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NUMERICAL MODELLING OF HEAT AND MASS TRANSFER IN THE COUNTERFLOW WET-COOLING TOWER FILL

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Abstract: *The article deals with the numerical modelling of heat and mass transfer in the counterflow wet-cooling tower fill. Due to the complexity of this phenomenon the simplified model based on the set of four ODEs was chosen. Boundary condition for outlet water temperature are based on experimentally obtained Merkel number correlation. The numerical solution of chosen model was performed using Runge-Kutta method combined with shooting method. The results are compared with data available in the literature.*

Keyworlds: ***evaporative cooling, wet cooling tower, Merkel number***

1. Introduction

In the counterflow wet-cooling tower fill of film type water flows vertically down through the fill as a liquid film. Air is driven by a tower draft or fan and flows vertically in the opposite direction. Heat and mass transfer occurs at the water and air interface. Evaporation and convective heat transfer cool the water, what leads to increase of air humidity and temperature.

2. Mathematical models

Due to the complexity of two phase flow occurring in the wet-cooling tower fill the one dimensional models of heat and mass transfer are used (e.g. Kröger (2004); Williamson (2008); Klimanek and Białecky (2009)). These models are based on few assumptions which allow to create simplified one dimensional models. The first assumption talks about neglecting of the effects of horizontal temperature gradient in the liquid film, horizontal temperature gradient in air temperature and humidity (e.g. Williamson (2008)). The second one states that temperatures and humidity are represented only by averaged value for each vertical position. We are also assuming that at the interface of two phases, there is a thin vapour film of saturated air at the water temperature (e.g. Williamson (2008)).

This chapter is based on the works of Kröger (2004); Williamson (2008) and mainly on the work of Klimanek and Białecky (2009). The derivation of every one dimensional model of heat and mass transfer in the fill is based on balance laws. We have four variables in this problem: t_a temperature of air, t_w temperature of water, x specific humidity and $\dot{m}_w$ water mass flow rate. Mass balance of the incremental step of the fill is given by

$$\mathrm{d}\dot{m}_w + \dot{m}_a \mathrm{d}x = 0, \tag{1}$$

where $\dot{m}_a$ is mass flow rate of dry air. The change in water mass flow rate can be expressed using mass transfer coefficient α_m as

$$\mathrm{d}\dot{m}_w = \alpha_m (x''(t_w) - x)\mathrm{d}A, \tag{2}$$

where $x''(t_w)$ is saturated specific humidity at t_w and $\mathrm{d}A$ is infinitesimal contact area. The energy balance can be written in the form

$$\dot{m}_a \mathrm{d}h_{1+x} = \dot{m}_w \mathrm{d}h_w + h_w \mathrm{d}\dot{m}_w, \tag{3}$$

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where h_{1+x} is enthalpy of air water vapour mixture and h_w is enthalpy of water. The change in total enthalpy can be evaluated using interface parameters similarly like in the case of mass balance

$$\dot{m}_a \mathrm{d}h_{1+x} = \alpha(t_w - t_a)\mathrm{d}A + h_v(t_w)\mathrm{d}\dot{m}_w, \tag{4}$$

where α is heat transfer coefficient and $h_v(t_w)$ is enthalpy of water vapour.

2.1. Merkel's model

The simplest model of heat and mass transfer in the fill is the Merkel's model which is based on previous equations and additional assumptions, see e.g. Williamson (2008). The first assumption is about neglecting the change of water flow rate in energy balance. Using this assumption we can derive from equation (3) simplified energy equation in the form

$$\mathrm{d}h_{1+x} = \frac{\dot{m}_w}{\dot{m}_a}\mathrm{d}h_w. \tag{5}$$

Second assumption states that air exiting the cooling tower fill is saturated and this state can be characterized only by its enthalpy. The last assumption of the Merkel's model states that Lewis factor $Le_f = 1$. Lewis factor is equal to the ratio of heat transfer Stanton number St to the mass transfer Stanton number St_m

$$Le_f = \frac{St}{St_m} = \frac{\alpha}{c_p \alpha_m}, \tag{6}$$

where c_p is constant pressure specific heat capacity of moist air

$$c_p = c_{p_a} + x c_{p_v}. \tag{7}$$

The effect of the Lewis factor is precisely discussed in the work of Kloppers and Kröger (2005). Using assumption $Le_f = 1$ we can get from equation (4) after additional modifications

$$\frac{\mathrm{d}h_{1+x}}{\mathrm{d}A} = \frac{\alpha_m}{\dot{m}_a}(h''_{1+x}(t_w) - h_{1+x}). \tag{8}$$

The driving force for heat and mass transfer has been then reduced to the enthalpy difference between the water surface and the air stream. Simplified energy equation (5) and equation (8) form the basis of the Merkel's model. Equation (5) can by modified to the form

$$\frac{\mathrm{d}h_{1+x}}{\mathrm{d}A} = c_{p_w}\frac{\dot{m}_w}{\dot{m}_a}\frac{\mathrm{d}t_w}{\mathrm{d}A}, \tag{9}$$

where c_{p_w} is specific heat capacity of water. Finally we can combine equation (9) and equation (8) to get

$$Me = \int_0^A \frac{\alpha_m}{\dot{m}_w}\mathrm{d}A = \int_{t_{w_o}}^{t_{w_i}} \frac{c_{p_w}\mathrm{d}t_w}{(h''_{1+x}(t_w) - h_{1+x})}, \tag{10}$$

where Me is non-dimensional Merkel number, t_{w_o} is water temperature at the outlet and t_{w_i} is water temperature at the inlet. More precise derivation of the Merkel's model together with details connected with numerical calculation of Merkel number and outlet water temperature from known Merkel number can be found e.g. in references (Kröger (2004); Williamson (2008); Hyhlík (2013)).

2.2. Model of Klimanek and Białecky (2009)

To derive the system of ordinary differential equations we have to choose independent variable. The model of Klimanek and Białecky (2009) is based on the selection of spatial coordinate z as independent variable contrary to Poppe's model (e.g. Williamson (2008)) which is based on the choice of water temperature t_w. The interface area can be expressed using variable z as

$$\mathrm{d}A = a_q A_{fr}\mathrm{d}z, \tag{11}$$

where a_q is the transfer area per unit volume and A_{fr} is the cross sectional area of the fill. We can derive equation for the change of the water mass flow rate from equation (2)

$$\frac{\mathrm{d}\dot{m}_w}{\mathrm{d}z} = \alpha_m a_q A_{fr}(x''(t_w) - x). \tag{12}$$

To obtain the equation for the change of specific humidity in the fill we can substitute equation (12) into equation (1)

$$\frac{\mathrm{d}x}{\mathrm{d}z} = \frac{\alpha_m a_q A_{fr}(x''(t_w) - x)}{\dot{m}_a}. \tag{13}$$

Enthalpy of moist air can be expressed like

$$h_{1+x} = c_{p_a} t_a + x(l_0 + c_{p_v} t_a), \tag{14}$$

where c_{p_a} and c_{p_v} are constant pressure specific heat capacities of dry air and water vapour and l_0 is latent heat of vaporisation. Differentiation of equation (21) leads to

$$\frac{\mathrm{d}h_{1+x}}{\mathrm{d}z} = (c_{p_a} + x c_{p_v})\frac{\mathrm{d}t_a}{\mathrm{d}z} + (l_0 + c_{p_v} t_a)\frac{\mathrm{d}x}{\mathrm{d}z}. \tag{15}$$

The left hand side of equation (15) can be substituted from equation (4), the last term can be expressed using (13) and after application of the definition of Lewis factor (6) we can get

$$\begin{aligned}\frac{\mathrm{d}t_a}{\mathrm{d}z} = \frac{\alpha_m a_q A_{fr}}{\dot{m}_a(c_{p_a}(t_a) + x\, c_{p_v}(t_a))}\,&[Le_f(t_w - t_a)\,(c_{p_a}(t_a) + \\ + x\, c_{p_v}(t_a)) + (c_{p_v}(t_w)t_w - c_{p_v}(t_a)t_a)&(x''(t_w) - x)]\,.\end{aligned} \tag{16}$$

By using equation (3) after substitution of equation (12) and equation (4) we derive equation for the change of water temperature

$$\begin{aligned}\frac{\mathrm{d}t_w}{\mathrm{d}z} = \frac{\alpha_m a_q A_{fr}}{\dot{m}_w c_w(t_w)}\,&[Le_f(c_{p_a}(t_a) + x\, c_{p_v}(t_a))(t_w - t_a) + \\ &+ (x''(t_w) - x)(c_{p_v}(t_w)t_w - c_{p_w}(t_w)t_w + l_0)]\,.\end{aligned} \tag{17}$$

The application of the Lewis factor in the previous equations simplified the problem to find experimentally only mass transfer coefficient α_m and calculate heat transfer coefficient α using known value of Lewis factor. The most commonly used formula for the calculation of Lewis factor is Bosnjakovic formula (Bosnjakovic (1965))

$$Le_f = 0.866^{2/3}\left(\frac{x''(t_w) + 0.622}{x + 0.622} - 1\right)\left[\ln\frac{x''(t_w) + 0.622}{x + 0.622}\right]^{-1}. \tag{18}$$

The driving force of evaporation process is $(x''(t_w) - x''(t_a)))$ in the case of supersaturation. The system of ordinary differential equations has to be changed. The equation for the water mass flow rate is

$$\frac{\mathrm{d}\dot{m}_w}{\mathrm{d}z} = \alpha_m a_q A_{fr}(x''(t_w) - x''(t_a)). \tag{19}$$

Equation for specific humidity has the form

$$\frac{\mathrm{d}x}{\mathrm{d}z} = \frac{\alpha_m a_q A_{fr}(x''(t_w) - x''(t_a))}{\dot{m}_a}. \tag{20}$$

The enthalpy of supersaturated air is

$$h_{1+x} = c_{p_a} t_a + x''(t_a)(l_0 + c_{p_v} t_a) + (x - x''(t_a))c_w(t_a)t_a. \tag{21}$$

Equations for the air and water temperature are derived similarly like in the case of under-saturated case. Lewis factor is calculated using saturation humidity $x''(t_a)$ unlike x in the case of under-saturated air. Air temperature distribution is calculated using equation

$$
\begin{aligned}
\frac{\mathrm{d}t_a}{\mathrm{d}z} = -\frac{\alpha_m a_q A_{fr}}{\dot{m}_a} &\left[c_{p_a}(t_a)Le_f(t_a - t_w) - x''(t_w)(l_0 + c_{p_v}(t_w)t_w)\right. \\
&+c_w(t_a)(Le_f(t_a - t_w)(x - x''(t_a)) + t_a(x''(t_w) - x''(t_a))) \\
&\left.+x''(t_a)\left(l_0 + c_{p_v}(t_a)Le_f(t_a - t_w) + c_{p_v}(t_w)t_w\right)\right] \\
/ \left[c_{p_a}(t_a) + c_w(t_a)x + \frac{\mathrm{d}x''(t_a)}{\mathrm{d}t_a}\right.&\left.(l_0 + c_{p_v}(t_a)t_a - c_w(t_a)t_a) + x''(t_a)(c_{p_v}(t_a) - c_w(t_a))\right].
\end{aligned}
\tag{22}
$$

The equation for water distribution is

$$
\begin{aligned}
\frac{\mathrm{d}t_w}{\mathrm{d}z} = \frac{\alpha_m a_q A_{fr}}{\dot{m}_w c_w(t_w)} &\left[(l_0 + c_{p_v}(t_w)t_w - c_w(t_w)t_w)(x''(t_w) - x''(t_a))\right. \\
&\left.+ Le_f(t_w - t_a)(c_{p_a}(t_a) + c_w(t_a)(x - x''(t_a)) + c_{p_v}(t_a)x''(t_a))\right].
\end{aligned}
\tag{23}
$$

3. Methodology of numerical simulations

The numerical solution of four ordinary differential equations mentioned in the previous section represents the boundary value problem. We know air temperature t_{ai} and specific humidity x_i at air inlet and water temperature t_{wi} and water mass flow rate $\dot{m}_i$ on the opposite site of the fill because air and water flows in the opposite direction. There is additional unknown set of parameters in the system of equations, i.e. $\alpha_m a_q A_{fr}$. These parameters have to be solved by using experimentally obtained characteristics of the fill. There are at least two possibilities how to solve this problem.

The fist possibility is based on the calculation of Merkel number of the model of Klimanek and Białecky (2009) using

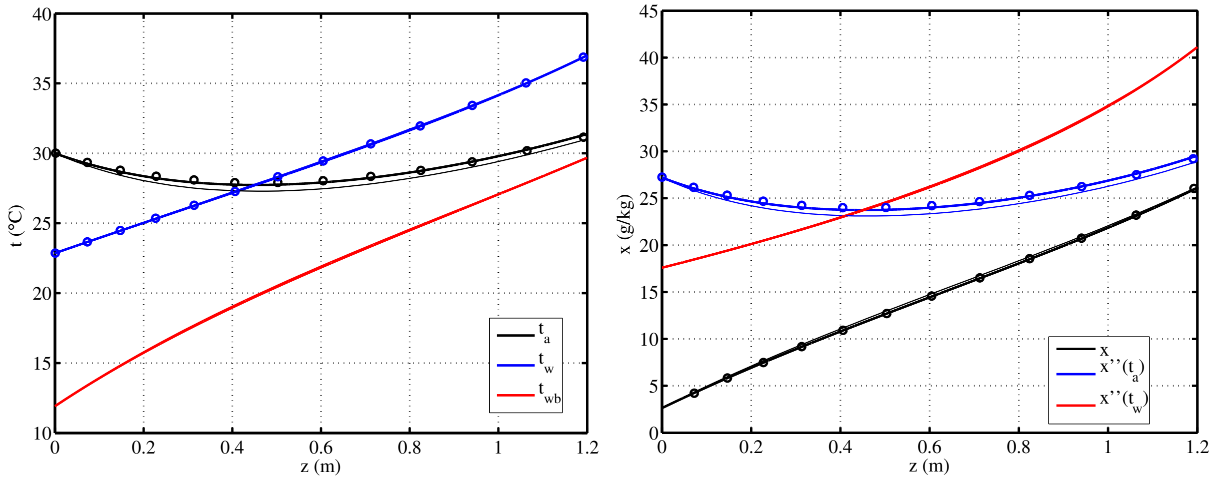
$$
\frac{\mathrm{d}Me_2}{\mathrm{d}z} = \frac{\alpha_m a_q A_{fr}}{\dot{m}_w}. \tag{24}
$$

We can adjust the set of parameters $\alpha_m a_q A_{fr}$ until we reach experimentally obtained value of Me_2. It has been shown by Klimanek and Białecky (2009) that the value of Me_2 calculated by using their model is practically equivalent with Merkel number calculated by using Poppe model. The Merkel number Me_2 has for about few percent higher value than classical Merkel number calculated using Merkel's model (10). The second possibility is to calculate the outlet water temperature t_{wo} using Merkel's model and adjust $\alpha_m a_q A_{fr}$ until we obtain prescribed inlet water temperature t_{wi}. This approach is probably most appropriate because the standard Merkel's model is almost exclusively used in the cooling tower industry and the characteristics of the fill are available as a function of Merkel's model Merkel number (10). Outlet mass flow rate can be adjusted using simple iteration $\dot{m}_{wo} = \dot{m}_{wi} - \dot{m}_a(x_o - x_i)$ and the product $\alpha_m a_q A_{fr}$ can be adjusted using regula falsi method.

4. Results

The first test case was taken from the reference Klimanek and Białecky (2009). The calculation was performed for a fill of height $H = 1.2$ m and cross-sectional area $A_z = 1$ m^2. The air and water mass flow rates are equal to $\dot{m}_a = \dot{m}_w = 3.0$ kg/s. Inlet water temperature is $t_{wi} = 37$°C. Air inlet temperature is $t_{ai} = 30$°C and specific humidity at air inlet is $x_i = 2.62$ g/kg. This parameters correspond to hot and very dry atmospheric conditions. Reference Klimanek and Białecky (2009) does not contain ambient pressure for this particular case but the value of standard atmospheric pressure $p = 101325$ Pa looks relevant and is used.

Figures 1 and 2 show results of the numerical simulations and grid sensitivity study of dependence of outlet parameters is shown in the table 1. There is an intersection in the figure 1 between air temperature curve and water temperature curve. In the bottom half of the fill air temperature decreases and in the upper half slightly increases. The decrease of the air temperature in the bottom part of the fill is also connected with the decrease of saturation humidity in the same part of the fill. The water temperature is monotonously decreasing due to the cooling process. Water is also cooled in the bottom part of the fill



(a) Air temperature t_a, water temperature t_w, adiabatic saturation temperature t_{wb}

(b) Specific humidity x, saturation humidity at the air temperature $x''(t_a)$, saturation humidity at the water temperature $x''(t_w)$

Figure 1: The first test case - distribution of temperature and specific humidity compared with data taken from (Klimanek and Białecky (2009)); solid lines correspond to calculation and circles to data from the reference; thin lines correspond to calculation on coarse grid and thick lines on finest grid.

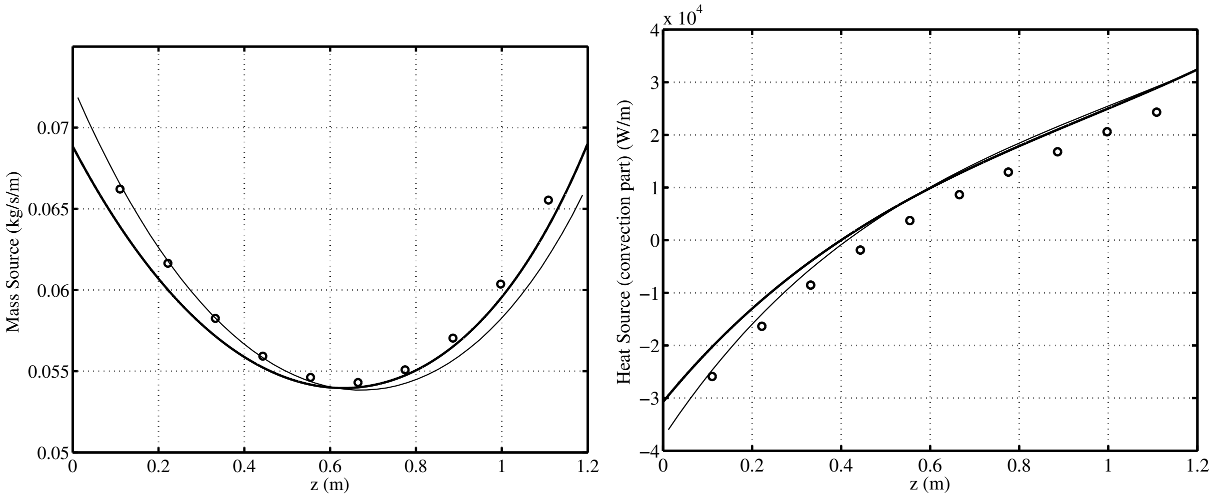
Table 1: Grid independence study test for the test case according to Klimanek and Białecky (2009).

	50 cells	100 cells	500 cells	1000 cells	5000 cells	10000 cells
t_{wo} (°C)	22.86	22.86	22.86	22.86	22.86	22.86
Me_2 (1)	1.871	1.871	1.864	1.863	1.862	1.862
t_{ao} (°C)	31.02	31.20	31.35	31.36	31.38	31.38
x_o (g/kg)	26.20	26.15	26.11	26.11	26.10	26.10
$x''(t_{ao})$ (g/kg)	28.92	29.24	29.49	29.52	29.54	29.55
$\dot{m}_{wo}$ (kg/s)	2.929	2.929	2.929	2.929	2.929	2.929

because the evaporative cooling dominates over convective heat transfer. The distributions of the density of heat and mass sources which are depicted on the figure 2 confirms the intensity of evaporative cooling against the convective heat transfer. The negative value of the density of heat source in the bottom part of the fill corresponds to the air cooling in this fill part.

The Merkel number according to Klimanek and Białecky (2009) is $Me = 1.8613$ and is very close to the values shown in the table 1. From the above mentioned table is visible that the results are practically the same for the different grid sizes, but from the figure 1 is visible that the solution on coarse grid shows little differences against the fine grid and the reference solution of Klimanek and Białecky (2009). There is a good agreement in the distribution of the density of mass source with reference solution of Klimanek and Białecky (2009), where in the bottom part of the fill is better correspondence with the solution on coarse grid and in the upper part is better correspondence with solution on fine grid. The calculated distribution of the convective part of the density of heat source is slightly overestimated against the reference data as shown in the figure 2.

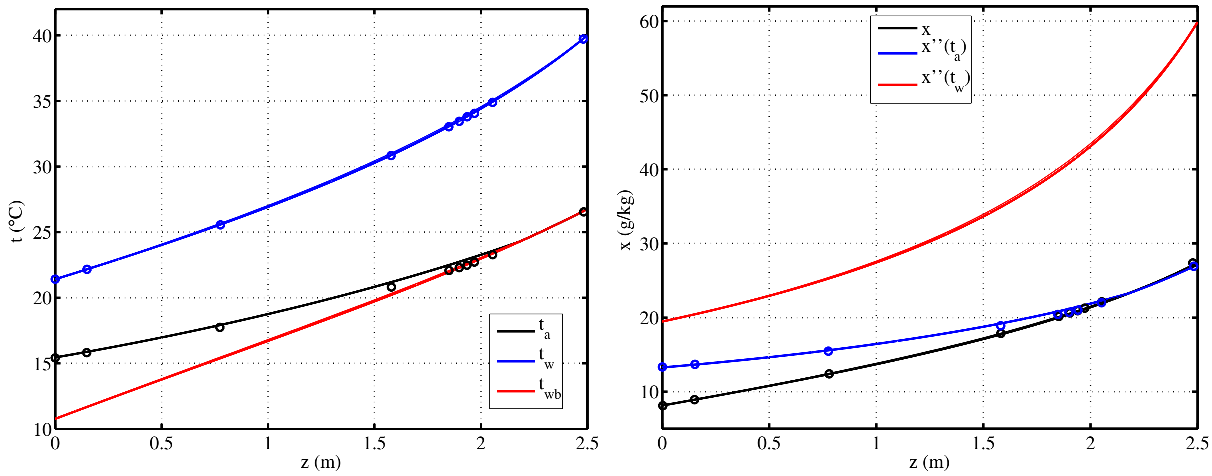
The second test case was taken from the reference Kloppers (2003). This test case is used also by Klimanek and Białecky (2009) as the reference solution. The calculation was performed for a fill of height $H = 2.5$ m. The air inlet mass flow rate is equal to $\dot{m}_a = 16672.19$ kg/s and water inlet mass flow rate is $\dot{m}_w = 12500$ kg/s. Inlet water temperature is $t_{wi} = 40$°C. Air inlet temperature is $t_{ai} = 15.45$°C and specific humidity at air inlet is $x_i = 8.127$ g/kg. The atmospheric pressure is $p = 84100$ Pa. Outlet water temperature is known from the reference $t_{wo} = 21.41$°C. Poppe Merkel number is $Me = 1.5548$ in this case. The air temperature is monotonously increasing in this case unlike



(a) Density of mass source

(b) Density of heat source

Figure 2: The first test case - distribution of mass and heat source compared with data taken from (Klimanek and Białecky (2009)); solid lines correspond to calculation and circles to data from the reference; thin lines correspond to calculation on coarse grid and thick lines on finest grid.



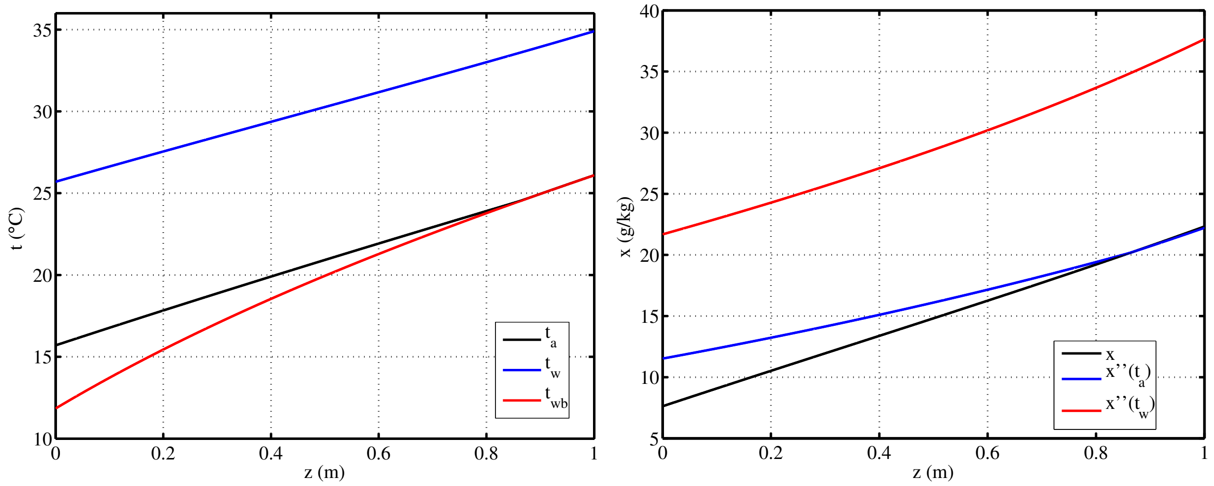
(a) Air temperature t_a, water temperature t_w, adiabatic saturation temperature t_{wb}

(b) Specific humidity x, saturation humidity at the air temperature $x''(t_a)$, saturation humidity at the water temperature $x''(t_w)$

Figure 3: Second test case - distribution of temperature and specific humidity compared with data taken from (Klimanek and Białecky (2009)); solid lines correspond to calculation and circles to data from the reference.

Table 2: Grid independence study test for the test case according to Kloppers (2003) and comparison with data of Klimanek and Białecky (2009) and Kloppers (2003).

	50 cells	500 cells	5000 cells	10000 cells	Kloppers	Klimanek
t_{wo} (°C)	21.41	21.41	21.41	21.41	21.41	21.41
Me_2 (1)	1.597	1.545	1.538	1.538	1.5548	1.5548
t_{ao} (°C)	26.77	26.77	26.76	26.76	26.71	26.71
x_o (g/kg)	27.34	27.38	27.38	27.39	27.89	27.67
$x''(t_{ao})$ (g/kg)	27.18	27.17	27.15	27.15	27.18	27.18
$\dot{m}_{wo}$ (kg/s)	12179.7	12179.1	12178.9	12178.9	12170.5	12174.2



(a) Air temperature t_a, water temperature t_w, adiabatic saturation temperature t_{wb}

(b) Specific humidity x, saturation humidity at the air temperature $x''(t_a)$, saturation humidity at the water temperature $x''(t_w)$

Figure 4: Third case - distribution of temperature and specific humidity; thin lines correspond to calculation on coarse grid and thick lines on finest grid.

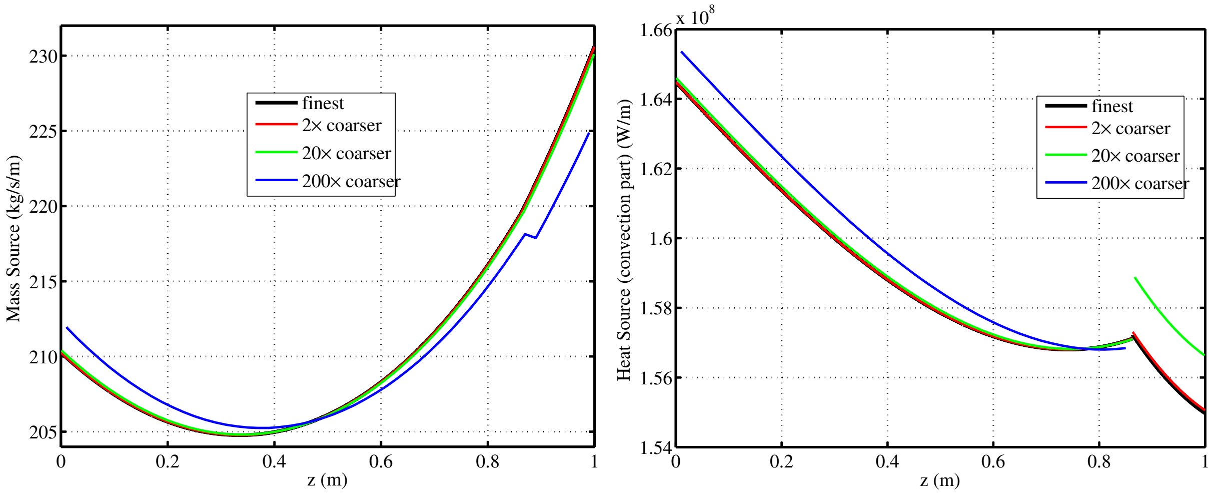
the first test case. Water temperature is decreasing through the whole fill depth similarly like in the first case. The distributions of temperature and specific humidity is depicted in the figure 3. From the figure 3 is possible to observe supersaturation in the upper part of the fill, it means the air temperature is equal to adiabatic saturation temperature and specific humidity of air is slightly higher then saturation specific humidity, i.e. there is an intersection between specific humidity and saturation specific humidity. The calculated distributions are compared with reference solution of Klimanek and Białecky (2009). From the point of view of comparison with reference solution, it seams that the supersaturation occurs little bit earlier in the reference solution of Klimanek and Białecky (2009), but differences are very small. The table 2 shows grid independence study for this case and the comparison of results with the solution by Kloppers (2003) and the solution by Klimanek and Białecky (2009). It is possible to recognize that results are practically mesh independent from the coarse grid, but there is small difference between obtained solution and solutions from references. Calculated Merkel number is slightly lower, air outlet temperature is higher, specific humidities are lower and mass flow rate is little bit higher. It is almost impossible to distinguish the influence of grid from the figure 3 because the differences are almost smaller then the solid line thickness.

The third test case was inspired by one case used by Zúñiga-González (2005). The calculation was performed for a fill of height $H = 1$ m. The air inlet mass flow rate is equal to $\dot{m}_a = 14333$ kg/s and water inlet mass flow rate is $\dot{m}_w = 17200$ kg/s. Inlet water temperature is $t_{wi} = 34.9$°C. Air inlet temperature is $t_{ai} = 15.7$°C and specific humidity at air inlet is $x_i = 7.622$ g/kg. The atmospheric pressure is $p = 98100$ Pa. Outlet water temperature is known from the reference $t_{wo} = 25.7$°C.

The temperature and humidity distributions exhibit similar behaviour like in the previous case. There are monotone changes in variables in the figure 4 and supersaturation in the upper part of the fill. The solution is practically grid independent as shown in in the table 3, where the values are mesh independent practically from the coarse grid. However the mesh independence of the solution is questionable, when the figure 5 is taken into consideration. The behaviour which is possible to observe in the distribution of the density of mass source is natural. The solution on coarse grid is slightly different against other solutions which are almost identical. The problem can be identified on the distribution of the convective part of heat source. There is a discontinuity on coarse grids in the place where supersaturation starts. This discontinuity disappear on finer grids. The problem is that there is no distinguishable presence of numerical errors calculated using method of Dormand and Price (1980) in this case. The relative error norms of calculated variables exhibit typical behaviour and values similarly like in the previous test cases. The distribution of relative error norm of air temperature and relative error norm of specific humidity is shown in the figure 6.

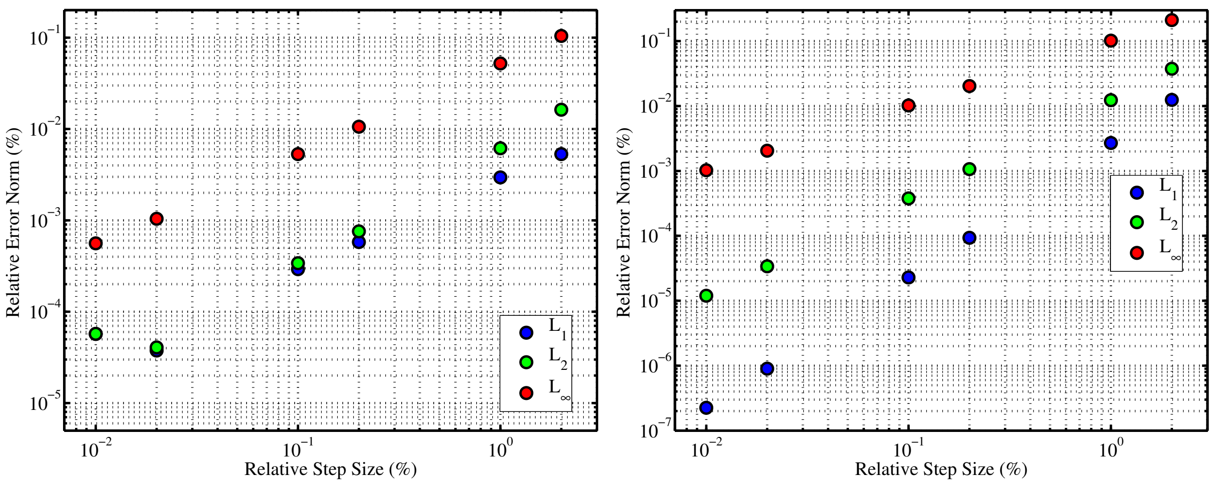
Table 3: Grid independence study test for the third test case.

	50 cells	100 cells	500 cells	1000 cells	5000 cells	10000 cells
t_{wo} (°C)	25.7	25.7	25.7	25.7	25.7	25.7
Me_2 (1)	0.874	0.875	0.875	0.875	0.875	0.875
t_{ao} (°C)	26.12	26.10	26.10	26.10	26.10	26.10
x_o (g/kg)	22.29	22.30	22.32	22.32	22.32	22.32
$x''(t_{ao})$ (g/kg)	22.25	22.22	22.22	22.22	22.22	22.22
$\dot{m}_{wo}$ (kg/s)	16989.7	16989.6	16989.4	16989.4	16989.3	16989.3



(a) Density of mass source (b) Density of heat source

Figure 5: Third case - distribution of mass and heat source.



(a) Relative air temperature error norm (b) Relative specific humidity error norm

Figure 6: Third case - relative error norms.

5. Conclusions

The results of the numerical solution was discussed for three test cases. The first two test cases was chosen to compare results with data from references by Klimanek and Białecky (2009) and Kloppers (2003). Klimanek and Białecky (2009) have mentioned that their results are equivalent with results obtained using Poppe method by Kloppers (2003). The results presented in this article exhibit slight differences from the reference solutions. The main difference is presented in the second case where it seams that supersaturation exhibits little bit later and it leads to differences shown in the table 2. The differences are probably caused by different choice of saturated vapour pressure equation in references and in this article. The difference is also connected with using the more precise equation for the calculation of specific humidity in this article. The previous two sentences are based on assumption that the work of Klimanek and Białecky (2009) is based on the same thermodynamics equations as are presented in the book of Kröger (2004) and this is not possible to recognize from their article. The third test case is shown because of the problematic presence of the discontinuity on the distribution of convective part of the density of heat source. It has been shown that the discontinuity is vanishing with the grid refinement. The presence of discontinuity does not correspond with the increase of error norm. Unfortunatelly, the article by Klimanek and Białecky (2009) does not mention the distribution of the density of convective part of heat source for the case where supersaturation occurs. They have mentioned only general fact that adaptive step size control technique can increase accuracy of the integration with small additional computational effort.

6. Acknowledgement

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