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ON EFFECTIVE IMPLEMENTATION OF THE NON-PENETRATION CONDITION FOR NON-MATCHING GRIDS PRESERVING SCALABILITY OF FETI BASED ALGORITHMS

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Abstract: *The point of this work is to extend our results obtained for elastic contact problems to the contact problems with non-matching grids which necessarily emerge, e.g., in the solution of transient contact problems or in contact shape optimization. We want to get good approximation and the constraint matrix B with nearly orthogonal rows. We consider both standard engineering approaches such as node to segment, or mortar elements. We give simple bounds on the singular values of the resulting matrix B and results of numerical experiments, including both the academic examples and some problems of practical interest. We conclude that the normalized orthogonal mortars proposed by Wohlmuth can be used to approximate the non-penetration conditions in a way that complies with the requirements of the FETI methods.*

Keywords: ***contact problems, Mortar elements, TFETI, scalability.***

1. Introduction

Mathematical models of contact include the inequalities which make the contact problems strongly nonlinear. In spite of this, a number of interesting results have been obtained by modifications of the methods that were known to be scalable for linear problems, in particular of the FETI domain decomposition method introduced by Farhat and Roux for parallel solution of linear problems. Using this approach, a body is partitioned into non-overlapping subdomains, an elliptic problem with Neumann boundary conditions is defined for each subdomain, and intersubdomain field continuity is enforced via Lagrange multipliers. The Lagrange multipliers are evaluated by solving a relatively well conditioned dual problem of small size that may be efficiently solved by a suitable variant of the conjugate gradient algorithm. Later Farhat (Mandel and Roux) introduced a “natural coarse problem” whose solution was implemented by auxiliary projectors so that the resulting algorithm became scalable.

It has been soon observed that duality based domain decomposition methods may also be successful for the solution of variational inequalities that describe equilibrium of a system of elastic bodies in unilateral contact. Recently, we obtained the theoretical results that guarantee the scalability also for contact problems, see Dostál et all (2010, 2012, 201x); Sadowská et all (2011).

The scalability results were originally proved for matching grids. In this case, the boolean matrix B which imposes the “gluing” conditions and non-penetration conditions has nearly orthogonal rows, which turns out to be a key ingredient of the proofs of optimality. By nearly orthogonal we mean that the matrix B has singular values distributed in a given positive interval that does not depend on the discretization parameter. For linear problems, B can be effectively reduced to the matrix with orthogonal rows; this was used by Klawonn and Widlund to improve the estimates of the rate of convergence. The orhogonalization of constraints that they use comprises multiplication of constraints that is not admissible for inequalities that describe the non-penetrations.

The point of this paper is to extend the results mentioned above to the contact problems with non-matching grids which necessarily emerge, e.g., in the solution of transient contact problems or in contact

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shape optimization. We want to get both good approximation and B with nearly orthogonal rows. We consider both standard engineering approaches such as node to segment, (see Wriggers (2005)) or mortar elements (see Wohlmuth (2001, 2011); Laursen et all (2005)). We give simple bounds on the singular values of the resulting matrix B and results of numerical experiments, including both the academic examples and some problems of practical interest such as the the gears in fig. 1. We conclude that the normalized orthogonal mortars proposed by Wohlmuth can be used to approximate the non-penetration conditions in a way that complies with the requirements of the FETI methods.

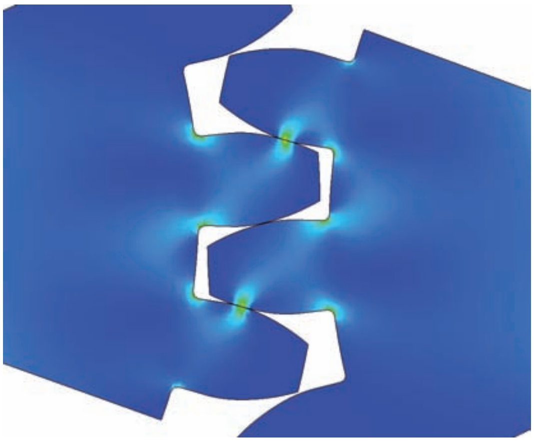

Fig. 1: von Misses stress in the real world example

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