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SIMULATION OF HOST ROCK CRACKING DUE TO PRESSURE CHANGES IN MAGMA CHAMBER

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Abstract: *Calderas are volcanic structures, which can be dangerous but also beneficial at the same time. Therefore, it is important to understand complex processes leading to a formation of the calderas. For this purpose, numerical simulations using the finite element method can be employed very well. In our paper, we look for a simplified, yet still realistic concept of modeling of a host rock and a magma chamber. Results indicate that our approach is able to capture phenomena, which are described by geologists in field studies. This work shows that it is possible to model a complex volcanological process using a relatively simple concept.*

Keywords: ***numerical modeling, quassibrittle, caldera, host rock, magma chamber***

1. Introduction

Calderas are volcanic depressions caused by rupturing of a magma chamber roof as a consequence of a pressure evolution inside a magma chamber. One can easily mistake calderas for apical craters, which can be indeed very similar at the first glance. However, the difference is that the craters result from a collapse and a subsequent blocking of a volcanic conduit without affecting the roof of the magma chamber and they have many times smaller diameter than the calderas.

A formation of a caldera is usually accompanied by volcanic eruptions. These eruptions represent a serious risk because even a single eruption can kill thousands of people in a few moments, as described by e.g. Sigurdsson (2000); Blong (2003); Witham (2005). On the other hand, volcanic processes can be also beneficial for the society as they may contribute to formation of ore deposits, may serve as geothermal energy resources and may produce fertile soils. Another reason why scientists pay attention to calderas is that they can partially reveal vast magmatic processes ongoing under the surface. These processes are usually hidden and we can observe just surface demonstrations like volcanoes and their eruptions.

2. Geological Phenomena Associated with Caldera Formation

Generally, volcanic processes can be divided into pre-eruptive and eruptive. The pre-eruptive processes (e.g. Martí and Folch (2005); Gudmundsson (1998); Burov and Guillou-Frottier (1999)) are the ones leading to the volcanic eruptions (e.g. cooling, differentiation, mixing, degasification, magma chamber rupture etc.).

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The eruptive processes can be observed inside the magma chamber and in the conduit during a magma discharge, and in the atmosphere during the eruption. The processes in all these parts are physically interconnected.

In the present study we focus our attention on the fracture of a magma chamber roof that eventually leads to its collapse and formation of a caldera. Therefore, only phenomena essential to this process are taken into account in the proposed model. We consider a magma income in a form of a fresh magma injection from lower stratas of the Earth. An increasing volume of magma causes an overpressure in the magma chamber and when the tensile strength of the surrounding rock is exceeded, cracks occur. An eruption is assumed to take place when the cracks propagating from the chamber reach the Earth surface. The subsequent depletion of the chamber causes a decrease of the magma pressure, which results in the loss of magma chamber roof support and its caving in.

3. Model

To numerically simulate the above-mentioned phenomena, we construct a finite element model of a magma chamber embedded in a large body of a host rock. As the focus is on the host rock fracture, it is modeled as a quasibrittle solid. The magma is assumed to act on the host rock as a nearly incompressible fluid. However, its flow within the chamber as well as into the formed cracks are not modeled, as they are considered having minor effect on the crack initiation. Instead, magma is represented as a nearly incompressible elastic region, whereas the injection and release of magma is modeled by applying eigenstrain (expansion and contraction, respectively) to it. Furthermore, the interface between the magma and the host rock is assumed to transfer only pressure, but no tension and shear. All these phenomena can be well represented by the material models implemented in the ATENA 2D software (Červenka et al. (2012)), which is thus used for the calculations discussed hereafter. Specifically, we use version 4.3.1.0.

3.1. Host Rock

Obviously, the host rock is the most important material because this is the medium in which the crack development and consequently the collapse take place. For this purpose, a material model originally developed for concrete called *3D Nonlinear Cementitious 2 User* is utilized. The constitutive model (Červenka et al. (2012); Červenka and Papanikolaou (2008)) combines quasibrittle approach for fracture in tension and plasticity for compression. The fracture model is based on the classical smeared crack formulation and the crack band concept. It employs the Rankine failure criterion, a user-defined softening relationship (linear softening is employed in our case) and it can be used as a rotated or fixed crack model (fixed crack is used in this study). The hardening/softening plasticity model is based on the Menétrey-Willam failure surface.

Considering that in the particular studied case the host rock is an igneous volcanic rock rhyolite, the relevant material characteristics were adopted from the literature and are summarized in Table 1.

3.2. Magma and Interface Elements

For simplicity's sake, magma is not represented as a fluid but it is modeled using as an elastic isotropic material. A density of the rhyolite magma is taken from Shervais (2001). Considering that the magma melt has a high gas content, the value of its Young's modulus is assumed to

Table 1: Physical and mechanical properties of rhyolite and magma

material / material model	ρ [kg/m^3]	E [GPa]	ν [-]	f_c [MPa]	f_t [MPa]	source
rhyolite / 3D Nonlin. Cem. 2	2700	80	0.25	180	10	Zhao (2007)
magma / elastic isotropic	2200	8	0.49	—	—	

be one tenth of that of the solid rhyolite. The Poisson's ratio is set equal to 0.49 to simulate the near-incompressibility of the fluid, while still allowing us to use a standard FE formulation. The parameters of the magma are listed in Table 1.

Since a fluid magma should not transfer any significant shear and/or tension to the surrounding rock, interface elements are inserted along the boundary between magma and the host rock. The interface is assumed to follow Mohr-Coulomb failure criterion with very low cohesion and tensile strength and zero friction coefficient. To prevent an overlapping of the neighboring domains, relatively high initial stiffnesses (normal and tangential) are assigned to the interface elements. The particular values of the interface parameters are summed up in Table 2. They were estimated on a trial and error basis so as to satisfy the requirements that a potential separation occurs in the interface elements and not the host rock, that the shear transferred to the host rock is negligible and that numerical instabilities do not occur during the calculations.

Table 2: Properties of interface elements

normal stiffness K_{nn} [MN/m^3]	tangential stiffness K_{tt} [MN/m^3]	tensile strength f_t [MPa]	cohesion C [MPa]	friction coefficient ϕ [-]
2×10^8	2×10^8	5	0.5	0

A graphical interpretation of the parameters assigned to the interface elements is shown in Figure 1.

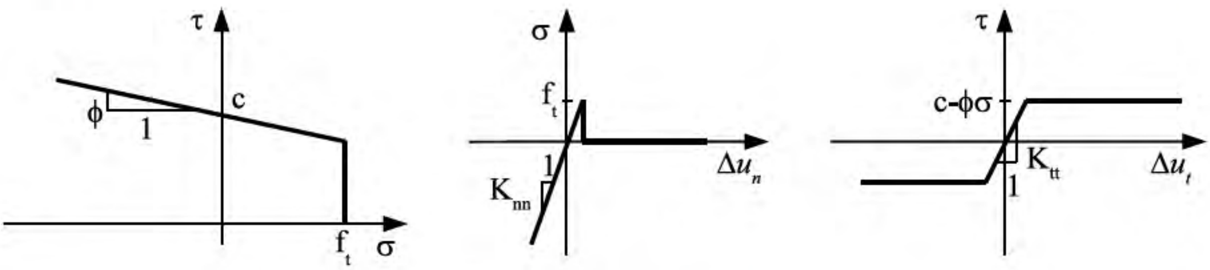


Figure 1: Failure condition and stress-displacement laws

3.3. Geometry

The volcanic structure is modeled as a magma chamber completely filled with magma located 5 km beneath the Earth surface. Although in reality a plan view of a magma chamber may be round but irregular, we idealize it as circular which allows us to exploit the simplification resulting from axial symmetry. We consider a sill-like magma chamber, which is frequently occurs in the nature and is characterized by a flat shape. Therefore the vertical section of the chamber is modeled as na oval composed from a rectangle and a semi-circles connected to its sides. The diameter of the chamber is 22 km and the height is 5 km. This magma chamber is surrounded by a host rock. Dimensions of the whole host rock body are chosen as 200 km in diameter and 37.5 km the height in order to minimize the effect of boundary conditions. The configuration is shown in Figure 2.

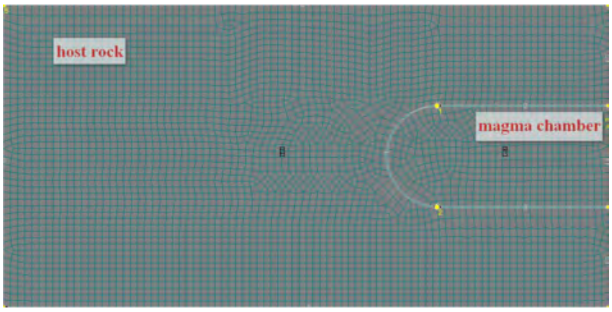


Figure 2: Magma chamber surrounded by rhyolite host rock

The analyzed domain is discretized by 4-noded quadrilateral finite elements (ref fig). The typical size of the finite elements assigned to the host rock near the magma chamber is 350 m. These unusually big elements are a consequence of huge dimensions of the system. Nevertheless, further from the magma chamber, the mesh is coarser. The size of elements in the magma subdomain is also about 350 m to preserve the regularity of the mesh in a transition zone from magma to the host rock.

3.4. Boundary Conditions and Loading

The following boundary conditions are applied in the model: The vertical sides, both the outer one and the one at the symmetry axis are assigned zero radial displacement and are allowed free vertical displacement. Both components of displacement are constrained at the bottom side. The upper surface is traction-free.

As briefly described in Section 2., loading is applied in three stages. First, a self-weight is applied. The self-weight is applied in 5 increments. In each of these increments, a small amount of uniform expansion eigenstrain is applied to the magma chamber to counter its elastic deformation, which would otherwise result in deflection and cracking of the chamber roof. The second stage corresponds to the injection of fresh magma from the lower strata. This process is simulated by increasing the expansion eigenstrain assigned to the magma domain. The chamber expansion is terminated once there is a crack in the roof connecting it to the surface. After that, the final stage takes place — the volume of magma inside the chamber decreases due to the eruption. To simulate the release of magma, shrinkage is applied to the magma body. The final stage goes on until an initiation of the roof collapse occurs. Typically, the total amount of eigenstrain applied in stage one was -0.031%, in stage two 0.29% and in stage three -1.33%.

4. Results and Discussion

A parametric study was carried out to identify the appropriate value of the critical crack opening displacement δ for the host rock, at which the cohesive stress of a crack is completely released. It is noted that in the context of the employed fracture model, this value can be interpreted as the total opening of a set of parallel cracks within a crack band, which is calculated based on the element size (Červenka et al. (1995)). Considering the typical element size used (∼350 m), it is plausible to assume that several cracks actually form along this length.

Figure 3 shows the effect of the parameter δ on the calculated vertical surface displacement after magma release, which can be interpreted as the a shape of the caldera. It is seen that with low values of δ (0.5 and 1 m), which correspond to relatively brittle cracking, instabilities occur, which result in uneven surface displacements. On the contrary, smooth surface deflection is obtained with more ductile material (δ equal to 2 and 5 m). From these two values, the lower one was chosen for the further calculations as it was considered as more physically realistic.

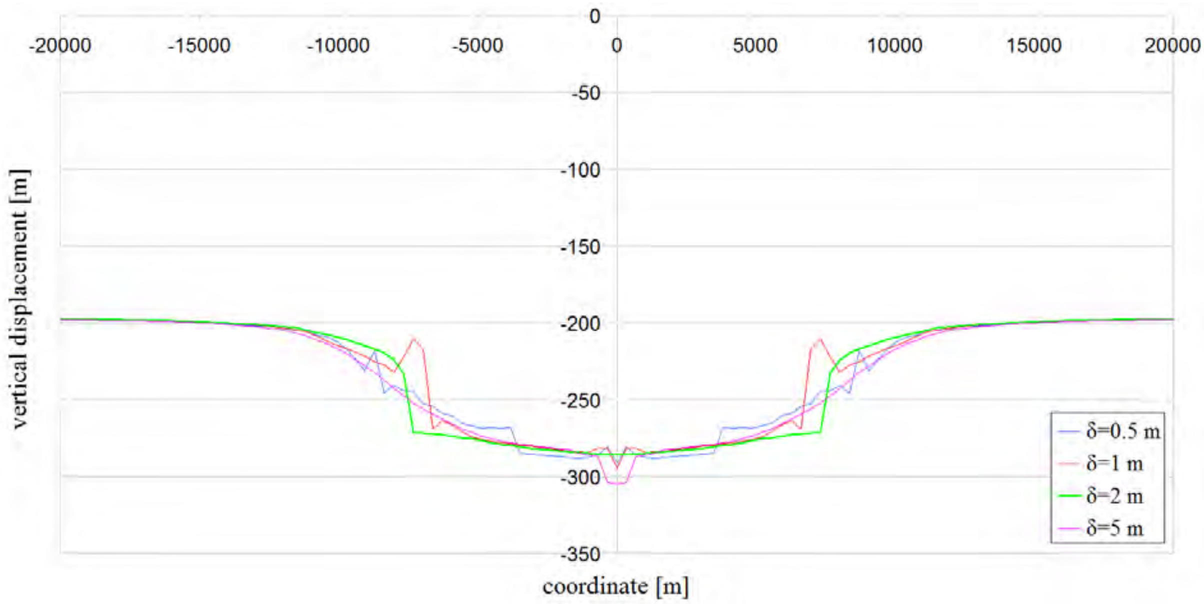
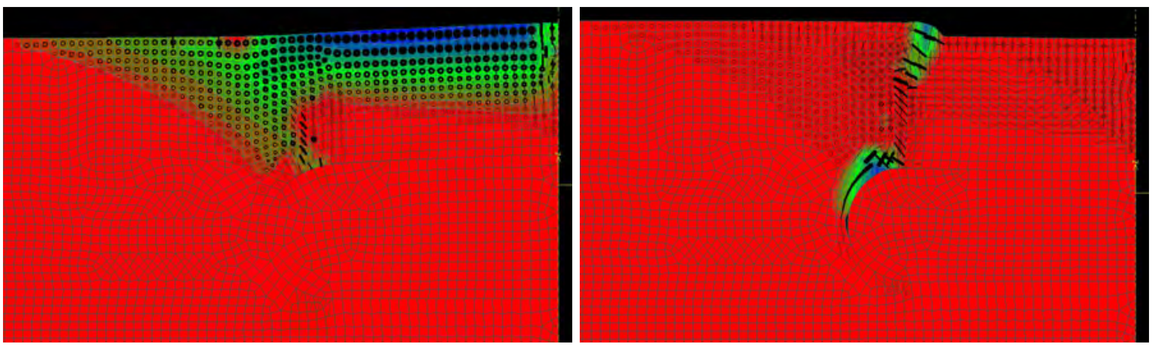


Figure 3: Influence of critical crack opening δ on shape of caldera

The result of the magma chamber expansion can be seen in Figure 4a. Numerous radial cracks occur, especially in the vicinity of a central vent. This phenomenon can be really observed in the nature. It has been also successfully simulated by analogue models (modeled in laboratory using real materials like flour, sand etc.).



(a) inflation — magma injection (10 × magnified displacements)

(b) deflation — magma eruption (5 × magnified displacements)

Figure 4: Main stages in simulation of caldera formation

When the chamber is consequently contracted, we can see in Figure 4b a localized zone of the cracks close to a margin of the chamber. The magma chamber roof then subsides as a coherent block without any considerable deformation or cracking. This fact can be also observed in Figure 3, which does not show only the symmetric part of the system. However, important is that this type of the collapse is well recognized and described by the geologists (e.g. Steven and Lipman (1976)) who call it *the piston-like collapse*.

5. Conclusions

This work was focused on finding the suitable concept for the modeling of the magma chamber and the host rock. Two main conditions were a reliability of the model and a reasonability of the results. Despite the simplifications, the proposed approach provided very satisfactory results. These are the conclusions, which can be made based upon the results:

- the biggest influence on the solution have strengths and critical crack opening of the host rock, and the properties of the interface elements (all the parameters are summarized in Tables 1 and 2),
- the above mentioned modeling approach, together with the particular dimensions, lead to the piston-like collapse, which is described in many field studies and analogue models,

The analysis was not supposed to predict the exact values of the magma chamber roof displacements or the erupted magma volume. The aim was to reproduce the fracturing and deformation of the magma chamber surroundings, which can be observed in the nature. The further development of the model should include a fluid-solid interaction and thermal-mechanical phenomena, because many of the analyzed parameters can be temperature-dependent. Also, the effect of an additional loading caused by the weight of the erupted material should be considered. In the future, a study dealing with a relationship between the type of the collapse and the dimensions of the magma chamber (and its position in a crust) will be carried out.

Acknowledgment

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