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MODELING OF FRESH CONCRETE FLOW USING XFEM

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Abstract: *Modeling of fresh concrete flow is interesting problem from both theoretical and practical point of view and its application to self compacting concrete casting simulations is a subject of active research with important practical aspects. Practical importance is especially in application to self compacting concrete, which is highly actual. It is usually modeled in eulerian description of motion as a problem of two immiscible fluids (concrete as a Bingham fluid and air as a Newtonian fluid). Due to different physical properties of these fluids, there are discontinuities of velocity and pressure fields at the interface. In this paper, the eXtended Finite Element Method (XFEM) is used allowing the standard FE approximation space with tailor made functions across the interface to resolve the discontinuities.*

Keywords: ***Flow, concrete, XFEM, Bingham model, level set***

1. Introduction

This paper deals with eXtended Finite Element Method (XFEM) and its implementation in flow problems. Especially, it is focused on its application to fresh concrete flow. In modeling of flow problems using standard Finite Element Method (FEM) fluid is usually considered as a single homogeneous continuous medium. There are in principle three ways, how to describe the motion of continuous medium. In Lagrangian description, motion of each point is described in the framework of reference configuration. This approach, usually used in structural mechanics, is not suitable for fluid description, because of large deformations which require frequent re-meshing. In Eulerian description the motion is connected to actual configuration and therefore convective term is present and the Navier-Stokes equations governs the motion of the fluid. In this case, computation can be done on a fixed grid and no re-meshing is needed. On the other hand, one needs to use some stabilization due to convective terms and also LBB condition has to be satisfied. The advantages of both approaches have been combined in Arbitrary Lagrangian Eulerian formulation which is often used to model fluid-structure interactions. In the present work, Eulerian formulation is used. Modeling of fresh concrete flow in the context of Eulerian formulation is typically done using so called immiscible fluids concept (as a free surface flow), first proposed in (Chessa and Belytschko (2003)). For example, in case of fresh concrete flow, one fluid represents concrete and the other one represents air. Since both fluids are immiscible, the interface between them can be always captured. There are different possibilities, how to track the interface. In FEM context, Volume Of Fluid (see Gopala and Van Wachem (2008)) and level-set method (Sethian (1999), Osher and Fedkiw (2003)) are suitable choices. Since the flow is modeled using XFEM, the level-set method is used in this work. The level-set method describes interface as a zero level set of higher dimensional function. Usually, that function is chosen as a signed distance function. Motion of the interface is then governed by simple convective equation. Extended Finite Element Method enriching the standard continuous approximation of velocity and pressure fields by discontinuous enrichment functions along the interface is then used to discrete governing equations.

2. Governing equations

As was mentioned before, problem is described by Navier-Stokes equations. In this work, only 2D flow is considered. Let $\Omega \subset R^2$ be open set with boundary $\partial\Omega$. Boundary $\partial\Omega$ is decomposed to four mutually disjoint parts Γ_D, Γ_N, $\Gamma_{SWF}$and Γ_{OUT}, on which we prescribe Dirichlet boundary condition,

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Neumann boundary condition, so called ”slip with friction” boundary condition, ”penetration with resistance” (Volker (2002)) and so called ”do nothing” boundary condition. The whole problem can be formulated as follows:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \left(\boldsymbol{u} \cdot \boldsymbol{\nabla} \right) \boldsymbol{u} - \boldsymbol{\nabla} \cdot \boldsymbol{\sigma} - \rho \boldsymbol{b} = \boldsymbol{0} \quad \text{in } \Omega \tag{1}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \quad \text{in } \Omega \tag{2}$$

$$\boldsymbol{u} = \boldsymbol{g} \quad \text{on } \Gamma_D \tag{3}$$

$$\boldsymbol{u} = \boldsymbol{h} \quad \text{on } \Gamma_N \tag{4}$$

$$\boldsymbol{u} \cdot \boldsymbol{t} + \beta^{-1}\, \boldsymbol{n} \cdot (\boldsymbol{\tau} - p\boldsymbol{\delta}) \cdot \boldsymbol{t} = 0 \quad \text{on } \Gamma_{SWF} \tag{5}$$

$$\boldsymbol{u} \cdot \boldsymbol{n} + \alpha\, \boldsymbol{n} \cdot (\boldsymbol{\tau} - p\boldsymbol{\delta}) \cdot \boldsymbol{n} = 0 \quad \text{on } \Gamma_{SWF} \tag{6}$$

$$\boldsymbol{n} \cdot (\boldsymbol{\tau} - p\boldsymbol{\delta}) = 0 \quad \text{on } \Gamma_{OUT}. \tag{7}$$

Unknown fields are then velocity $\boldsymbol{u}$ and pressure p. Density ρ, body forces $\boldsymbol{b}$ and functions $\boldsymbol{g}$ and $\boldsymbol{h}$ are prescribed. Parameters β and α in equations (5) and (6) are assumed to be constant. Outer normal vector to the boundary is denoted as $\boldsymbol{n}$, tangent vector as $\boldsymbol{t}$. Standard decomposition of stress tensor $\boldsymbol{\sigma}$ into deviatoric stress $\boldsymbol{\tau}$ and hydrostatic pressure p is used. Strain rate tensor (8) is defined as symmetric part of velocity gradient:

$$\boldsymbol{D} = \frac{1}{2} \left(\boldsymbol{\nabla u} + (\boldsymbol{\nabla u})^T \right). \tag{8}$$

Constitutive law for air can be considered as one-parameter (viscosity μ) Newtonian fluid (9). It is well known that fresh concrete flow can be described by at least two parameters. The first one is yield stress τ_0 which introduces minimal stress necessary for concrete flow. The second parameter, plastic viscosity, μ_{pl} governs the main flow. Despite its simplicity, practical simulations have proved, that it is a suitable choice for describing fresh concrete behavior. The Bingham model (10) is described by following equations:

$$\boldsymbol{\tau} = \mu \boldsymbol{D} \tag{9}$$

$$\begin{cases} \boldsymbol{\tau} = \left[\mu_{pl} + \frac{\tau_0}{\sqrt{\boldsymbol{J_2^e}}} \right] \boldsymbol{D} & ;\, |\boldsymbol{J_2}| \le \tau_0 \\ \boldsymbol{D} = \boldsymbol{0} & ;\, |\boldsymbol{J_2}| \ge \tau_0 \end{cases} \tag{10}$$

where $\boldsymbol{J_2^e}$ is the second invariant of deviatoric strain tensor and $\boldsymbol{J_2}$ is second invariant of deviatoric stress tensor, which is defined as:

$$\boldsymbol{J_2} = \frac{1}{2} \boldsymbol{\tau} : \boldsymbol{\tau} \tag{11}$$

The second invariant of strain rate tensor is defined similarly.

3. Description of the interface

Generally speaking, there are two major approaches for description of the interface. So called interface tracking and interface capturing methods. First group of methods uses deforming mesh to track the interface and describes the interface in explicit manner. As it was mentioned before, in flow problems it is usual to use fixed grid and describe the motion in eulerian sense. Therefore, interface capturing methods have been developed (Sethian (1999)).They describes the interface in some implicit sense. One of the most often used methods of this group is so called level-set method which is applied in this work as well. In this method, interface is represented as a zero level set of some scalar function ϕ. Here, ϕ has been chosen as a signed distance function, which is defined by following property:

$$\phi(\boldsymbol{x}) = \pm \min_{\boldsymbol{x}^* \in \Sigma} ||\boldsymbol{x} - \boldsymbol{x}^*|| \tag{12}$$

The sign in the definition depends on which fluid occupies the point x, Σ denotes the interface between both fluids. Since the interface is changing in time, the level-set representation has to be updated at each time step. Motion of the interface is governed by level-set transport equation:

$$\frac{\partial \phi}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \phi = 0 \qquad \text{in } \Omega \text{ x } [0, T] \tag{13}$$

where $\boldsymbol{u}$ is convective velocity of the fluid. Of course, to solve (13), the proper boundary and initial conditions are needed.

4. Spatial dicretization and XFEM

Since both fluids in our model have different physical properties (density and viscosity), there are discontinuities in velocity and pressure fields along the interface. In general, one can distinguish between two types of discontinuities: strong and weak discontinuities. Strong discontinuity is present when there is a jump in a function. Weak discontinuity arises, when there is a jump in derivative of the function. Example of both types of discontinuities is typically two phase flow with surface tension, where the jump in pressure field (strong discontinuity) and jump in derivative of velocity field, or jump in strain rate tensor (weak discontinuity) occur. Proper description of discontinuities in terms of standard FEM is impossible because functions from approximation space are continuous. It is possible to refine mesh in sub-domains where one expects some discontinuities in the solution. In the case of two phase flow, such a solution is not efficient because the interface is evolving in time and therefore frequent re-meshing would be necessary. Contrary to this, treatment of discontinuities is very easy and natural using XFEM. The main idea behind XFEM is to enrich approximation space with tailored global (defined in whole domain) functions which can describe discontinuities in the solution. Choice of these functions depends on solved problem and on our a priori knowledge of solution. In our case, approximation of unknown function in XFEM has following form:

$$\boldsymbol{u^h(x,t)} = \underbrace{\sum_{i \in I} N_i(\boldsymbol{x}) \boldsymbol{u_i(t)}}_{standard\ FE\ approx.} + \underbrace{\sum_{i \in I^*} M_i(\boldsymbol{x}) \boldsymbol{a_i(t)}}_{enrichment} \tag{14}$$

where $N_i(\boldsymbol{x})$ is standard FE shape function belonging to node i, I is the set of all nodes in computational domain Ω, $M_i(\boldsymbol{x})$ is enrichment function belonging to node i and I^* is the set of enriched nodes (which is subset of I). Note, that enrichment function $M_i(\boldsymbol{x})$ is defined as multiplication of proper global shape function, which stores ”the knowledge” behind enrichment and ”partition of unity” (PU) function:

$$M_i(\boldsymbol{x}) = N_i(\boldsymbol{x})[\psi(\boldsymbol{x}) - \psi(\boldsymbol{x_i})] \qquad \forall \text{ i} \in I^* \tag{15}$$

In (15), ψ is so called global enrichment function, which is defined on whole domain Ω and $N_i(\boldsymbol{x})$ is standard FE shape function. In general, one can use any set of functions with PU property instead of FE shape functions. Note, that global enrichment function ψ is ”shifted” to ensure Kronecker-δ property hold. In two phase flow, when interface is described by level-set method, the enrichment functions for strong (16) and weak (17) discontinuities can be constructed easily as:

$$\psi_{sign}(\boldsymbol{x}) = sign(\phi(\boldsymbol{x})) = \begin{cases} -1 & : \phi(\boldsymbol{x}) \leq 0 \\ 0 & : \phi(\boldsymbol{x}) = 0 \\ 1 & : \phi(\boldsymbol{x}) \geq 0 \end{cases} \tag{16}$$

for strong discontinuity and

$$\psi_{abs} = abs(\phi(\boldsymbol{x})) \tag{17}$$

for weak discontinuity. As was proposed in (Fries (2003)), enrichment function in form 17 can leads to sub-optimal convergence because of presence of parasite terms in blending elements (elements in which only some nodes are enriched). In this paper, so-called ”Ramp function”, first published in (Fries (2003)), is used to overcome problems in blending elements.

Modeling of fresh concrete flow is a two fluid problem without the surface tension. Therefore, both velocity and pressure fields are enriched by ”abs-enrichment” function. Namely:

$$\boldsymbol{u^h(x,t)} = \sum_{i\in I} N_i(\boldsymbol{x})\boldsymbol{u_i(t)} + \sum_{i\in I^*} N_i(\boldsymbol{x})\left[abs(\phi(\boldsymbol{x})) - abs(\phi(\boldsymbol{x_i})\right]\boldsymbol{a_i(t)} \tag{18}$$

$$p^h(\boldsymbol{x},t) = \sum_{i\in I} N_i(\boldsymbol{x})p_i(t) + \sum_{i\in I^*} N_i(\boldsymbol{x})\left[abs(\phi(\boldsymbol{x})) - abs(\phi(\boldsymbol{x_i})\right]b_i(t) \tag{19}$$

Provided that proper function spaces are defined (see Tezduyar and Osawa (2000)), weak formulation of (1) - (7) states as follows: find $\boldsymbol{u^h} \in S_u^h$ and $p^h \in S_p^h$ such that $\forall \boldsymbol{w^h} \in V_u^h, \forall q^h \in V_p^h$:

$$\begin{aligned}
&\int_\Omega \rho \boldsymbol{w^h}\frac{\partial \boldsymbol{u^h}}{\partial t}d\Omega + \int_\Omega \rho \boldsymbol{w^h}\cdot(\boldsymbol{u^h}\cdot\boldsymbol{\nabla u^h})d\Omega + \int_\Omega \boldsymbol{\nabla w^h} : \boldsymbol{\tau(u^h)}d\Omega - \int_\Omega \boldsymbol{w^h}\cdot p^h d\Omega \\
&- \int_\Omega \boldsymbol{w^h}\cdot \boldsymbol{b}d\Omega - \int_{\partial\Omega} \boldsymbol{w^h}\cdot(\boldsymbol{\tau} - p\delta)\cdot \boldsymbol{n}dS + \int_\Omega q^h\boldsymbol{\nabla}\cdot\boldsymbol{u^h}d\Omega \\
&+ \sum_{el}\left[\int_{\Omega_e} \tau_{SUPG}(\boldsymbol{u^h}\cdot\boldsymbol{\nabla w^h})\cdot\left(\rho\frac{\partial \boldsymbol{u^h}}{\partial t} + \rho\boldsymbol{w^h}\cdot(\boldsymbol{u^h}\cdot\boldsymbol{\nabla u^h}) - \boldsymbol{\nabla}\cdot\tau(\boldsymbol{u^h}) + \nabla p^h - \boldsymbol{b}\right)d\Omega_e\right] \\
&+ \sum_{el}\left[\int_{\Omega_e} \tau_{PSPG}\frac{1}{\rho}\nabla q^h\cdot\left(\rho\frac{\partial \boldsymbol{u^h}}{\partial t} + \rho\boldsymbol{w^h}\cdot(\boldsymbol{u^h}\cdot\boldsymbol{\nabla u^h}) - \boldsymbol{\nabla}\cdot\tau(\boldsymbol{u^h}) + \nabla p^h - \boldsymbol{b}\right)d\Omega_e\right] \\
&+ \sum_{el}\left[\int_{\Omega_e} \tau_{LSIC}\boldsymbol{\nabla}\cdot\boldsymbol{w^h}\rho\boldsymbol{\nabla}\cdot\boldsymbol{u^h}d\Omega_e\right] = 0
\end{aligned} \tag{20}$$

Terms in the first two lines follows from standard Galerkin discretization, the third line represents stabilization term due to convection effects, the fourth line provides PSPG stabilization for elements not satisfying LBB condition and the last line provides another stabilization in higher velocity flow. Stabilization parameters $\tau_{SUPG}, \tau_{PSPG}, \tau_{LSIC}$ are chosen according to (Tezduyar and Osawa (2000)). Note, that in that work, finite elements linear in both velocity and pressure were used and therefore terms with $\boldsymbol{\nabla\tau}$ vanishes. Moreover, due to relatively small flow velocity of concrete, only PSPG stabilization is needed, because linear element does not satisfy the LBB condition.

Level-set function, as a scalar function, is discretized by the same shape functions as pressure:

$$\phi^h(\boldsymbol{x}) = \sum_{i\in I} N_i(\boldsymbol{x})\phi_i \tag{21}$$

5. Temporal discretization and solving scheme

After spatial discretization, we have system of non-linear ordinary differential (in time) equations, which has in general form:

$$(\boldsymbol{M} + \boldsymbol{M_\delta})\,\boldsymbol{a} + (\boldsymbol{N(u)} + \boldsymbol{N_\delta(u)}) + (\boldsymbol{K} + \boldsymbol{K_\delta})\,\boldsymbol{u} + \boldsymbol{K}_\mu\boldsymbol{u} + (\boldsymbol{G} + \boldsymbol{G_\delta})\,\boldsymbol{p} = \boldsymbol{F} + \boldsymbol{F_\delta} \tag{22}$$

$$\boldsymbol{G^T u} + \boldsymbol{M_\epsilon a} + \boldsymbol{N_\epsilon(u)} + \boldsymbol{K_\epsilon u} + \boldsymbol{G_\epsilon p} = \boldsymbol{E} + \boldsymbol{E_\epsilon} \tag{23}$$

Terms $\boldsymbol{M}, \boldsymbol{N(u)}, \boldsymbol{K}, \boldsymbol{G}, \boldsymbol{F}, \boldsymbol{E}$ in (22) and (23) follows from standard Galerkin discretization and represents time dependent term, convective term, diffusive term, term connected to pressure and terms represents boundary conditions. Terms with δ subscript are due to the SUPG stabilization, terms with ϵ are due to the PSPG stabilization and $\boldsymbol{K_\mu}$ follows from LSIC stabilization.

5.1. Solution scheme

Solution scheme introduced in (Patzák and Bittnar (2009)) can be described as follows:

1. Temporal discretization by θ-scheme.

$$\begin{aligned} &\boldsymbol{a}^{\boldsymbol{t}+\boldsymbol{\Delta t}} = \boldsymbol{a}^{\boldsymbol{t}} + \boldsymbol{\Delta a} \\ &\frac{\boldsymbol{u}^{\boldsymbol{t}+\boldsymbol{\Delta t}} - \boldsymbol{u}^{\boldsymbol{t}}}{\Delta t} = \alpha \boldsymbol{a}^{\boldsymbol{t}+\boldsymbol{\Delta t}} + (1-\alpha)\boldsymbol{a}^{\boldsymbol{t}} \\ &\boldsymbol{p}^{\boldsymbol{t}+\boldsymbol{\Delta t}} = \boldsymbol{p}^{\boldsymbol{t}} + \boldsymbol{\Delta p} \end{aligned} \tag{24}$$

2. Evaluation (prediction)

$$\begin{aligned} &\boldsymbol{u} = \boldsymbol{u}^{\boldsymbol{t}} + \boldsymbol{\Delta a}^{\boldsymbol{t}} \\ &\boldsymbol{a} = \boldsymbol{a}^{\boldsymbol{t}} \\ &\boldsymbol{p} = \boldsymbol{p}^{\boldsymbol{t}} \end{aligned} \tag{25}$$

3. Computing of velocity and pressure increments

$$\begin{aligned} &\boldsymbol{M}^* \boldsymbol{\Delta a} - \boldsymbol{G}^* \boldsymbol{\Delta p} = \boldsymbol{R} \\ &(\boldsymbol{G}^{\boldsymbol{T}})^* \boldsymbol{\Delta a} + \boldsymbol{G}_{\boldsymbol{\epsilon}} \boldsymbol{\Delta p} = \boldsymbol{Q} \end{aligned} \tag{26}$$

where

$$\begin{aligned} &\boldsymbol{M}^* = \boldsymbol{M} + \boldsymbol{M}_{\boldsymbol{\delta}} + \alpha \Delta t \left(\frac{\partial \boldsymbol{N}}{\partial \boldsymbol{u}} + \frac{\partial \boldsymbol{N}_{\boldsymbol{\delta}}}{\partial \boldsymbol{u}} + \boldsymbol{K} + \boldsymbol{K}_{\boldsymbol{\delta}} \right) \\ &\boldsymbol{G}^* = \boldsymbol{G} + \boldsymbol{G}_{\boldsymbol{\delta}} \\ &(\boldsymbol{G}^{\boldsymbol{T}})^* = \boldsymbol{M}_{\boldsymbol{\epsilon}} + \alpha \Delta t \left(\frac{\partial \boldsymbol{N}_{\boldsymbol{\epsilon}}}{\partial \boldsymbol{u}} + \boldsymbol{K}_{\boldsymbol{\epsilon}} + (\boldsymbol{G}^{\boldsymbol{T}}) \right) \\ &\boldsymbol{R} = \boldsymbol{F} + \boldsymbol{F}_{\boldsymbol{\delta}} - \left[(\boldsymbol{M} + \boldsymbol{M}_{\boldsymbol{\delta}})\, \boldsymbol{a} + (\boldsymbol{N}(\boldsymbol{u}) + \boldsymbol{N}_{\boldsymbol{\delta}}(\boldsymbol{u})) + (\boldsymbol{K} + \boldsymbol{K}_{\boldsymbol{\delta}})\, \boldsymbol{u} + \boldsymbol{K}_{\mu} \boldsymbol{u} + (\boldsymbol{G} + \boldsymbol{G}_{\boldsymbol{\delta}})\, \boldsymbol{p} \right] \\ &\boldsymbol{Q} = \boldsymbol{E} + \boldsymbol{E}_{\boldsymbol{\epsilon}} - \left[\boldsymbol{G}^{\boldsymbol{T}} \boldsymbol{u} + \boldsymbol{M}_{\boldsymbol{\epsilon}} \boldsymbol{a} + \boldsymbol{N}_{\boldsymbol{\epsilon}(\boldsymbol{u})} + \boldsymbol{K}_{\boldsymbol{\epsilon}} \boldsymbol{u} + \boldsymbol{G}_{\boldsymbol{\epsilon}} \boldsymbol{p} = \boldsymbol{E} + \boldsymbol{E}_{\boldsymbol{\epsilon}} \right] \end{aligned} \tag{27}$$

4. Evaluation of velocity and pressure

$$\begin{aligned} &\boldsymbol{a} \leftarrow \boldsymbol{a} + \boldsymbol{\Delta a} \\ &\boldsymbol{p} \leftarrow \boldsymbol{p} + \boldsymbol{\Delta p} \\ &\boldsymbol{u} \leftarrow \boldsymbol{u} + \Delta t \alpha \boldsymbol{\Delta a} \end{aligned} \tag{28}$$

5. Repeat steps 2.-4. until convergence is reached.

6. Solve the level set equation with computed velocity field $\boldsymbol{u}$. This is done using positive explicit scheme described for example in (Barth and Sethian (1998)).

7. Proceed with next time step.

6. Numerical example

Implementing of XFEM in combination with moving interface by level-set method is quite complicated task and there is a lack of suitable benchmark tests. To illustrate the prototype implementation of XFEM and its application, the application to structural behavior of composite cantilever with circular inclusions is presented. The geometry and structured FE mesh are shown in $Fig.1$ together with stress magnitude contours. Note, that the discretization is based on structured, regular grid and is not capturing the circular

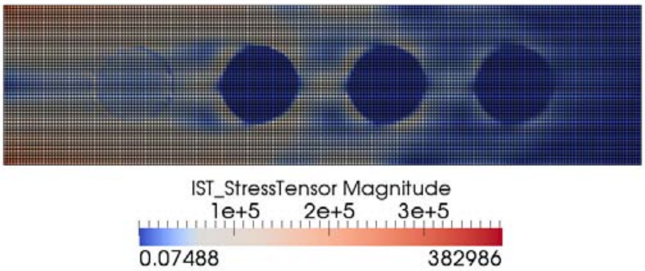

Fig. 1: Cantilever with weakened holes - stress magnitude contours

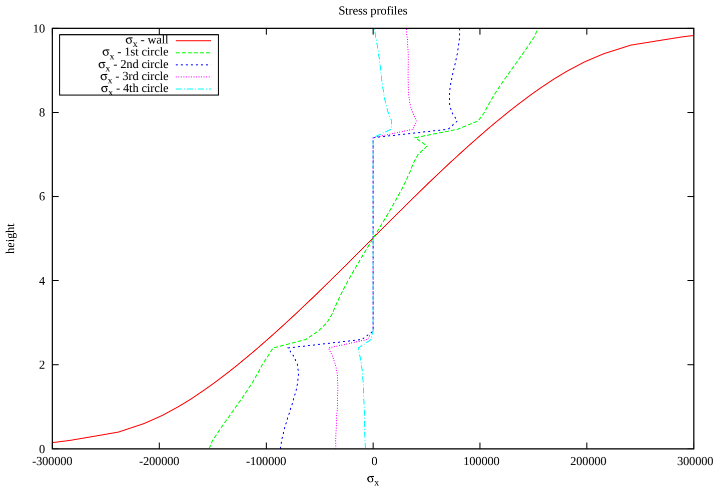

Fig. 2: The stress profile

inclusion geometry which is captured by introducing weak discontinuity enrichment in velocity field with kinks located at material interfaces. Material of cantilever is linear elastic with Young modulus $E = 3 \cdot 10^4 MPa$, weakened holes have Young modulus $E = 0.1MPa$. Poisson ratio is equal to 0.3. Since there are different material properties, so-called ”abs enrichment” was used, because of weak discontinuity in displacement (or strong discontinuity in strains and stresses). Constant continuous load with intensity 1 kN/m has been prescribed on the top surface of the beam. In $Fig.1$, contours of stress magnitude are shown. It can be seen, that in weakened holes the stress is nearly zero as the material has very small Young Modulus. In $Fig.2$, normal stress profiles at the restraint, resp. at the cut through each hole center is plotted shown. Again, it can be seen, that the stress is concentrated near the hole, while the hole itself is not under the stress.

7. Conclusions

In this paper, the numerical model for fresh concrete casting simulations was presented. The problem is treated as flow of a two immiscible homogeneous fluids with different physical properties. The concrete is considered as two-parametric Bingham fluid, the air is modeled as a standard Newtonian fluid. The interface between both fluids is descrideb in sense of level-set method. Extended finite element method is then used to resolve description of discontinuities in velocity and pressure fields across the interface. Since there is no surface tension in this problem, presenting discontinuity is only weak and therefore, so-called ”abs enrichment” is used. The prototype XFEM implementation is illustrated on structural analysis of composite cantilever with circular inclusions.

Acknowledgments

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