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INFLUENCE OF THE DESIGN PARAMETERS ON THE RESPONSE OF THE STEP SKEW JOINT WITH A KEY

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Abstract: *Nowadays the numerical and experimental analysis of the step skew joint with a key is made. The article is focused on influence of some design parameters on the response of this type of joint, especially on geometry of the joint, mainly skew angle and position and material of a key. Some material parameters and results of numerical modeling same as comparison of numerical and experimental analysis will be shown.*

Keywords: ***Historical trusses, vertical step skew joint, numerical modeling, distribution of forces***

1. Introduction

Timber is one of the oldest building materials. It was used mainly for roofing of different types of buildings in Ancient World. One of the oldest well-preserved constructions is the gothic trusses. Considering many fires during the ages there are not many gothic trusses in Czech Republic. The gothic trusses are all wooden carpentry joints. Nevertheless much more historical trusses are from baroque, where the binding elements can be made from metal. This change is very important in the roof behavior.

For transferring forces between truss members the carpentry joints in traditional historical trusses are used. Forces are primary transferred by direct contact and by friction. It is necessary to protect initial (historical) shape of construction and conserve originality of members as much as possible during the reconstruction of historical trusses. On the basis of this only damaged part of the truss is cut of and replaced with a new material. Vertical splice step joint (Fig. 1) is one of the most common carpentry joint which is used for replacing parts of the trusses (adjusting existing undamaged member) (Gerner, 2003).

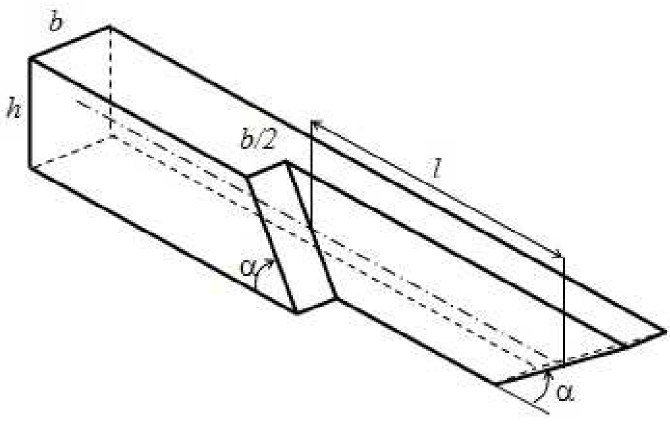

Fig. 1: Half of vertical splice skew joint

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2. Forces in the joint

As was mentioned, typical utilization of step joint is in a roof trusses. One dimension is more than 10times bigger than other two in truss members. On the basis of this it is possible to simplify plane stress in members. It is sufficient to observe normal stress, which is parallel with fibers in a wood (L direction) and shear stresses in a plane of section perpendicular to fibers (T and R direction) (Majano, Hughes & Fernández-Cabo, 2009; Požgaj, 1997) during joint analysis. Solving equilibrium state of construction is not complicated and it is possible to design truss members from initial forces due to this equilibrium state. Nowadays it is not easy to find out publication with behavior of step joints like Paris, M.A., Sordié, C. (2010) and Sangree, R.H., Schafer, B.W. (2009) did with other type of joints.

Step joint loaded only by bending moment helps to better understanding of distribution of forces in the joint. By Fajman (2013) it is possible to simplify distribution of forces on step joint according to fig. 2.

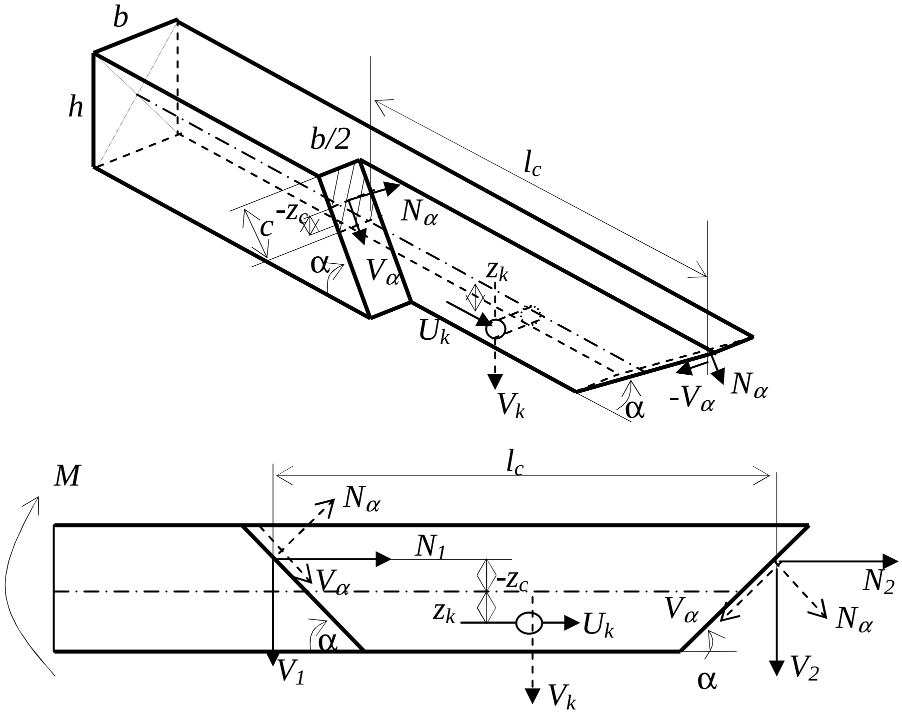

Fig. 2: Force distribution in the joint

In fig. 2 α is skew angle, N_α is compressive force, V_α is friction force, N is normal force, V is shear force, U_k and V_k are forces on a key, z_k is eccentricity of a key, z_c is eccentricity of normal force and l_c is a length between points of applications of normal forces.

Relationship between skewed forces and local system of coordinate forces is shown in equation (1).

$$\begin{aligned} N &= N_\alpha \cdot \sin\alpha + V_\alpha \cdot \cos\alpha \\ V &= -N_\alpha \cdot \cos\alpha + V_\alpha \cdot \sin\alpha \end{aligned} \tag{1}$$

Normal force N_α and friction force V_α have their application point on the step end of the joint. These two forces resists to the loading. For the bending moment loaded part of construction it is possible to write two force conditions of equilibrium (2) and one momentum condition of equilibrium (3). In equations (2) and (3) symmetry of construction about vertical axes was used. Symmetry causes $N_1 = N_2 = N$ and $V_1 = -V_2 = V$.

$$\rightarrow 2N + U_k = 0 \qquad \uparrow 0 = 0 \tag{2}$$

$$-M - 2N \cdot (-z_c) + V \cdot l_c + U_k \cdot z_k = 0 \tag{3}$$

Problem of equations of equilibrium is that a shear force on a key U_k is not known same as forces N_α and V_α on the step end of the joint and size of contact pressed area A_c. Shear force V_α is depended on a normal force and on a coefficient of friction, which can be found in literature or can be determinate by an experiments. Relationship between shear force and coefficient of friction reduces number of unknowns to three for equations (2) and (3). For exact value of coefficient of friction of timber used during joint analyses experiments was made at The Institute of Theoretical and Applied Mechanics Academy of Sciences CR (ITAM). From these experiments the coefficient of friction was set to 0.44.

After applying some assumptions (Fajman 2013), relation between resultant of forces at a compressive area of step end and force in the key is shown in equation (4). Intensity of M, N, V can be get from force analyses done by software. Equations (2, 3, 4) are valid for linear the same as material non-linear analyses.

$$z_c = \frac{-M + V \cdot l + U_k \cdot z_k}{-2N + 2V \cot \alpha} \tag{4}$$

Eccentricity of normal force is depended on the stress distribution on the joint end. The stress distribution is not known. At the basis of this, eccentricity of normal force can be chosen and interval of shear force on a key can be get from eccentricity variation. The intensity of shear force on a key is necessary for the design of a joint, especially for the design of the key.

3. Numerical modeling

Because experimental tests are very time-intensive, very often difficult for making and expensive, it is necessary to have numerical model strong enough to observe changes in distribution of forces after changes in design. When a numerical model is working experimental tests serves as assurance of computed behavior. Experimental tests scheme, which were made at ITAM is shown at fig. 3.

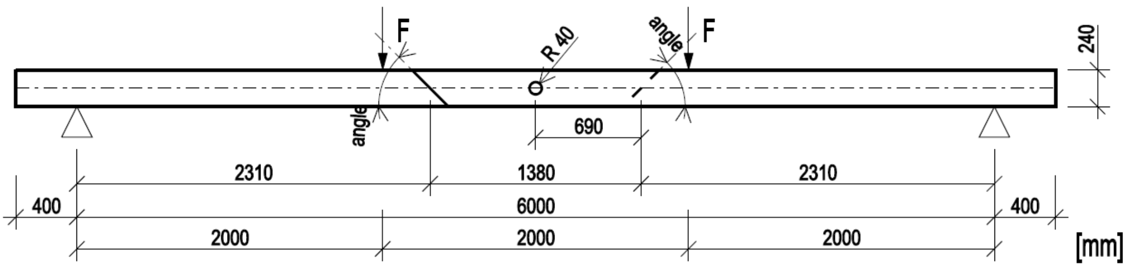

Fig. 3: Joint in the middle of span

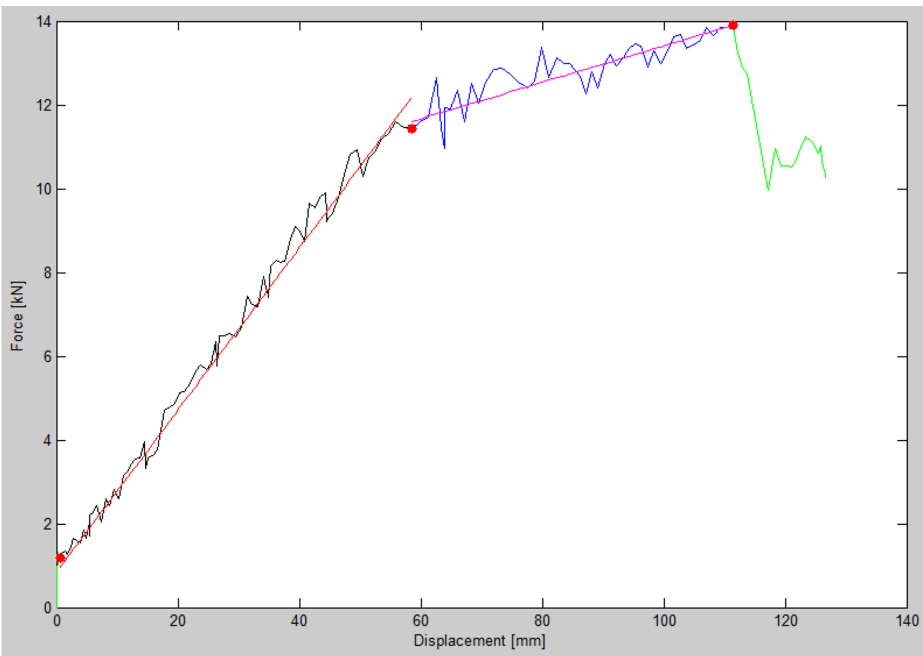

Fig. 4: Simplification of force-displacement relationship by two lines

Despite of absence of orthotropic material model, which is suitable for wood, ATENA 3D was chosen for numerical modeling. Orthotropy of material can be reasonable substituted by smeared reinforcement in ATENA. Smeared reinforcement strengthens physical properties for example modulus of elasticity in direction in which is used. More material models for description of behavior of step joint were used. At first nonlinear material model was used. But in this case there were no satisfactory results from numerical model. Than the linear material model, which is suitable for the beginning of application of load was used. The most satisfactory results of chosen material models are shown at fig. 5.

In simple way, behavior of joint during the loading can be described by two lines what shows fig. 4. Red points at the beginning and at the end of diagram at fig. 4 are the points where the strange behavior is cut off. Strange behavior at the beginning of loading has been promoted by bearing of experimental arrangement components. Strange behavior at the end of loading diagram is caused by data measured after joint collapse (ITAM 2012). Red point in the middle of diagram shows established limit of linear elasticity. At the basis of this simplification the bilinear material model was used in ATENA. Comparison of force-displacement relationship between experimental tests and numerical models is shown in fig. 5. Each material model has some advantages and unfortunately some disadvantages. None of them is sufficient for description of real behavior of truss with the joint.

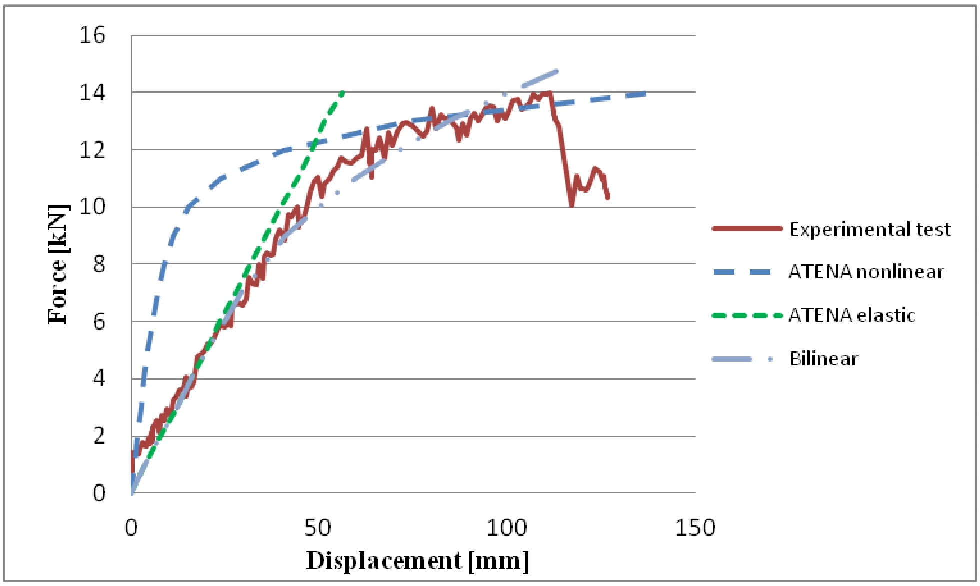

Fig. 5: Force-displacement relationship in the middle of the span

Wood is an anisotropic material which needs own set of material properties in each direction. Problem is set up the material using the smeared reinforcement in ATENA. To describe homogeneous material only one set of the material properties can be used in ATENA material model (Šobra, Fajman, 2013).

In spite of all problems with material models and unsatisfactory results from ATENA, numerical modeling shows two different behavior of joint during the loading. In the first case, all joint is moving downwards. In the second one, joint is moving downwards but due to normal force joint ends goes upwards (fig. 6). This behavior should be respected in the analytical force analysis using the equation (4).

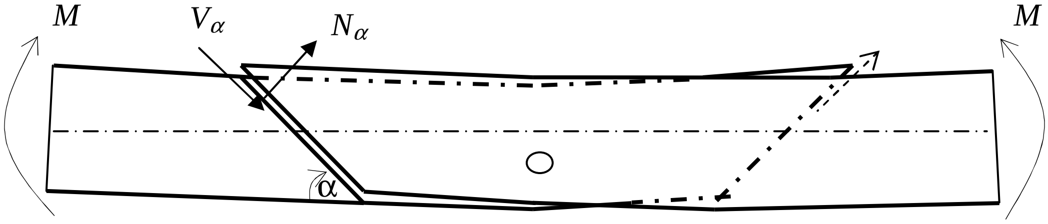

Fig. 6: joint ends upward displacement

4. Conclusions

Even through unsatisfactory results of numerical modeling, the numerical model shows two types of behavior during the loading. At the basis of this the analytical computation shows, that the forces in the joint are deeply wedded with coefficient of friction and size of contact pressed area. Size of contact area is depended on the skew angle and can be determinate by experiments or should be considered by some values at the basis of type of expected stress distribution. Whole joint end could not be pressed, this assumption follows from the construction of the joint. Eccentricity of normal force is linked with this assumption and with the way of stress distribution on the joint end.

Intensity of forces is lower in case of higher eccentricity of the key. Higher eccentricity means larger press area for same loading. Advantageous influence of the key is minimized by lower skew angle. For lower skew angle is lower intensity of forces as shows tab. 1. Unfortunately for lower skew angle the length of the joint is higher which is not demanded for designers.

If the stiffness of a key is lower, all joint moves downwards included the joint ends. The same as higher eccentricity of the key, lover stiffness of the key is better for intensity of forces. It should be noticed that timber in thin part of the joint could be cracked due to high eccentricity of the key.

From the experiments it is not possible to measure forces in the key at the step joint and nowadays numerical models are not apposite and not confirmed due to this assumption, yet.

Tab. 1: Intensity of shear force at the key in.

Skew angle	Moving of joint ends	
	Down	**Up**
35°	11.6 – 12.6 kN	17.5 – 19.2 kN
45°	17 – 18.6 kN	24.5 – 27 kN
63°	31.8 – 33.6 kN	47 – 52 kN

Values in tab. 1 are for boundary conditions: construction loaded by bending moment M = 20 kNm, coefficient of friction μ = 0.1, length of step joint center line l = 1.5 m and cross section dimension 0.2 x 0.24 m.

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