18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 1311–1317
Paper **#311**

APPLICATION OF DYNAMIC RELAXATION METHOD IN ANALYSIS OF CABLE MEMBRANE STRUCTURES

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Abstract: *The article is focused on analysis of the cable membrane structure mainly the dynamic relaxation method and parameters which influence the stability and the speed of computation.*

Keywords: ***Dynamic relaxation method, cable membrane structures, iteration parameters.***

1. Motivation

Light cable membrane construction founds its utilization mainly on the structures where it is necessary to cover large areas like warehouses, exhibition areas and stadiums. Thanks to the modern design, many cable membrane structures were built in the last twenty years.

There are many causes for their permanently higher utilization. They can be transported with very low costs because they are very light. Their lightness also causes that large areas should be covered under good costs for area unit. They can be prefabricated which leads to effective build considering usage of material. One of the most noticeable aspects is design. Cable membrane structures are highly visible. In case of design they are significant architectonical elements. The examples of cable membrane structures are shown in Figure 1 and Figure 2.

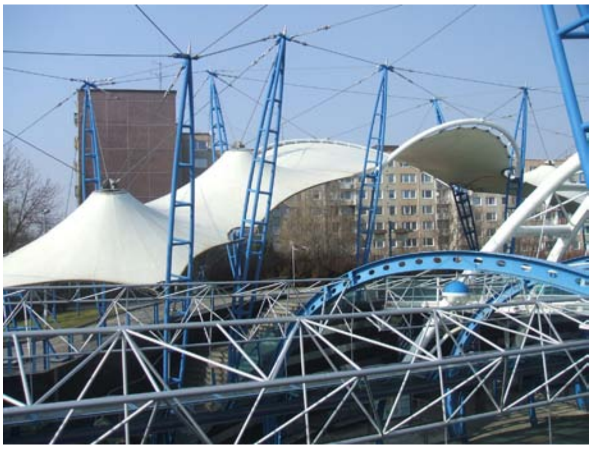

Fig. 1: Tram station K Barrandovu, Prague

Fig. 2: Munich Olympic Stadium

2. Design of cable membrane structures

By Topping and Iványi (2007) there are several steps which are necessary for design cable membrane structure. At the beginning shape definition, discussion about general shape of structure between client and architect is necessary. After the shape definition the engineering model can be created and the main parameters for *form finding* are defined. During the form finding process the equilibrium state of

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cable membrane structure is found and the specific boundary conditions are obtained from this process. The final equilibrium state is found by optimizations methods. Using geometrical optimization the surface is described by a mathematical surface equation. In the second type of optimization, equilibrium form finding methods, the equilibrium state is numerically computed. Equilibrium form finding methods may be employed to analyse greater and more complicated constructions and structures with unconventional shapes. In the next step, the response of construction to loading is *analysed*. Now, the appropriate shape of structure is known and it is necessary to make the *cutting pattern generation* and *design the details*. This minimizes the wastage of material during the production plan parts of construction from roll of material.

The aim of this article is to introduce the main methods which can be used for the cable membrane structure analysis.

3. Cable membrane structures analysis

Various methods are used for cable membrane structure analysis. One of the simplest methods is *grid method*. When the horizontal forces are in equilibrium, the height of grid points can be calculated from the equilibrium of vertical forces. System of linear equations is a set considering all nodes of grid.

Further, there are simply numerical methods, *finite difference method* and *finite element method*. Nowadays it is possible to solve constructions with arbitrary irregular shape and prestressed construction using these methods.

From many various methods there are two others suitable. *Force density method* which is based on the constant ratio between the force in the element and the length of the element. At last the *dynamic relaxation method* which is highly used for the form finding and analysis of construction.

4. Dynamic relaxation

Dynamic relaxation is not used for finding dynamic response of construction but it is used for static problems using a fictitious dynamic analysis. In this method the motion of construction from the time of loading to the state of equilibrium is traced step by step. From the motion it is possible to determine the curve of the construction without compile the matrix of stiffness. This characteristic leads to the conclusion that the dynamic relaxation is a method, which is suitable for highly nonlinear problems.

The method is a direct application of Newton's second law of motion ($F = M.a$). During the static analysis of construction the fictitious damping is used. The proportional, frequently critical damping factor is mostly applied. Iteration to the static solution is relatively fast when critically damped or overdamped construction is used. The influence of various damping factors is shown in Figure 3. Speed of iteration also depends on the fictitious masses. Because the masses are fictitious, their appropriate distribution between joints can accelerate the speed of calculation.

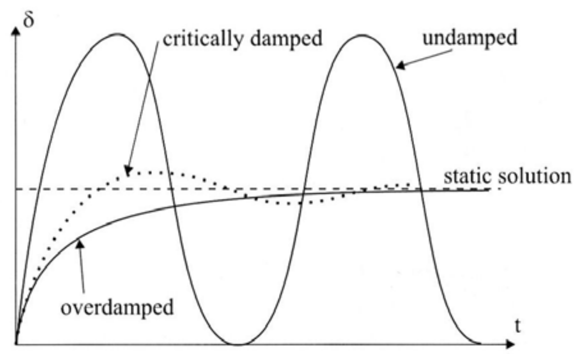

Fig. 3: One degree of freedom time – displacement trace

4.1. Numerical procedure

The Newton's second law of motion is presented in the equation which describes residual forces in time *t* and joint *i*. To the calculation of residual forces it is necessary to add the effect of prestress. The Equation (1) describes the calculation of residual forces in *x* direction:

$$R_{ix}^{t} = M_{ix}.\dot{v}_{ix}^{t} + C_{ix}.v_{ix}^{t} \tag{1}$$

where:

R_{ix}^{t} is the residual force at joint *i* at time *t*

M_{ix} is the fictitious mass at joint *i*

C_{ix} is the viscous damping factor for joint *i*

$\dot{v}_{ix}^{t}, v_{ix}^{t}$ are the acceleration and velocity at the time *t* at joint *i*.

By calculating the response of construction to the loading it is necessary to determine the acceleration and velocity at joint at the demanded time. The result of substituting the average velocity and the acceleration over the time step Δt into the Equation (1) is:

$$R_{ix}^{t} = \frac{M_{ix}}{\Delta t}.\left(v_{ix}^{(t+\Delta t/2)} - v_{ix}^{(t-\Delta t/2)}\right) + \frac{C_{ix}}{2}.\left(v_{ix}^{(t+\Delta t/2)} - v_{ix}^{(t-\Delta t/2)}\right) \tag{2}$$

The rearrangement of the Equation (2) enables to calculate the velocity at the new time step ($t+\Delta t$):

$$v_{ix}^{(t+\Delta t/2)} = v_{ix}^{(t-\Delta t/2)}\left(\frac{M_{ix}/\Delta t - C_{ix}/2}{M_{ix}/\Delta t + C_{ix}/2}\right) + R_{ix}^{t}\left(\frac{1}{M_{ix}/\Delta t + C_{ix}/2}\right) \tag{3}$$

In the next step the Equation (3) is used to calculate the current coordinates of joint *i*:

$$x_{i}^{(t+\Delta t)} = x_{i}^{t} + \Delta t.v_{ix}^{(t+\Delta t/2)} \tag{4}$$

The residual forces are calculated from the Equation (5), where T_{ix} represents the internal forces and F_{ix} represents the applied loading including prestress. The internal forces are calculated at the joints where the residuals are determined – Equation (6).

$$R_{ix}^{(t+\Delta t)} = F_{ix} + T_{ix}^{(t+\Delta t)} \tag{5}$$

$$T_{m}^{(t+\Delta t)} = \frac{EA_{m}}{l_{m}^{0}}\left(l_{m}^{(t+\Delta t)} - l_{m}^{0}\right) + T_{m}^{0} \tag{6}$$

where:

l_{m}^{0} is the internal initial length of link

$l_{m}^{(t+\Delta t)}$ is the current length of link at time *($t+\Delta t$)*

EA_{m} is the elastic modulus multiplied by the cross sectional area of the link *m*

T_{m}^{0} is the internal prestress in link

For calculating the current coordinates of joint *i* at the end of the first time step ($x_{i}^{(t+\Delta t)}$) it is necessary to set the initial conditions for time *t=0*: $v_{ix}^{0} = 0$. Substituting initial conditions to the Equation (3) enables to calculate the initial velocity at time $t = \Delta t/2$:

$$v_{ix}^{(\Delta t/2)} = \frac{R_{ix}^{0}}{2M_{ix}} \tag{7}$$

4.2. Dynamic relaxation method stability and convergence

The stability and convergence of the dynamic relaxation is influenced by the distribution of fictitious nodal mass, the damping factor and the time interval of the step. During the calculation fixed time step is often used and other factors are tuned until the required accuracy and stability of calculation is reached. When the time step Δt exceeds a critical value or fictitious masses are too low, numerical instability of the calculations will occur and the equilibrium state cannot be reached. This shortage can be eliminated by decreasing the time step or increasing the fictitious masses. Speed of convergence is partially affected by the damping factor. Critically damped or overdamped constructions have good speed of convergence.

From the previously stated it is obvious that the tuning of the calculation parameters (time step, fictitious masses and damping factor) is really an attractive area of interest since these parameters have a large influence on the speed of calculation. Beside these factors are specific for each construction.

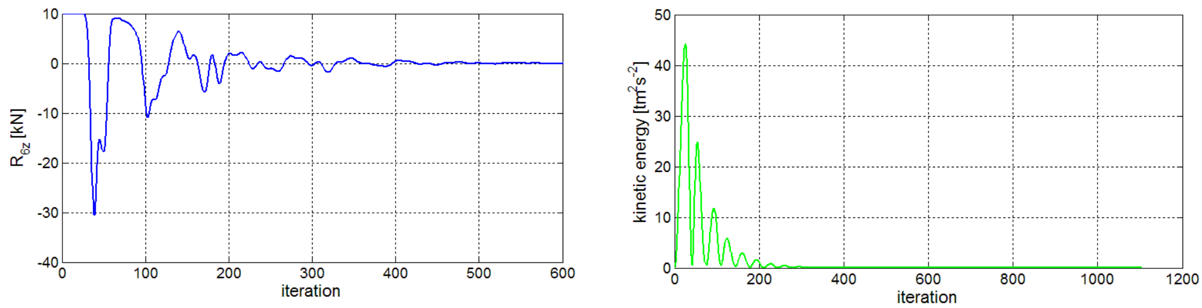

Fig. 4: Residual force and kinetic energy in the middle of span - damping factor 10 $t.s^{-1}$

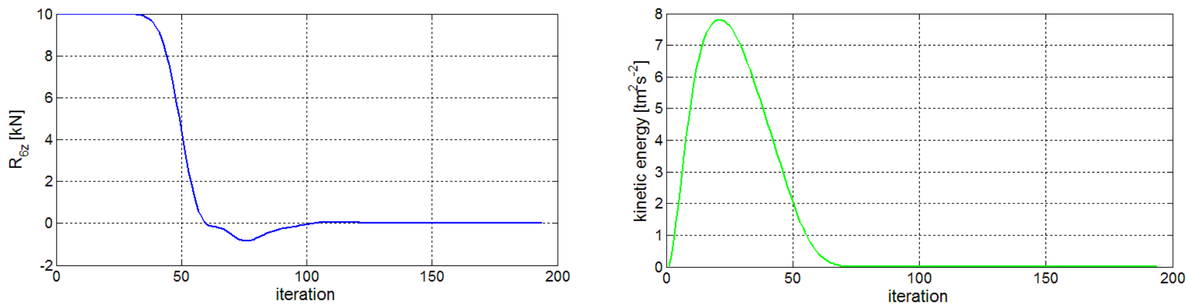

Fig. 5: Residual force and kinetic energy in the middle of span - damping factor 65 $t.s^{-1}$

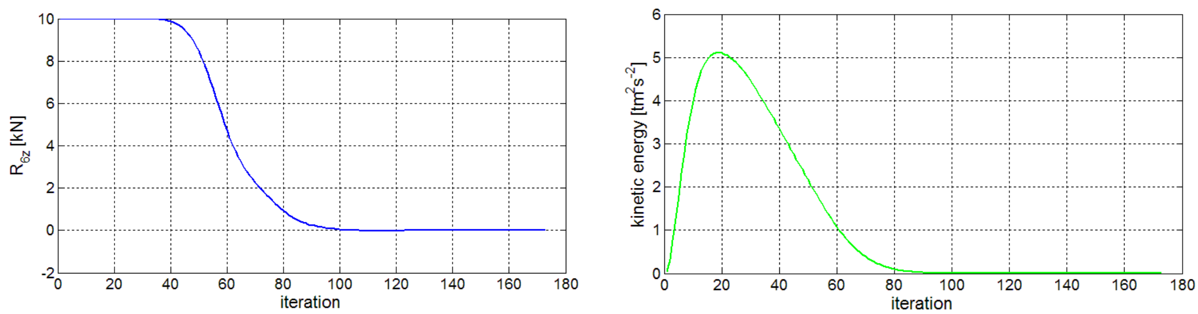

Fig. 6: Residual force and kinetic energy in the middle of span - damping factor 85 $t.s^{-1}$

The influence of the various damping factor on the iteration speed is shown in the Figure 4, Figure 5 and Figure 6. In Table 1 fixed parameters, which were used for the calculation of simple supported bar are shown. External distributed loading was equally spread between all joints.

Tab. 1: Basic construction characteristics

Span	l = 20 [m]
Yong's modulus of elasticity	$E = 210.10^6$ [kPa]
External load	$f_z = 5$ [kN/m]
Time step	$\Delta t = 0{,}01$ [s]
Cross section area	$A = 5.10^{-4}$ [m^2]

In Table 2 z coordinates in the middle of span there are shown. Influence of various calculation parameters on the speed of iteration is also obvious from Table 2. For comparison results calculated by deformation method are shown in Table 3.

Tab. 2: Z coordinate – dynamic relaxation method

n	M_{ix} [t]	M_{iz} [t]	Δt [s]	C_{iz} [$t.s^{-1}$]	Accuracy R_{zi}	Count of iteration	z coordinate [m]
10	5000	50	0,1	0,5	0,001	22374	0,7128
10	5000	50	0,1	10	0,001	1114	0,7128
10	5000	50	0,1	85	0,001	173	0,7128
10	5000	50	0,25	10	0,001	Unstable	-
10	5000	50	0,01	10	0,001	10679	0,7128
10	500	5	0,01	10	0,001	1101	0,7128
10	5000	5000	0,1	1000	0,001	1786	0,7128
10	5000	50	0,1	10	0,1	559	0,7122
10	5000	50	0,1	10	0,0001	1370	0,7128
50	5000	50	0,1	10	0,001	Unstable	-
50	5000	50	0,05	10	0,001	2095	0,7105
50	5000	50	0,05	40	0,001	1110	0,7105

where:

n is the number of elements on the bar

M_i is the fictitious mass in various directions (in x and y direction the same mass was considered)

Δt is the length of the time step

C_{iz} is the damping factor for joint i in z direction

Tab. 3: Z coordinate – deformation method

n	Count of iteration	z coordinate [m]
10	59	0,7128
50	17	0,7121
50	65	0,7105

5. Application and conclusion

The iteration speed and the accuracy of the method with various parameters were shown on the simple supported bar. For more illustrative functionality of the dynamic relaxation method the cable net was calculated. Figure 7 shows the shape of the construction. The construction is composed from 12 cable elements and 12 joints (joints 4, 5, 8 and 9 are unsupported). All cables have the same cross section area (A = $1{,}4645.10^{-4}$ m^2) and the same Young's modulus of elasticity (E = $8{,}2737.10^{7}$ kPa). Unsupported length of the elements 3, 4, 8, 11 is 30,419 m, unsupported length of other elements is 31,76 m. The distributed load was equally spread between the unsupported joints. The concentrated load at all unsupported joints is F = 35,56 kN. For the calculation of this example the deformation method and the dynamic relaxation was used. In both cases truss element was used. From Deng, Jiang and Kwan (2005) specification of this example was implemented.

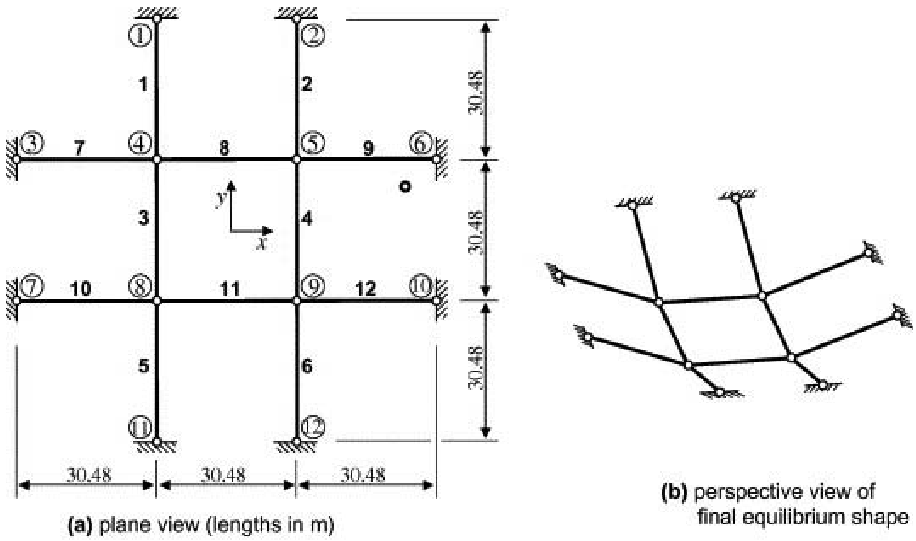

Fig. 7: Schema of cable net

After the calculation, some conclusions were set. First of all, both methods iterates to almost the same results (in Table 4 and Table 5 results from dynamic relaxation and deformation method are shown). However, the iteration speed is influenced by various factors which were assumed. Deformation method is faster than the dynamic relaxation one in case that the initial shape of construction is similar to equilibrium shape. Moreover, this behavior leads to another simple analysis of construction before deformation method is initiated. In opposite to the deformation method the dynamic relaxation one is faster in general. Almost arbitrary initial shape can be used and the dynamic relaxation converges to equilibrium shape. As it was mentioned, in dynamic relaxation method it is not necessary to compile the matrix of stiffness what leads to higher speed of iteration. In case of larger and more complicated construction this advantage should be more visible. On the other hand in dynamic relaxation the appropriate set up of the calculation parameters is difficult and nowadays the general approach to their set up is not known. Comparison of results in case of various calculating parameters is shown in Figure 8.

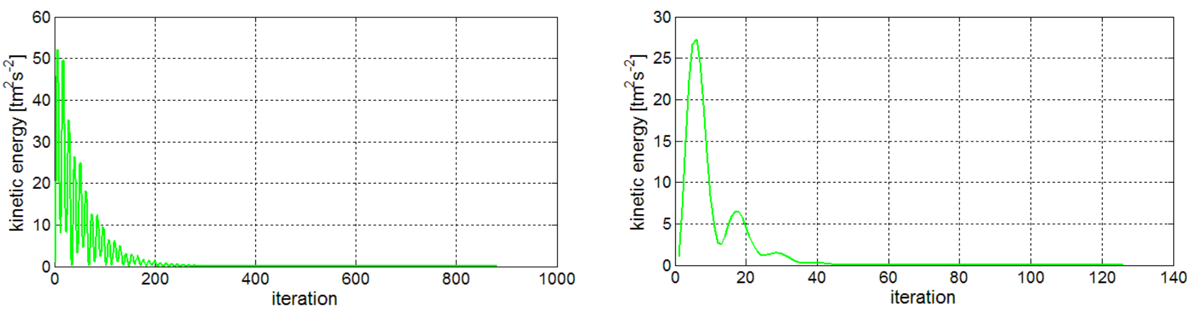

Fig. 8: Kinetic energy in joint 5 - damping factor 10 t.s^{-1} (left) and 75 t.s^{-1}

Tab. 4: Coordinates of joint 5 – dynamic relaxation method

M_{ix} [t]	M_{iy} [t]	M_{iz} [t]	Δt [s]	C_{iz} [$t.s^{-1}$]	Accuracy R_{zi}	Count of iteration	x coordinate [m]	z coordinate [m]
1000	1000	10	0,1	2	0,001	882	15,2804	9,5930
1000	1000	10	0,1	10	0,001	203	15,2804	9,5930
1000	1000	10	0,1	15	0,001	126	15,2804	9,5930
5000	5000	50	0,1	10	0,001	937	15,2804	9,5930
5000	5000	50	0,1	30	0,001	328	15,2804	9,5930
5000	5000	50	0,1	75	0,001	948	15,2804	9,5930

Tab. 5: Coordinates of joint 5 – deformation method

Count of iteration	x coordinate [m]	z coordinate [m]
13	15,2802	9,5917
20	15,2804	9,5930

Acknowledgement

The results presented in this paper are outputs of the research project "P105/11/1529 - Cable - membrane structures analyses" supported by Czech Science Foundation and project "SGS12/027/OHK1/1T/11 - Numerical modeling in mechanics of structures and materials" supported by CTU in Prague.

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