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AEROELASTIC SENSITIVITY ANALYSIS OF AIRLINER WING

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Abstract: *This paper describes the airliner wing flutter sensitivity analysis. The sensitivity coefficients define the influence of the structural parameters changes to the structure eigenvalue and flutter stability characteristics. Evaluated structural parameters represent the possible changes of the structure due to the installation of the smart high-lift devices at the leading and trailing edge region. In general, we can suppose the increasing of the mass and mass moment of inertia around the elastic axis and decreasing of the stiffness. Described effects are ordinarily considered destabilizing regarding the flutter. The main aim of the presented work is to evaluate the impact of components to the stability and to define the most critical regions or parameters.*

Keywords: ***aeroelasticity, flutter, eigenvalue, sensitivity***

1. Introduction

As a part of the 7th Framework Programme of the European Community, there was accomplished the project focused on research and development of the smart high-lift devices. These devices which allow the smooth changing of the airfoil geometry can help to optimize aerodynamic characteristics of modernized wings. This can increase the operational efficiency of new generation airliners.

Smart high-lift devices are placed at the leading or trailing region of a wing, outside the main wing-box. Whereas we can expect a minor influence of their component to the wing integral stiffness, the mass of smart components placed far from the wing elastic axis may have some influence to a wing flutter characteristics.

The subject of the presented work is a sensitivity analysis. Sensitivity coefficients are defined as rate of change of a response parameter (e.g. eigenvalue or flutter one) with respect to change of a structural parameter (stiffness, inertia). The final aim of the task is evaluation of critical areas or parameters with respect to a structure flutter behavior, possibly also formulation of recommendations for a wing structural design and critical values for particular parameters.

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2. Theoretical Background

Design sensitivity analysis computes the rates of change of structural response quantities (e.g. weight, strain, stress, modal frequency, dynamic response, flutter stability etc.) with respect to change of the design variables. Design variables are quantities which are changeable, related to the properties of a structure model. This relation may become either a linear combination of design variables:

$$(p_j = C_0 + \sum_i C_i x_i) \tag{1}$$

or a general, also non-linear function:

$$(p_j = f(\{x\},\{C\})) \tag{2}$$

Sensitivity coefficients are evaluated at a particular design characterized by the vector of the design variables $\vec{x}^0$, giving:

$$\lambda_{ij} = \left(\frac{\partial r_j}{\partial x_i}\right)_{\vec{x}^0} \tag{3}$$

where subscripts are used to indicate i-th design variable and the j-th response. Eqn. (3) is just the slope of the response with respect to the design variable as shown in fig.1.

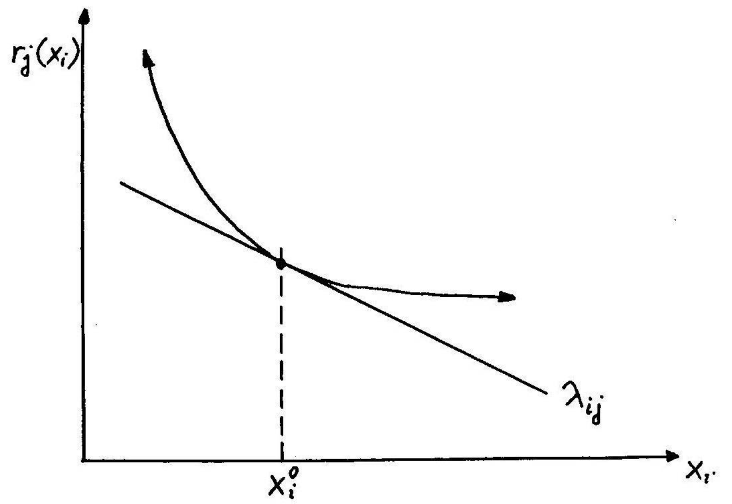

Fig.1: Graphical interpretation of the sensitivity coefficient

There were used two types of the design response in the presented task:

1) <u>Eigenvalue response sensitivity</u>

The eigenvalue equation is:

$$([K] - \lambda_n [M])\{\phi_n\} = 0 \tag{4}$$

where λ_n and ϕ_n are the n-th eigenvalue and eigenvector respectively. ***[K]*** is the structural stiffness and ***[M]*** is the structural mass matrix. The eqn.(4) can be differentiated with respect to the i-th design variable x_i:

$$\left([K]-\lambda_n [M]\right)\frac{\partial\{\phi_n\}}{\partial x_i}+\left(\frac{\partial [K]}{\partial x_i}-\lambda_n\frac{\partial [M]}{\partial x_i}\right)\{\phi_n\}=\frac{\partial \lambda_n}{\partial x_i}[M]\{\phi_n\} \tag{5}$$

When the eqn.(5) is premultiplied by $\boldsymbol{\phi_n}^{\boldsymbol{T}}$, the first term become zero and eqn.(5) can be then solved for the eigenvalue derivatives:

$$\frac{\partial \lambda_n}{\partial x_i}=\frac{\{\phi_n\}^T\left(\frac{\partial [K]}{\partial x_i}-\lambda_n\frac{\partial [M]}{\partial x_i}\right)\{\phi_n\}}{\{\phi_n\}^T[M]\{\phi_n\}} \tag{6}$$

In practice the solution of the eqn.(6) is based on the semi-analytical approach. The derivatives of the mass and stiffness matrices are approximated using the finite differences. Equation is solved for each retained eigenvalue referenced in the design model and for each design variable.

2) <u>Aeroelastic flutter response sensitivity</u>

Aeroelastic flutter stability equation is given by:

$$\left[M_{hh}p^2+\left(B_{hh}-\frac{1}{4}\rho\,\bar{c}\,V\frac{Q_{hh}^{\mathrm{Im}}}{k}\right)p+\left(K_{hh}-\frac{1}{2}\rho\,V^2\,Q_{hh}^{\mathrm{Re}}\right)\right]\{u_h\}=0 \tag{7}$$

The eqn. (7) represents the PK-method of the flutter solution, which is only method applicable for the design sensitivity purposes. $\boldsymbol{M_{hh}}$; $\boldsymbol{B_{hh}}$ and $\boldsymbol{K_{hh}}$ are modal mass, damping and stiffness matrices respectively, which are a function of Mach number ***(M)*** and reduced frequency ***(k)***. $\boldsymbol{Q_{hh}}^{\boldsymbol{Re}}$ and $\boldsymbol{Q_{hh}}^{\boldsymbol{Im}}$ are real and imaginary part of a complex aerodynamic matrix, which is also a function of parameters $\boldsymbol{M}$ and $\boldsymbol{k}$. ρ is an air density, $\bar{c}$ is a reference length and $\boldsymbol{u_h}$ is a modal amplitude vector. The eigenvalue $\boldsymbol{p}$ is given as:

$$p=\omega\left(\gamma\pm j\right) \tag{8}$$

and γ is a transient decay rate coefficient. Note that structural damping coefficient is:

$$g = 2.\gamma \tag{9}$$

Flutter sensitivity computes the rates of change of this transient decay rate coefficient γ with respect to changes of the design variables. Eqn.(7) is differentiated with respect to the design variables for the quantity:

$$\left(\frac{\partial\gamma}{\partial x_i}\right) \tag{10}$$

The solution is semi-analytic in nature with derivatives approximated using either forward differences or central differences.

3. Analytical Procedure

Analytical approach is based on FE analysis. The FE model must include separate elements for the wing-box, leading edge and trailing edge regions, thus the detailed FE model is envisaged. Considering the dynamic analysis, such model includes local modes which do not affect the global dynamics and the structure stability. Such modes make the analysis unclear. Also the effect of a computational time and disk-space saving may be significant. Therefore, the model is reduced by means of the standard Guyan reduction (also called static condensation). Obviously, there is a minor difference between the full and reduced model modes, since the reduction is based on the partition of the stiffness matrix.

The next step is a flutter analysis. The aerodynamic model for the simulation of the unsteady aerodynamic forces is based on the Doublett-Lattice Subsonic Lifting Surface Aerodynamic Theory. The theory was presented by Albano and Rodden in 1969, the theoretical basis is linearized aerodynamic potential theory. The lifting surfaces are modeled by the trapezoidal flat panels, which are parallel to the flow. Each of aerodynamic macroelements is divided into small trapezoidal lifting elements (boxes) in strips parallel to the free stream with the surface edges, fold lines and hinge lines on the box boundaries. The flutter stability is calculated by eqn.(7).

The purpose is to find the target flutter instability, which are the sensitivity coefficients calculated to. The flutter speed and frequency as well as the flutter shape and contributing modes are evaluated. The flutter calculation have a character of the non-matched analysis. In the non-matched analysis, there is used just one reference Mach number for the whole range of velocities. Aerodynamic forces are given from the model calculated for this reference Mach number. Analysis velocities do not match the Mach number, therefore the results have a character of artificial states. Such approach is frequently used in the flutter analysis, because it allows to evaluate the rate of reserve in the flutter stability with respect to the specific velocity (e.g. certification velocity). Also, it allows to perform a sensitivity analysis.

The final step is the sensitivity analysis. Sensitivity coefficients are calculated for a specific property of elements with respect to: 1) frequencies of the flutter major contributing modes and 2) to damping of the target flutter mode. We used the following types of the design variables:

1) stiffness characteristics: ***E*** - Young's modulus; ***G*** - shear modulus; ***E*** and ***G*** values linked by the equation:

$$\frac{E}{G} = 2\left(1 + NU\right) \tag{11}$$

2) inertia characteristics: ρ - density;
3) geometry characteristics (influencing both stiffness and inertia): T - shell element thickness.

Design variables were connected to the elements at the local level, it means that each element (with own property and material input) was specified as a separate design variable. The elements of the wing part out of the wing box were used as design variables. Further examples of the aeroelastic sensitivity analysis and optimization can be found in the referenced papers.

4. Application example - airliner wing / engine component model

The first application example represents the narrow body airliner wing with an engine on a pylon. The structural model is shown in the fig.2.

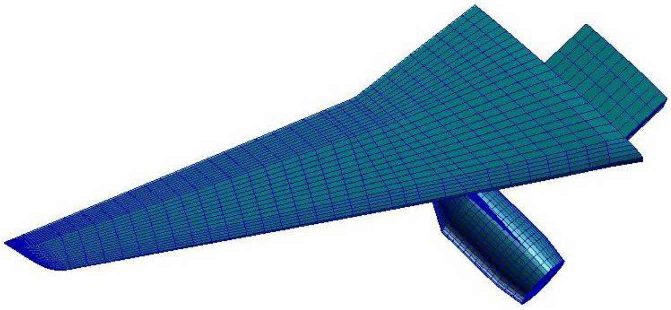

Fig.2: Airliner wing - structural model

The structural model includes main load carrying structural elements modeled by means of beam and plate elements. The residual structure inertia characteristics are included by means of concentrated mass elements. The aerodynamic model consist of a wing, pylon, engine and splitter. The wing is modeled by means of seven macroelements in order to hold the wing planform shape with enough accuracy. The pylon is modeled by one macroelement. The engine is modeled by means of cross-surface model. It includes two horizontal and two vertical macroelements with the root chord at the engine centerline. The splitter avoiding the boundary effect at the wing root is modeled via one macroelement. The density of panelization is made considering the importance of a particular part of the model with respect to the flutter stability (e.g. increasing the density spanwise from the root to the tip, increasing the density at the leading or trailing edge region). The aerodynamic model is presented in fig.3.

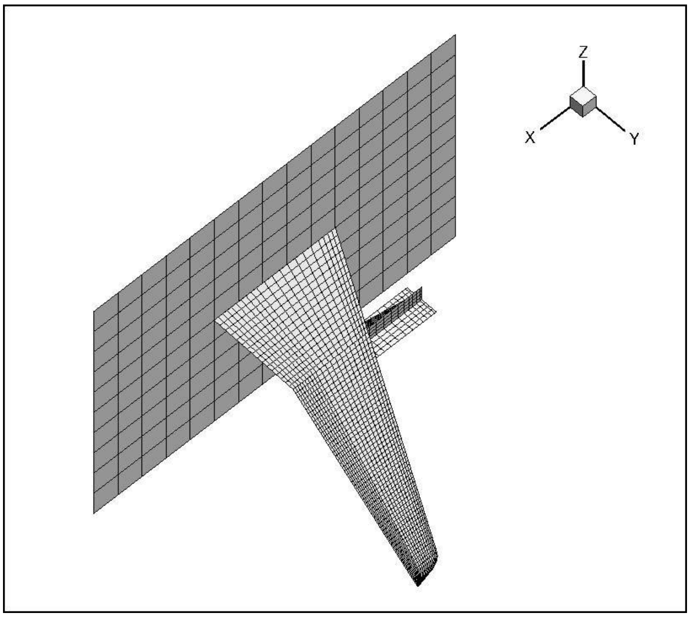

Fig.3: Airliner wing - aerodynamic model

The interpolation between the structural and aerodynamic model was realized by means of the surface splines. The spline function transforms the aerodynamic loads into the structural model and structural deformations into the aerodynamic model. The surface spline is based on the infinite plate. The spline surface function ***w(x,y)*** is a smooth function based on the discrete set of known points.

The modal characteristics were calculated by means of standard Lanczos method. The summary of the 8 lowest natural frequencies is given in the tab.1.

Tab. 1: Airliner wing natural frequencies

#	title	f_0 [Hz]
1	Wing 1st vertical bending	2.216
2	Engine vertical vibrations (y-axis)	2.551
3	Engine horizontal vibrations (x-axis)	3.756
4	Wing 2nd vertical bending	5.472
5	Wing 1st horizontal bending	8.884
6	Wing 3rd vertical bending	12.447
7	Wing 1st torsion	17.795
8	Wing 4th vertical bending	22.785

The flutter analysis of the reference state was performed by means of the PK method which is also applicable for the sensitivity analysis. The analysis included 14 modes. The structural damping was included via common value of the damping ratio of 1%. The density was considered $\rho = 1.225\ kg.m^{-3}$ (ISA value for $H = 0$). There was found the flutter state of the bending torsional flutter. The flutter velocity was $V_{FL} = 400.04\ m.s^{-1}$; the flutter frequency was $f_{FL} = 13.87\ Hz$. The flutter shape is presented in fig.4, the primary flutter mode was *#7* (1st wing torsion), the critical combination of the modes was: 1st and 3rd wing bending and 1st wing torsion (*#1; #6; #7*). The V-g-f diagram is presented in the fig.5.

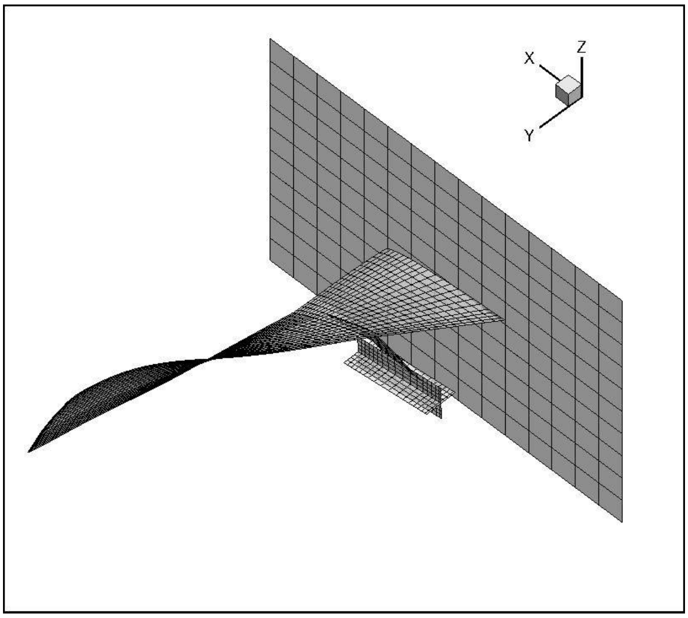

Fig.4: Airliner wing - flutter shape

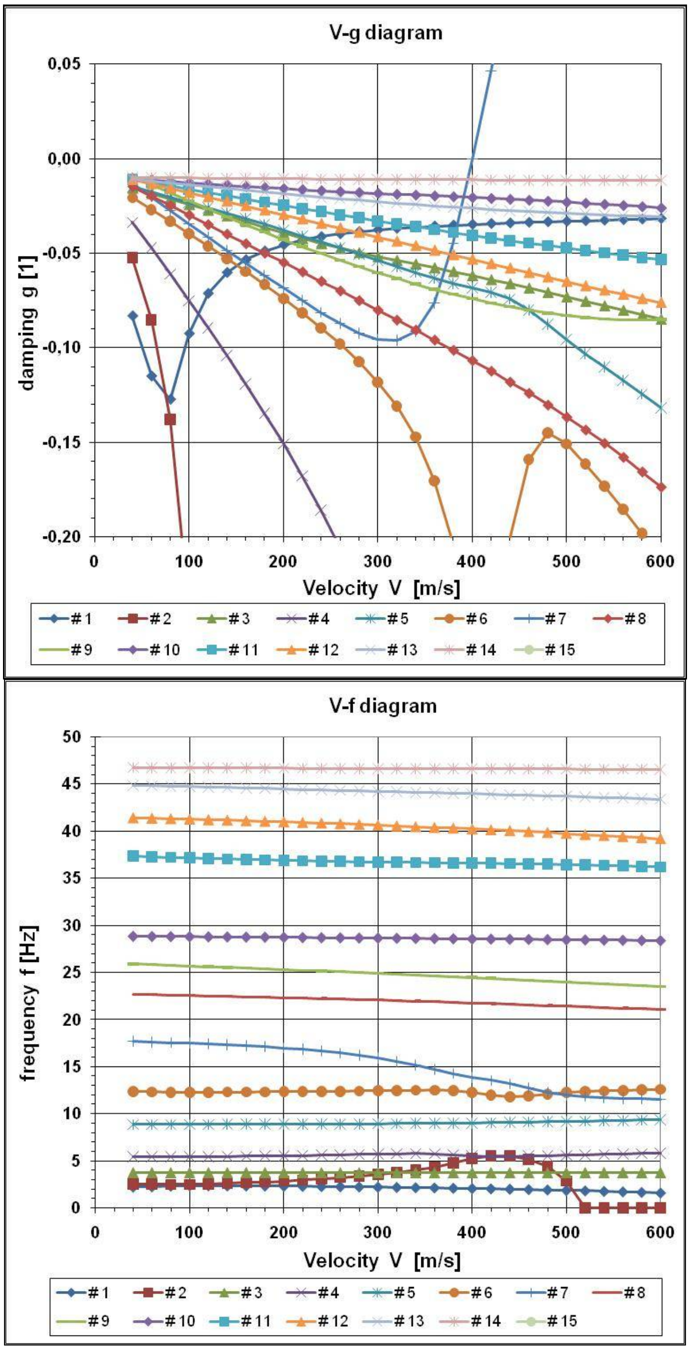

Fig.5: Airliner wing flutter - V-g-f diagram

The resulting sensitivities are presented as the normalized values. The normalization was performed with respect to the maximum value within the same type of the design variables (stiffness, inertia, geometry) and the same type of design response (eigenvalue, flutter).

The fig. 6 and 7 show the results. The fig.6 shows the normalized sensitivities of the leading edge upper skin thickness to the eigenvalue responses whereas fig.7 shows the sensitivities to the flutter responses. The inertia design variables have much higher sensitivities then stiffness ones. As apparent from the fig.7, flutter sensitivities are negative at the leading edge region (increasing of the design variable have a stabilizing effect). The maximal destabilizing effect has the increasing of the mass at the trailing edge region around the spanwise section *23*, which is approximately at 70% of the wing half-span.

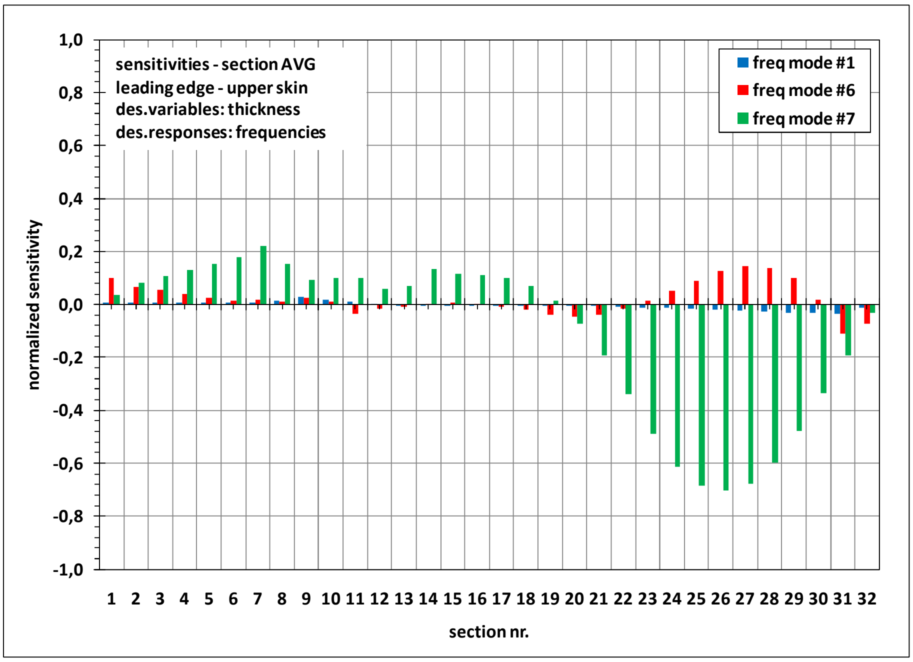

Fig.6: Airliner wing - eigenvalue response normalized sensitivities

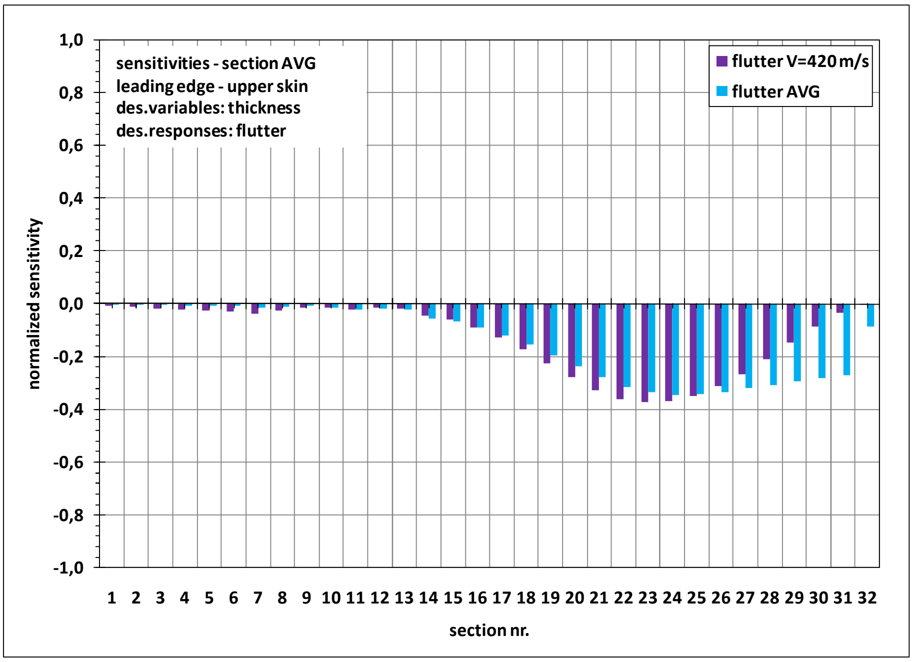

Fig.7: Airliner wing - flutter response normalized sensitivities

5. Application example - high aspect ratio wing component model

The second application example represents the high aspect ratio low swept wing. The structural model is shown in the fig.8.

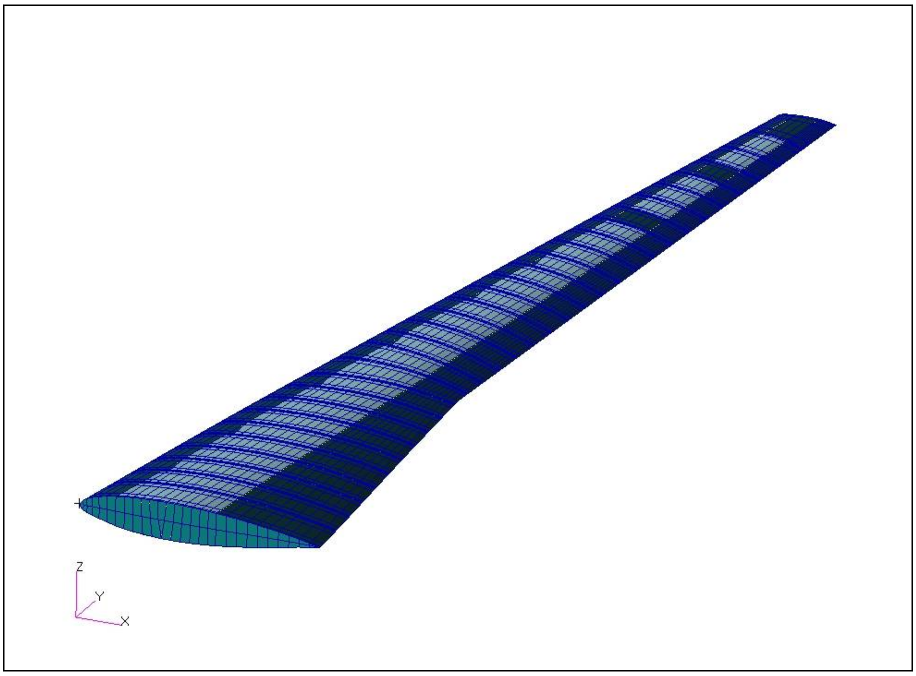

Fig.8: High aspect ratio wing - structural model

Structural model is the same type as the previous one. The aerodynamic model includes the wing and splitter. The wing model consists of two macroelements in order to hold the wing planform shape. The panelization rules and the interpolation method are the same as for the previous example as well. The aerodynamic model is shown in the fig.9.

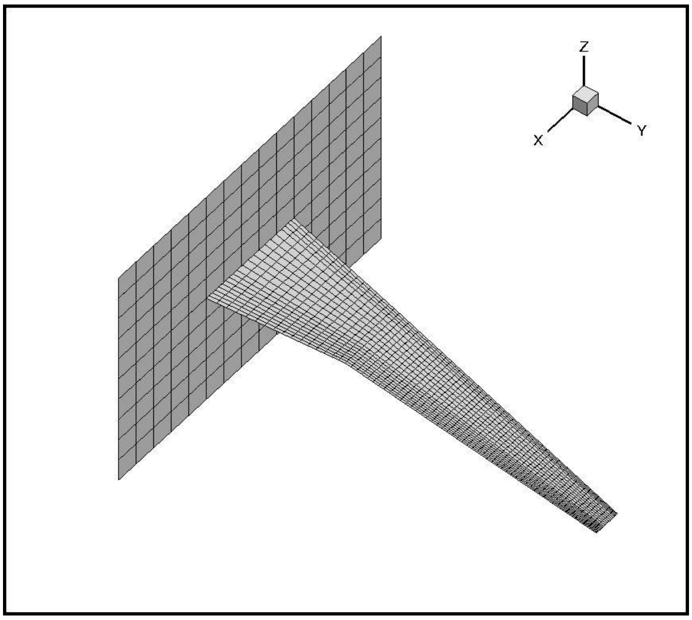

Fig.9: High aspect ratio wing - aerodynamic model

The modal and flutter analyses were performed also in the same way as described in the previous example. The modal characteristics are summarized in the tab.2, there are presented 8 lowest natural frequencies.

Tab. 2: High aspect ratio wing natural frequencies

#	title	f_0 [Hz]
1	Wing 1st vertical bending	2.744
2	Wing 2nd vertical bending	7.286
3	Wing 1st horizontal bending	11.309
4	Wing 3rd vertical bending	15.436
5	Wing 4th vertical bending	25.818
6	Wing 1st torsion	26.606
7	Wing 2nd horizontal bending	31.191
8	Wing 1st vertical bending	38.755

The flutter velocity of the bending torsional flutter was $\boldsymbol{V_{FL} = 379.8\ m.s^{-1}}$; the flutter frequency was $\boldsymbol{f_{FL} = 18.36\ Hz}$. The flutter shape is presented in fig.10. The primary flutter mode was *#5*, the critical combination of the modes was (*#1; #5; #6*). The V-g-f diagram is presented in the fig.11.

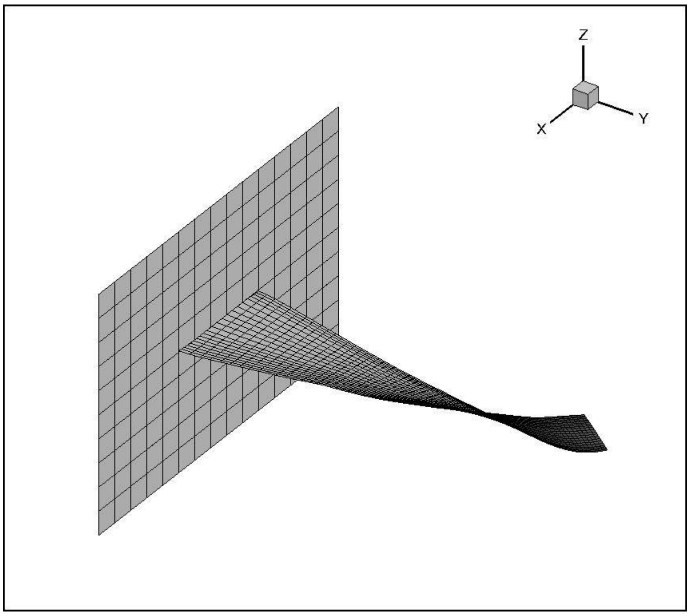

Fig.10: High aspect ratio wing - flutter shape

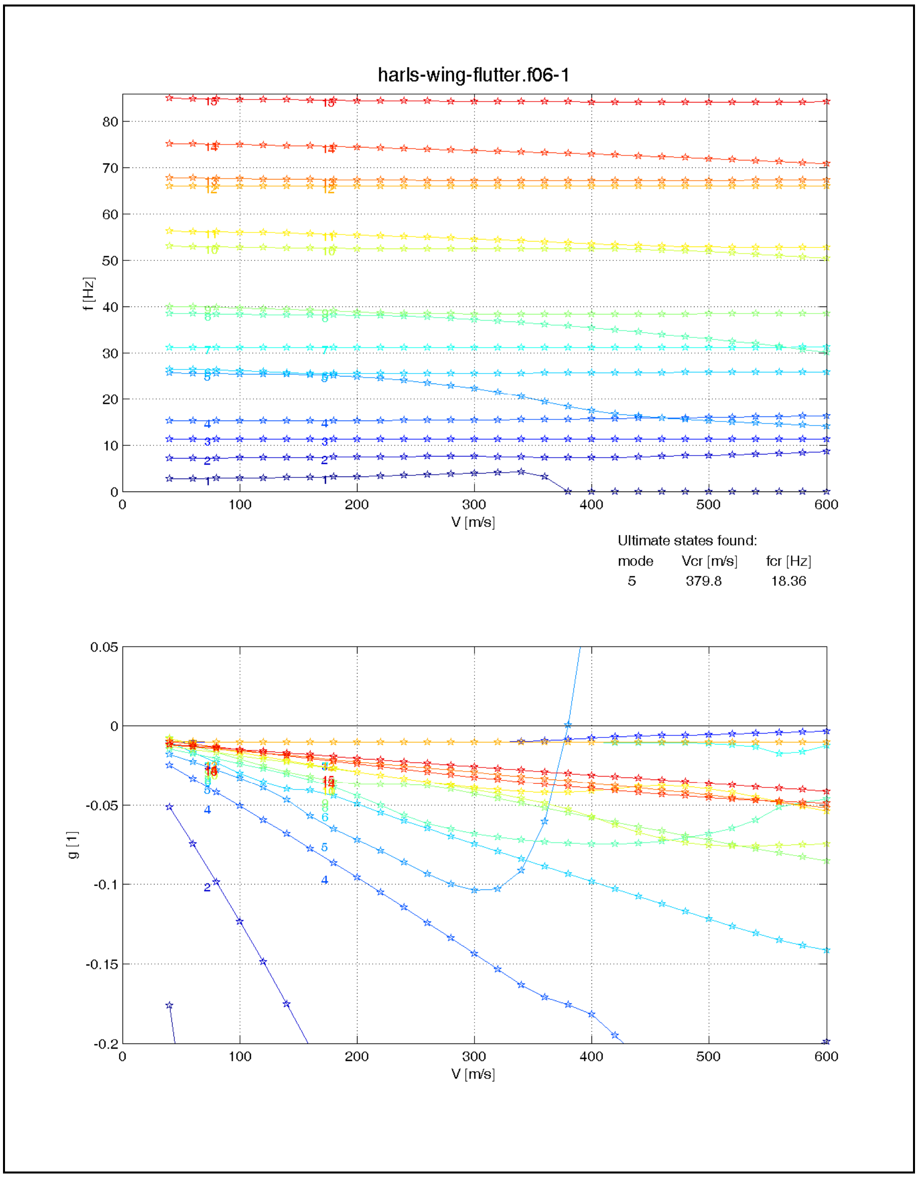

Fig.11: High aspect ratio wing - V-g-f diagram

The fig.12 and 13 show the results. The resulting sensitivities are presented as the normalized values as in the previous example. The results are the similar as the previous example, the inertia design variables have much higher sensitivities then the stiffness ones and flutter sensitivities are negative at the leading edge region and positive at the trailing edge region. Contrary to the previous example the highest sensitivities are at the wing tip region.

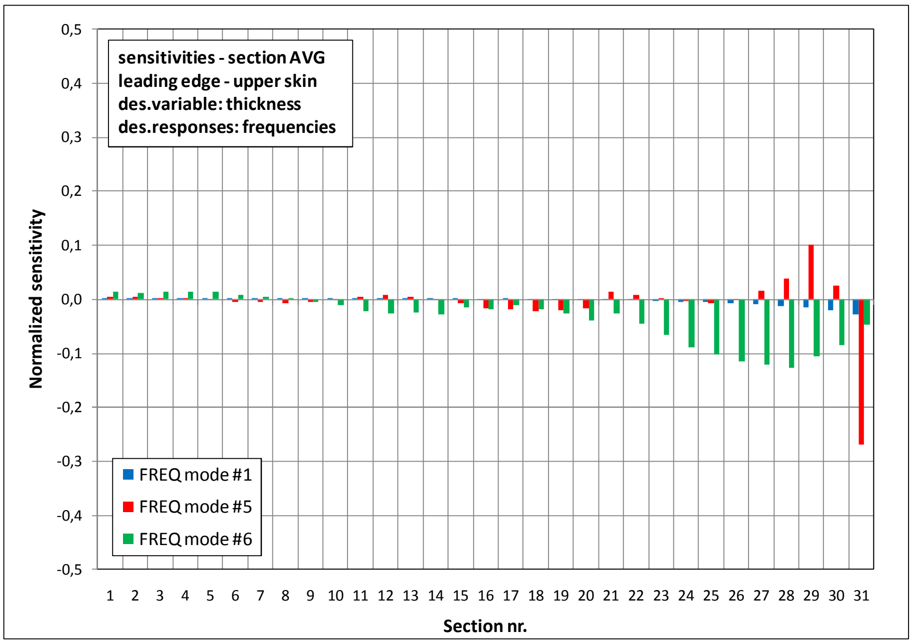

Fig.12: High aspect ratio wing - eigenvalue response normalized sensitivities

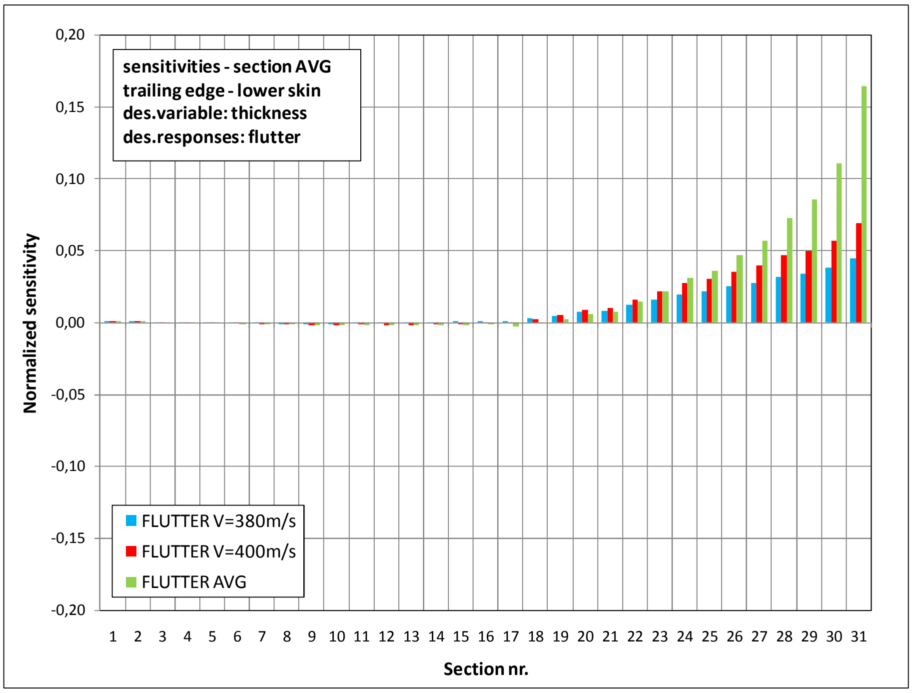

Fig.13: High aspect ratio wing - flutter response normalized sensitivities

6. Conclusion

Submitted paper presents aircraft wing flutter sensitivity analyses. The eigenvalue and flutter sensitivity coefficients are calculated for the leading and trailing edge region structural parameters. Structural stiffness and inertia parameters represent the possible structural changes due to installation of the smart high-lift devices. The sensitivity coefficients are calculated with respect to the natural frequencies of the flutter major modes and the flutter stability responses. The procedure is demonstrated on the two examples: the narrow-body airliner wing and the high aspect ratio wing. The most sensitive are inertia parameters, the critical region is the trailing edge region. In terms of the spanwise direction, the former example critical area is around 70-75% of half-span, whereas the latter example critical region is the wing tip.

7. References

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