as follows:

$$\sigma(CTOD_c) = \frac{f_t(w_c - CTOD_c)}{w_c}, \tag{2}$$

where tensile strength f_t and critical crack tip opening displacement w_c are parameters of the softening curve. As indicated, $CTOD_c$ is critical crack tip opening displacement (see e.g. Kumar & Barai, 2011). In this paper, w_c is a constant value (0.16 mm) for all the softening curves. The tensile strength value is estimated using the measured compression strength value f_{cu} using the following relationship (Červenka et al., 2012):

$$f_t = 0.24 f_{cu}^{\frac{2}{3}}. \tag{3}$$

When using a bilinear softening curve there are two cases:

In case I, ($CTOD_c \le w_s$) can be obtained as a $\sigma(CTOD_c)$ value according to the formula:

$$\sigma(CTOD_c) = f_t - (f_t - \sigma_s)\frac{CTOD_c}{w_s}, \tag{4}$$

where σ_s and w_s are respectively the ordinate and abscissa at the point of slope change of the bilinear softening curve. According to Petersson (1981), these values can be considered using the following formulas:

$$\sigma_s = \frac{1}{3} f_t, \text{ and } w_s = \frac{2}{9} w_c. \tag{5}$$

In case II, ($w_s \le CTOD_c \le w_c$) can be calculated as a $\sigma(CTOD_c)$ value using the following equation:

$$\sigma(CTOD_c) = \frac{\sigma_s}{w_c - w_s}(w_c - CTOD_c). \tag{6}$$

When using the exponential softening curve by Reinhardt et al. (1986) a $\sigma(CTOD_c)$ value can be obtained using the expression:

$$\sigma(CTOD_c) = f_t\left\{\left[1 + \left(\frac{c_1 CTOD_c}{w_c}\right)^3\right]\exp\left(\frac{-c_2 CTOD_c}{w_c}\right) - \frac{CTOD_c}{w_c}\left(1 + c_1^3\right)\exp(-c_2)\right\}, \tag{7}$$

where c_1 and c_2 are the material constants. For normal concrete these dimensionless parameters are the following: $c_1 = 3$ and $c_2 = 6.93$.

In the case when the exponential softening curve by Karihaloo (1995) is used the $\sigma(CTOD_c)$ value can be calculated using the following formula:

$$\sigma(CTOD_c) = f_t \exp\left(-\mu\frac{CTOD_c}{w_c}\right), \tag{8}$$

where μ is a material constant with the assumed value $\mu = 4.6052$ for $\sigma = 0.01 f_t$.

4. Results

The relative mean values of selected material properties (compressive strength, modulus of elasticity, effective crack elongation, and unstable fracture toughness) are introduced in Tab. 1: the 100% value for each material parameter represents the values of those parameters for the reference concrete without fibres *OB_REF*. The figures below show the arithmetic mean, standard deviation and coefficient of variation values of the parameters to be determined: compressive strength (Fig. 2), elasticity modulus (Fig. 3), effective crack elongation (Fig. 4), and unstable fracture toughness (Fig. 5).

Relative mean values of ratio K_{Ic}^{ini}/K_{Ic}^{un} are introduced in Tab. 2; 100% represents: (i) the value of ratio K_{Ic}^{ini}/K_{Ic}^{un} for the linear softening curve for the appropriate concrete, (ii) the value of ratio K_{Ic}^{ini}/K_{Ic}^{un} for the reference concrete *OB_REF* for each type of softening curve. Arithmetic mean, standard deviation and coefficient of variation values of the ratio K_{Ic}^{ini}/K_{Ic}^{un} are introduced in Fig. 6 for all considered softening curves.

Tab. 1: Relative mean values of selected material parameters in %.

	Concrete			
Parameter	***OB_REF***	***OB_FF*19**	***OB_FF*38**	***OB_FF*54**
f_c	100.0	95.3	78.1	83.4
E	100.0	53.5	57.6	43.1
$a_c - a_0$	100.0	114.4	134.6	117.0
K_{Ic}^{un}	100.0	102.6	107.5	97.4

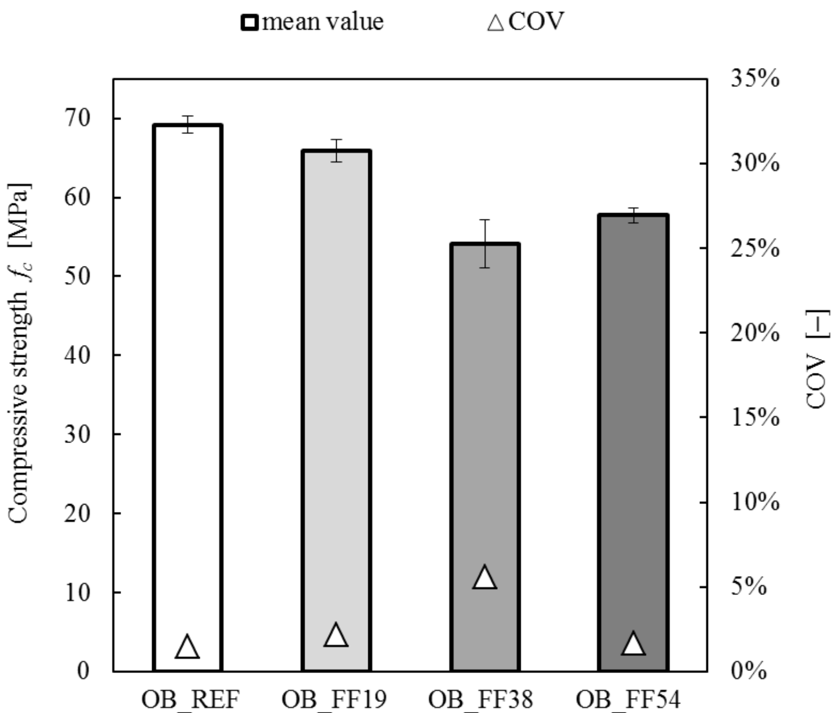

Fig. 2: Compressive strength f_c for the four concretes.

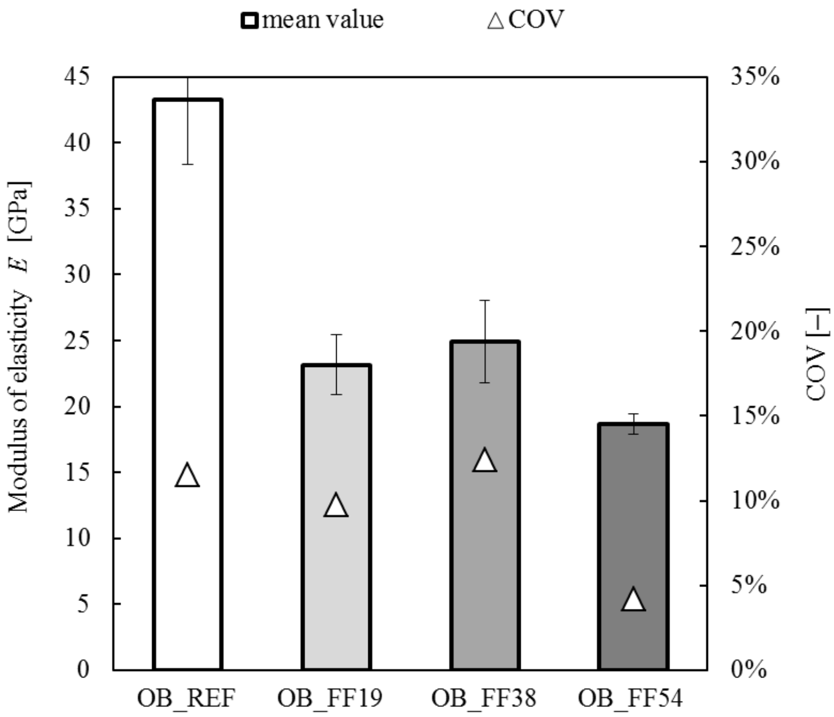

Fig. 3: Modulus of elasticity E for the four concretes.

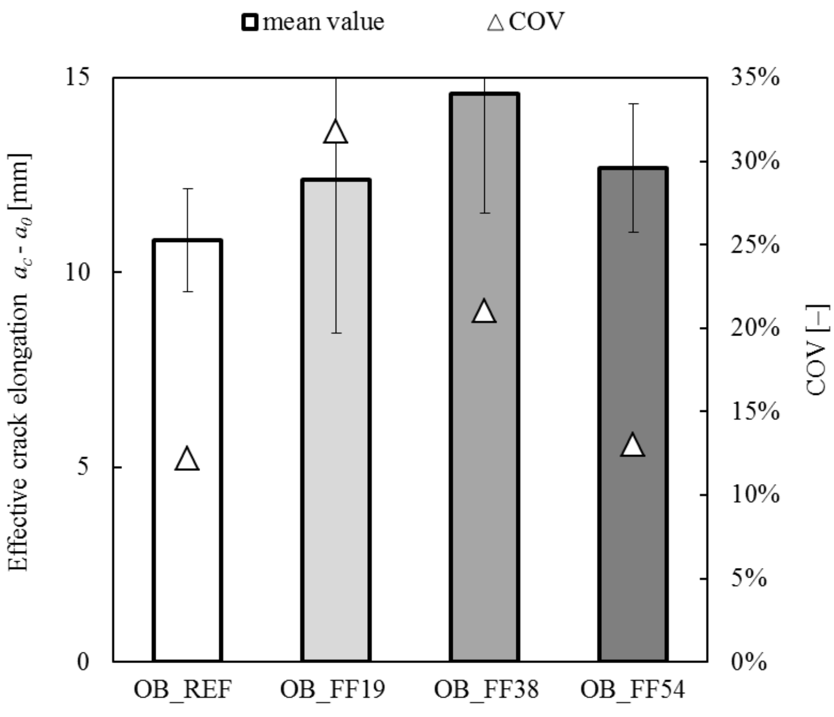

Fig. 4: Effective crack elongation $a_c - a_0$ for the four concretes.

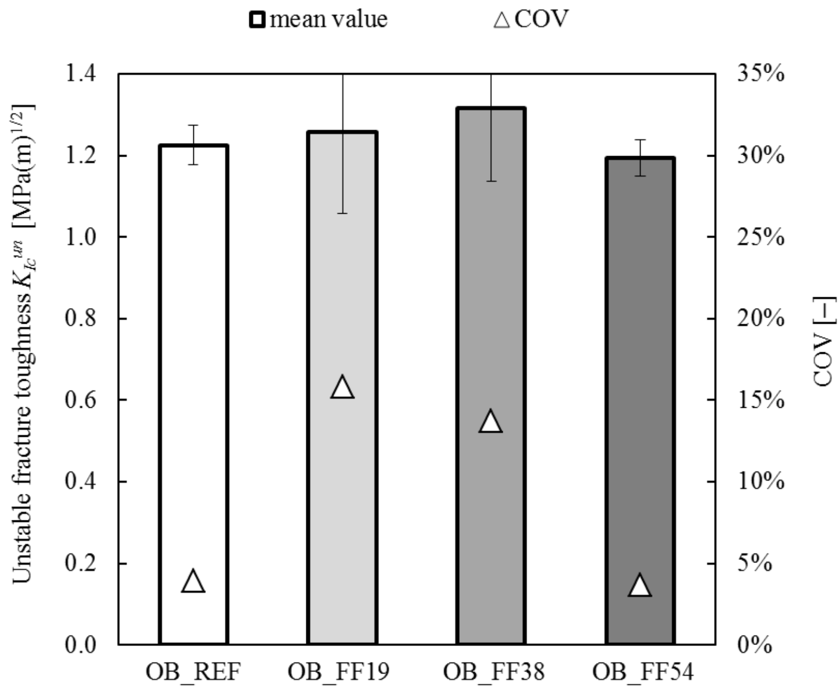

Fig. 5: Fracture toughness K_{Ic}^{un} for the four concretes.

Tab. 2: Relative mean values of ratio K_{Ic}^{ini}/K_{Ic}^{un} in %.

	Concrete			
Softening function	***OB_REF***	***OB_FF*19**	***OB_FF*38**	***OB_FF*54**
linear	100.0 | 100.0	100.0 | 98.8	100.0 | 106.9	100.0 | 103.3
bilinear	112.0 | 100.0	127.4 | 112.3	127.3 | 121.5	130.3 | 120.2
exp_R	126.6 | 100.0	142.1 | 110.9	140.1 | 118.4	142.7 | 116.4
exp_K	117.6 | 100.0	131.5 | 110.4	130.4 | 118.6	132.9 | 116.7

5. Conclusions

The conclusions can be divided into two parts: the first relating to the evaluation of concrete with/without fibres and second relating to the effect of the applied softening curve type on calculated results.

The presence of polypropylene fibres in the composite caused a reduction in the compressive strength values of 5 to 22 percent, and modulus of elasticity values were reduced by 46 to 57 percent. The largest reduction in compressive strength values was exhibited by concrete *OB_FF*38; in the case of elasticity modulus it was composite *OB_FF*54 that showed the largest fall. The effective crack elongation values of composites with fibres were from 14 to 35 percent higher in comparison with the reference concrete, the largest being in the case of concrete *OB_FF*38. The presence of fibres had no significant effect on the unstable fracture toughness values (composite *OB_FF*38 showed the largest relative increase, which was of less than 8 percent). In terms of resistance to stable crack propagation the addition of fibres appears to be a positive step – the highest relative increase in this resistance (over 20 percent) was reported by *OB_FF*38 concrete.

Using the selected softening curve has a significant effect on the determination of the resistance against stable crack growth for all investigated composites. Compared to a linear softening function, using a bilinear softening function leads to an increase in resistance of 12 to 30 percent, Karihaloo's exponential softening curve produced an increase of about 18 to 33 percent, and the highest increase was seen for Reinhardt's exponential softening curve: it was about 27 to 43 percent.

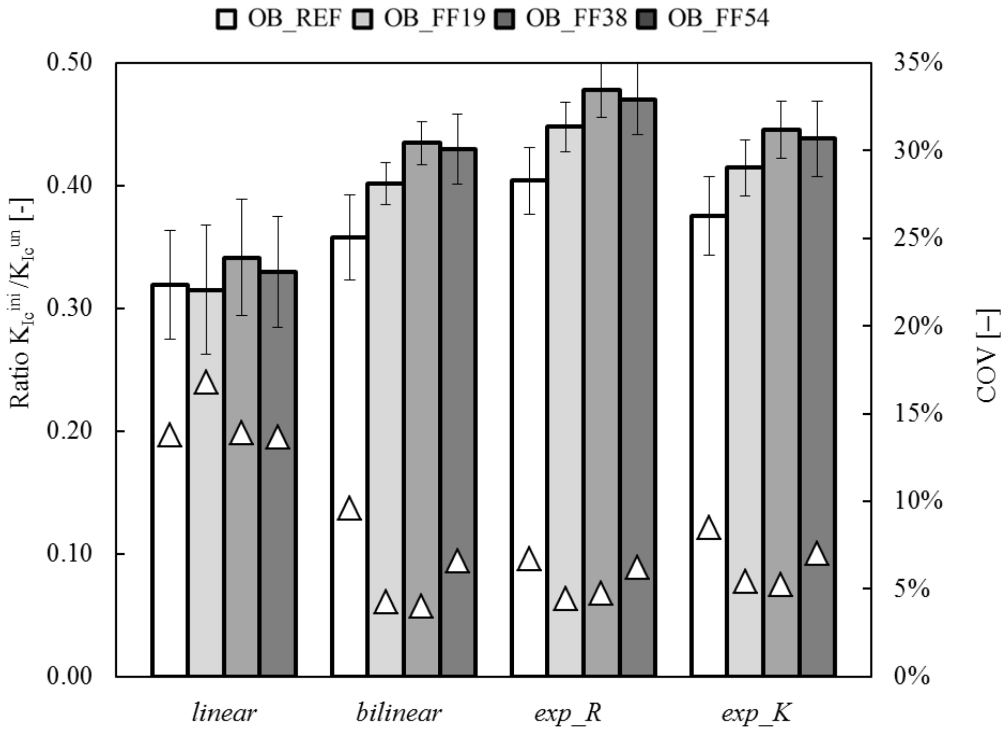

Fig. 6: Comparison of the effects of softening function types on calculated K_{Ic}^{ini}/K_{Ic}^{un} ratio for the four concretes.

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