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VIBRATIONS OF THE SLENDER ROD INDUCED BY THE TURBULENCE IN THE COOLANT FLOW

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Abstract: *The theoretical model is presented. The vibrations are proposed to be random nature and are caused by random pressure fluctuation in the turbulent boundary layer surrounding the rod. The mean square of the amplitude of rod deflection is expressed using frequency response function to distributed loading and the spatial correlation density of the pressure fluctuations. The general expressions is derived and applied to slender rod with boundary condition pinned-pinned.*

Keywords: ***Fundamental frequency, frequency response function, transverse forces, spatial correlation density.***

1. Introduction

In the operation of nuclear power plants the fuel assenblies are surrounded by the axial turbulent flow and in consequence of pressure fluctuations acting on its surface. The problem is this one of random vibrations. In general several types of forces exists as follows

- the pressure forces which are considered to be independent of rod motion.
- the damping forces which are dependent on the lateral velocity of the rod
- the inertie forces which are dependent on the acceleration
- the elastic restoring forces which are dependent on the stiffness.

Special type of kinematic excitation represents independent mutual motion of upper and lower supports. This one is not included in this model.

2. Basic equations

Under the suppositions discussed in the previous chapter it can be shown that the mean square of the amplitude of deflection at the centre of the rod may be expressed as follows (Thomson 1965)

$$\langle y^2(t)\rangle = \frac{1}{M\omega_1^4}\int_0^{+\infty} H(f)H^*(f)df \int_0^L\int_0^L R_f(x,x',f)\phi_1(x)\phi_1(x')\,dxdx' \tag{1}$$

Where M ….. mass of the rod

ω_1 …. fundamentals circular frequency of the rod to distubuted loading

$H(f)$ … frequency response function of the rod to distributed loading

$H^*(f)$… komplex conjugate of $H(f)$

L …. length of the rod

ϕ_1 …. fundamentals mode of the rod

x, x' …. space variables

t …… time

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$R_f = \langle F(x,t)F'(x',t)\rangle$ where $F(x,t)$ and $F(x',t)$ denote values of the transverse forces at points x and x′ respectively at time t and the symbol $\langle\ \rangle'$ denotes time average per unit bandwith in this case at frequencyf. The term per unit bandwith is related to frequency under analysis.

For the evaluation of the spatial correlation R_f we will concentrate attention on the component of vibration in the plane containing cross section A-A, see Fig.1: Let us suppose unity of the rod length and homogeneous pressure field surrounding the rod. If we denote pressure diference between any point on the rod surfaře and a point diametrally oposite as „p“ and let positive „p“ is associated with the force directed toward the upper half on the figure then the force directed upward in the interval $d\theta$ is $F = pD/2\cos\theta\, d\theta$. Similarly the force on the interval $d\theta'\, is\, F' = pD_0/2\cos\theta'\, d\theta'$.

Based on the principles of integral calculus it is evident that the time mean square of the force F acting on the rod in the plane containing cross section A-A per unit rod length and per unit bandwith is

$$\langle F(x,t)F(x,t')\rangle' = \frac{D_0^2}{4}\int_{-\pi/2}^{\pi/2}\int_{-\pi/2}^{\pi/2}\langle\rho(x,\theta,t)\rho(x,\theta',t)\rangle'\cos\theta\cos\theta'\, d\theta\, d\theta' \tag{2}$$

Reavis (1967) postulated that exists a such positive value of α that we can suppose following approximation

$$\langle p(x,\theta,t)\,(p/x,\theta',t)\rangle' = \langle p^2(x,t)\rangle' e^{-\alpha|\theta-\theta'|} \tag{3}$$

Substituting (3) into (2) as the result we obtain

$$\langle F^2(x,t)\rangle' = \frac{D_0^2}{4}\langle p^2(x,t)\rangle' \int_{-\pi/2}^{+\pi/2}\int_{-\pi/2}^{+\pi/2} e^{-\alpha|\theta-\theta'|}\cos\theta'\cos\theta\, d\theta'\, d\theta \tag{4}$$

$$= \langle p^2(x,t)\rangle D_0^2\psi_D^2 \tag{5}$$

where

$$\begin{aligned}\psi_D^2 &= \frac{1}{4}\int_{-\pi/2}^{\pi/2}\int_{-\pi/2}^{\pi/2} e^{-\alpha|\theta-\theta'|}\cos\theta'\cos\theta\, d\theta\, d\theta' \\ &= \frac{1}{4(\alpha^2+1)}\left\{\alpha\pi + \frac{2}{\alpha^2+1}\left[1+e^{-\alpha\pi}\right]\right\}\end{aligned} \tag{6}$$

and represent effective rod diameter squared to actual rod diameter squared.

In eq. (5) the force $\langle F^2(x,t)\rangle'$ is expressed in terms of $\langle\rho^2(x,t)\rangle'$. It means that the peripherally distributed pressure loading is converted into a concentrated force loading.

From the supposition of the homogeneous boundary layer we can deduce that equation (3) depend not only on angles $|\theta-\theta'|$ but also on the coordinates $(x-x')$. It means that the following equation may be written

$$\frac{\langle F(x,t)F(x',t)\rangle'}{\langle F^2(x,t)\rangle'} = \frac{\langle p(x,t)\rho(x',t)\rangle'}{\langle p^2(x,t)\rangle'} \tag{7}$$

Based on the Gorman experiments (Gorman 1969) equation (7) may be rewritten as follows

$$\frac{\langle p(x,t)p(x',t)\rangle'}{\langle p^2(x,t)\rangle} = e^{-\alpha|x-x'|}\cos\beta(x-x') \tag{8}$$

where parameter α is different from α in equation (6). The following values are valid: $\alpha = 0.6,\ \beta = 1.0$.(Gorman 1969). Putting eq. (8) into eq. (1) and after simple rearrangement we can written

$$\int_O^L\int_O^L R_f(x,x',t)\varphi_1(x)\varphi_1(x')dx\,dx' = \\ = D_0^2\psi_D^2\int_O^L\int_O^L\langle p(x,t)p(x',t)\rangle\varphi_1(x)\varphi_1(x')dx\,dx' \tag{9}$$

and as the result we obtain

$$\langle y^2(t)\rangle = \frac{D_0^2\Psi_D^2}{M\omega_1^4}\langle p^2(x,t)\rangle'\int_0^{+\infty}\mathrm{H}(f)\mathrm{H}^*(f)df\int_0^L\int_0^L e^{-\alpha|x-x'|}\cos\beta(x-x')* \\ *\varphi_1(x)\varphi_1(x')dx\,dx' \tag{10}$$

3. Application on the slender rod with boundary condition pinned pinned

In this case the double integral in eq. (10) takes the form

$$\Psi_L^2 = \int_O^L\int_O^L e^{-\alpha|x-x'|}\cos\beta(x-x')\sin\frac{\pi x}{L}\sin\frac{\pi x'}{L}dx\,dx'$$

and finally

$$\langle y^2(t)\rangle = \frac{L^2D_0^2}{M\omega_1^4}\Psi_D^2\Psi_L^2\langle\rho^2(x,t)\rangle\int_O^{+\infty}\mathrm{H}_1(f)\mathrm{H}_1^*(f)df \tag{11}$$

Based on given boundary conditions and eq. (5), the integral in eq. (11) takes the form

$$\mathrm{I} = \int_O^{+\infty}\frac{\Omega^4 + 4D^2\Omega^2\omega^2}{\left[(\Omega^2-\omega^2)+i2D\Omega\omega\right]\left[(\Omega^2-\omega^2)-i2D\Omega\omega\right]}d\omega \tag{12}$$

The general glutation of eq. (12) is given in (4) and takes the form

$$\mathrm{I}_n = \int_O^{+\infty}\frac{\mathrm{P}_n(x)}{Q_n(x)Q_n(-x)}dx \tag{13}$$

where

$$\mathrm{P}_n|x| = A_0x^{2nL} + A_1x^{2n-4} + \ldots + A_{n-1}$$

$$Q_n|x| = B_0x^2 + B_1x^{n-1} + \ldots + B_n$$

For $n = 2$ the following expression is valid (Prudnikov 1981)

$$\mathrm{I}_2 = -\frac{\Pi i}{B_o}\frac{A_0 B_2 - A_1 B_0}{B_1 B_2 - B_3 B_0}$$

Since $B_0 = -1, \quad B_1 = i2D\omega, \quad B_2 = \Omega^2, \quad A_0 = 4D^2\Omega^2, \quad A = \Omega$

As the result obtain

$$\mathrm{I}_2 = \frac{\Omega}{4D}\left(1 + 4D^2\right) \doteq \frac{\Omega}{4D} \quad \mathit{for}\ D \langle\langle 1$$

And finally

$$\left\langle y^2(t)\right\rangle = \frac{L^2 D_0^2}{M\omega_1^4}\Psi_D^2\Psi_L^2\left\langle \rho^2(x,\mathrm{H})\right\rangle \frac{\pi f_1}{4D} \tag{14}$$

Conclusions

This paper represent theoretical analysis of the effects of boundary layer turbulence on fuel rod vibrafon. Some results of experimental investigations has been used. We will continue in this effort as follows

- Numercial evaluation of the values Ψ_L^2
- Assessment of mean square of the rod amplitude for the fuel rod of fuel assembly TVSA-T
- Application on the fuel assembly TVSA-T of NPP Temelin.

Preliminary numercial assessment showed that $\left\langle y(t)\right\rangle_{rms} = 3.04\,\mu m$

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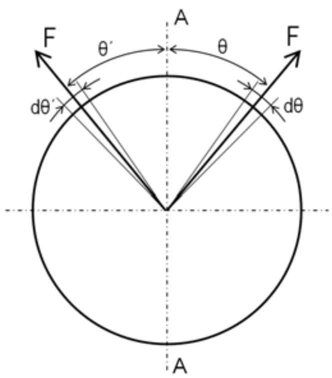

Fig. 1: Cross section A-A