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TRANSVERSE VIBRATING BEAM LOADED BY MOVING LOAD

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Abstract: *Simplest model for prediction effects of pedestrians to the structures is the harmonic force, which is fitted in the most efficient point on the pedestrian bridge. This article is focused on alternative modeling of human – structure interaction.*

Keywords: ***Vibrating beam, moving load, Finite Difference Method.***

1. Introduction

In this paper is presented numerical solution of the simply supported transverse vibrating beam with continuous weight per unit length. The beam is exposed to moving force with constant magnitude. It is the simplest model for analyze the human-structure interaction. This load acting at the structure only in the distance d_p, which is the length of human step, and in the time, which corresponds with velocity of moving load and step length. It means, that the structure is loaded by impulses of force moving with velocity v.

2. Equation for the analysis

Transverse vibration of beam with continuous mass distribution is described by fourth order partial hyperbolic differential equation

$$EI\frac{\partial^4}{\partial x^4}w_{(x,t)}+\mu\frac{\partial^2}{\partial t^2}w_{(x,t)}+2\mu\omega_b\frac{\partial}{\partial t}w_{(x,t)}=\delta_{(x-vt)}F \tag{1}$$

where EI is the bending stiffness, μ is the mass per unit length, ω_b is damping angular frequency and δ is the Dirac's (delta) function.

Area of solution is the rectangle $\Psi=[0;L]\times[0;T]$ with boundary and initial conditions. If we consider simply supported beam, the boundary conditions are:

$$w_{(0,t)}=0 \ \wedge \ w_{(L,t)}=0 \tag{2}$$

$$\left.\frac{d^2w_{(x,t)}}{dx^2}\right|_{x=0}=0 \ \wedge \ \left.\frac{d^2w_{(x,t)}}{dx^2}\right|_{x=L}=0 \tag{3}$$

Second condition is expressed as a static condition it means, the bending moment at the both edges of the beam is equal to null. Initial conditions are revolved, that the beam is inactive at the beginning of analysis.

$$w_{(x,0)}=0 \ \wedge \ \left.\frac{\partial w_{(x,t)}}{\partial t}\right|_{t=0}=0 \tag{4}$$

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2.1. Differential replacements for derivations in spatial and time domain

Numerical expression of derivation could be carried out thanks to the Taylor-McLaurin's expansion, which predicts, that derivations goes to the next formulas:

$$\frac{\partial^4 w_{(x,t)}}{\partial x^4} \approx \frac{w_{j+2}^i - 4w_{j+1}^i + 6w_j^i - 4w_{j-1}^i + w_{j-2}^i}{\xi^4} \tag{5}$$

$$\frac{\partial^2 w_{(x,t)}}{\partial t^2} \approx \frac{w_j^{i+1} - 2w_j^i + w_j^{i-1}}{\tau^2} \tag{6}$$

$$\frac{\partial w_{(x,t)}}{\partial t} \approx \frac{w_j^{i+1} - w_j^{i-1}}{2\tau} \tag{7}$$

2.2. Discretization of solved area Ψ

It is assumed, that solution of PDE (1) will not be fulfilled in each point of the interval $[0;L]$ respectively $[0;T]$, but only in finite number of nodes. Number of this nodes is given by length of spatial and time step. The length of time step τ has to be very fine in comparison with spatial step ξ according to stability of FDM.

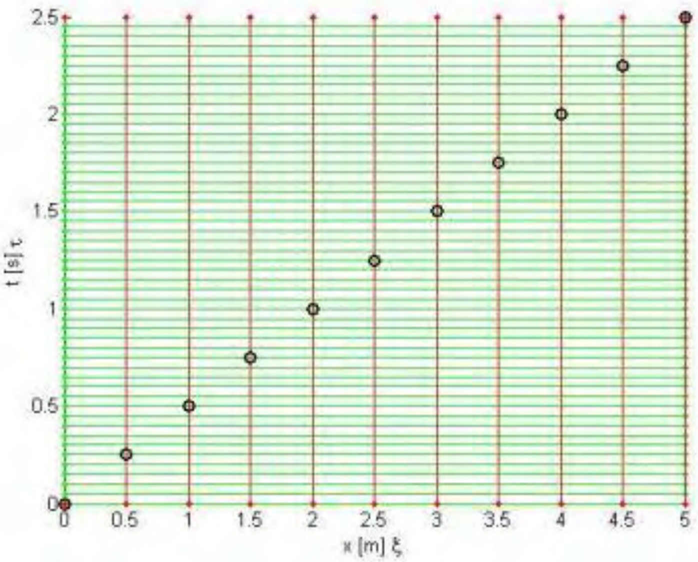

Fig. 1: Area Ψ discretes in time and space coordinates

In the (Fig. 1) are nodes of the meshed area Ψ with force applied to the structure. If we assume constant velocity of moving load $v = 2ms^{-1}$ and constant length of human step $d_p = 0,5m$, we are able to compute times of each steps and time T, which is the time, when force is leaving the structure.

2.3. Solution of differential equation

Numerical solution of equation (1) is realized by Finite Difference Method (FDM), where are the derivations substituted by differences, obtained from Taylor - McLaurin expansion. The differential equation (1) is transformed to the system of algebraic equations corresponding to the difference scheme, which was used.

The displacement at the $(i+1)$ th time layer and on the j th space layer is expressed for example as:

$$w_j^{i+1} = \frac{1}{2\mu(1+\omega_b\tau)} \left\{ \begin{aligned} & 2\mu(\omega_b\tau - 1) w_j^{i-1} + 4\mu w_j^i + \\ & + \frac{2\tau^2}{\xi^4} EI \left(-w_{j+2}^i + 4w_{j+1}^i - 6w_j^i + 4w_{j-1}^i - w_{j-2}^i \right) \end{aligned} \right\} \tag{8}$$

where ξ is the spatial step and τ is the time step

2.4. Numerical example

Parameters for numerical solution describes footbridge A8 across Opatovická street in Prague. The footbridge acting as simply supported beam. Parameters used in numerical solution: $EI = 3,83 \cdot 10^6 kNm^2$, $L = 25m$, $\mu = 5,3 t/m$, $\omega_b = 0,238 s^{-1}$, $F = 1,8kN$

Angular frequency ω_b was evaluate from the formula: $\omega_b = \vartheta \cdot f_{01} = 0,088 \cdot 2,70$ where ϑ means logarithmic damping decrement and f_{01} is first bending natural frequency. The logarithmic damping decrement was found out experimentally. F is constant, time-independent force, moving across beam.

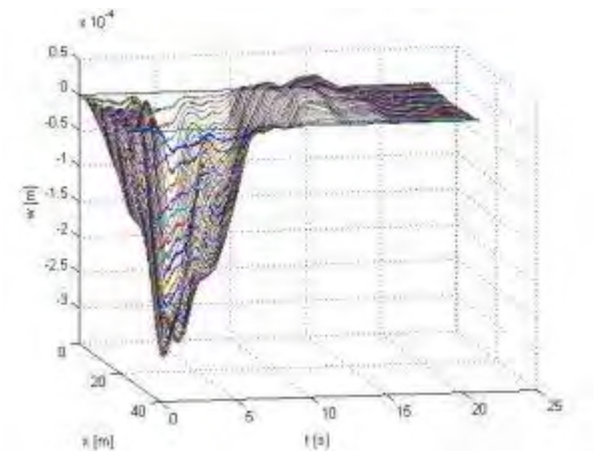

Fig. 2: Transverse vibration of beam in the spatial-time coordinates – deformed mesh

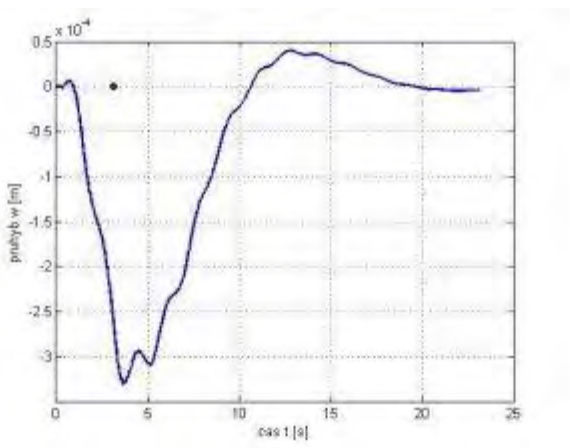

Fig. 3: Vibrating of beam center

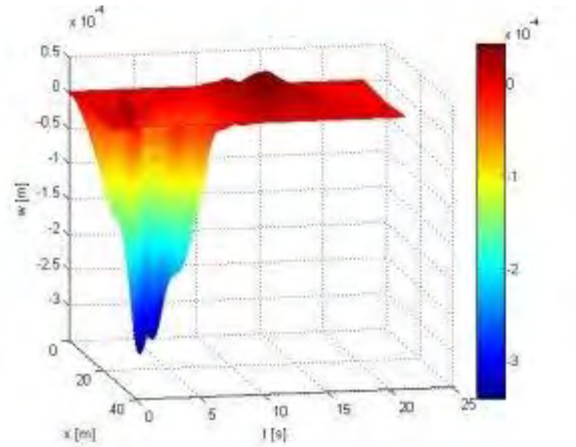

Fig. 4: Transverse vibration of beam in the spatial-time coordinates - compute displacements

3. Conclusion

Stability of Finite Difference Method is dependent on bending stiffness. For increasing EI has to be chosen fine time step τ in contrast to the spatial step ξ. This is the essential reason why this method is not usable for real structures, because computing time increases exponentially with decreasing of step in time domain. Results from simulation, presented in this paper, are in the conjunction with presuppositions. Constant force F, acting in the contact point and varying in the spatial domain with velocity v, describes pedestrian, who is running along the structure. Negligible deviations in results are caused by fact, that force in time domain is varying during walking or running.

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References

Frýba, L. (1989) Kmitání těles a konstrukcí způsobené pohybujícím se zatížením. *ACADEMIA*, ISBN 80-200 0039-9.