18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 709–718
Paper **#56**

CRACK ANALYSIS IN MAGNETOELECTROELASTIC SOLIDS

S. Krahulec*, J. Sladek, V. Sladek***, P. Stanak******

Abstract: *The paper discusses about crack analysis in magnetoelectroelastic solids. 2-D crack problems are considered. There are applied various electromagnetic boundary conditions on the crack-faces. The definition of the electromagnetic boundary conditions on the crack-faces plays an important role in the crack analysis of magnetoelectroelastic materials. Two extreme cases - the fully permeable and the fully impermeable crack surfaces are analysed. The finite element method is applied to solve crack boundary value problems. The coupling among magnetic, electrical and mechanical fields is adequately considered in magnetoelectroelastic solids subjected to external loads.*

Keywords: ***magnetoelectroelastic composites, crack opening displacement***

1. Introduction

Smart materials are widely used in practical engineering applications. It can be observed coupling effect between mechanical and electric fields (piezoelectric), mechanical and magnetic fields (piezomagnetic), mechanical and electric and magnetic fields (magnetoelectroelactic), etc. An electric potential is produced when a piezoelectric element is under stress or strain loading. This effect is called direct piezoelectric effect. When a mechanical deformation is produced by an electric field, it is a converse piezoelectric effect (Song at al., 2006). Magnetoelectric materials induce the polarization by a magnetic field, or conversely induce magnetization by an electric field Nan (1994). These materials are promising for a wide range of applications, such as four-state memories, magnetic field sensors and magnetically controlled optoelectric devices. It is important to analyze magnetoelectroelastic material for fracture resistance, because it is brittle. The electric and magnetic boundary conditions on the crack-faces are determined by the measure of shielding of the electric and magnetic fields. Thus, it is important to define the electromagnetic boundary conditions on the crack-faces. There are frequently considered two extreme cases. The first it is the fully permeable crack. This type does not shield the electric and magnetic field. The second one is fully impermeable crack, which shields the electric and magnetic field completely. The boundary value problems with cracks can be solved by several methods. For example finite element method (Enderlein at al., 2005), boundary element method (García-Sánchez et al., 2007) and meshless method (Sládek at al., 2008).

2. Basic equations of magnetoelectroelasticity

The coupling of the mechanical, electrical and magnetic fields in magnetoelectroelastic solids (Nan, 1994) is given by the following constitutive equations

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - d_{kij}H_k \tag{1}$$

* Ing. Slavomír Krahulec: Institute of Construction and Architecture, Slovak Academy of Science, 845 03 Bratislava, Slovakia, e-mail: slavomir.krahulec@savba.sk

** Prof. Ing. Ján Sládek, DrSc.: Institute of Construction and Architecture, Slovak Academy of Science, 845 03 Bratislava, Slovakia

*** Prof. RNDr. Vladimír Sládek, DrSc.: Institute of Construction and Architecture, Slovak Academy of Science, 845 03 Bratislava, Slovakia

**** Ing. Peter Staňák: Institute of Construction and Architecture, Slovak Academy of Science, 845 03 Bratislava, Slovakia, e-mail: peter.stanak@savba.sk

$$D_j = e_{jkl}\varepsilon_{kl} + h_{jk}E_k + \alpha_{jk}H_k \tag{2}$$

$$B_j = d_{jkl}\varepsilon_{kl} + \alpha_{kj}E_k + \gamma_{jk}H_k \tag{3}$$

where

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4}$$

$$E_i = -\psi_{,i} \tag{5}$$

$$H_i = -\mu_{,i} \tag{6}$$

The strain tensor ε_{ij}, electric field vector E_i and magnetic intensity vector H_i are related to independent variables - displacement , electrical potential and magnetic potential denoted by u_i , ψ and μ, respectively. σ_{ij}, D_i, B_i represent the stress tensor, the electric displacements, and the magnetic inductions, respectively. Material parameters are the elastic coefficients c_{ijkl} , dielectric permittivities h_{jk} and magnetic permeabilities γ_{jk} . Finally, e_{kij} , d_{kij} and α_{jk} are the coefficients for the piezoelectric, piezomagnetic, and magnetoelectric coupling, respectively.

For plane-deformation problems the constitutive equations can be written in a matrix form (Parton and Kudryavtsev, 1988)

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{33}\\ \sigma_{13}\end{bmatrix} = \begin{bmatrix}c_{11} & c_{13} & 0\\ c_{13} & c_{33} & 0\\ 0 & 0 & c_{44}\end{bmatrix}\begin{bmatrix}u_{1,1}\\ u_{3,3}\\ u_{1,3}+u_{3,1}\end{bmatrix} - \begin{bmatrix}0 & e_{31}\\ 0 & e_{33}\\ e_{15} & 0\end{bmatrix}\begin{bmatrix}-\psi_{,1}\\ -\psi_{,3}\end{bmatrix} - \begin{bmatrix}0 & d_{31}\\ 0 & d_{33}\\ d_{15} & 0\end{bmatrix}\begin{bmatrix}-\mu_{,1}\\ -\mu_{,3}\end{bmatrix} =$$

$$= \mathbf{C}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix} - \mathbf{L}\begin{bmatrix}E_1\\ E_3\end{bmatrix} - \mathbf{K}\begin{bmatrix}H_1\\ H_3\end{bmatrix} \tag{7}$$

$$\begin{bmatrix}D_1\\ D_3\end{bmatrix} = \begin{bmatrix}0 & 0 & e_{15}\\ e_{31} & e_{33} & 0\end{bmatrix}\begin{bmatrix}u_{1,1}\\ u_{3,3}\\ u_{1,3}+u_{3,1}\end{bmatrix} + \begin{bmatrix}h_{11} & 0\\ 0 & h_{33}\end{bmatrix}\begin{bmatrix}-\psi_{,1}\\ -\psi_{,3}\end{bmatrix} + \begin{bmatrix}\alpha_{11} & 0\\ 0 & \alpha_{33}\end{bmatrix}\begin{bmatrix}-\mu_{,1}\\ -\mu_{,3}\end{bmatrix} =$$

$$= \mathbf{G}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix} + \mathbf{H}\begin{bmatrix}E_1\\ E_3\end{bmatrix} + \mathbf{A}\begin{bmatrix}H_1\\ H_3\end{bmatrix} \tag{8}$$

$$\begin{bmatrix}B_1\\ B_3\end{bmatrix} = \begin{bmatrix}0 & 0 & d_{15}\\ d_{31} & d_{33} & 0\end{bmatrix}\begin{bmatrix}u_{1,1}\\ u_{3,3}\\ u_{1,3}+u_{3,1}\end{bmatrix} + \begin{bmatrix}\alpha_{11} & 0\\ 0 & \alpha_{33}\end{bmatrix}\begin{bmatrix}-\psi_{,1}\\ -\psi_{,3}\end{bmatrix} + \begin{bmatrix}\gamma_{11} & 0\\ 0 & \gamma_{33}\end{bmatrix}\begin{bmatrix}-\mu_{,1}\\ -\mu_{,3}\end{bmatrix} =$$

$$= \mathbf{R}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{33}\\ 2\varepsilon_{13}\end{bmatrix} + \mathbf{A}\begin{bmatrix}E_1\\ E_3\end{bmatrix} + \mathbf{M}\begin{bmatrix}H_1\\ H_3\end{bmatrix} \tag{9}$$

Governing equations for magnetoelectroelastic body under static loading conditions are given by the force equilibrium and the scalar Maxwell's equations as

$$\sigma_{ij,j} + X_i = 0 \tag{10}$$

$$D_{j,j} - \Pi = 0 \tag{11}$$

$$B_{j,j} = 0 \tag{12}$$

where X_i and Π represent the body force vector and the volume density of free charges, respectively.

The Dirichlet boundary conditions of magnetoelectroelastic body are given as follows

$$\begin{aligned} u_i &= U_i, \quad \text{on} \quad \partial\Omega_u \\ \psi &= V, \quad \text{on} \quad \partial\Omega_v \\ \mu &= A, \quad \text{on} \quad \partial\Omega_a \end{aligned} \tag{13}$$

where U_i, V and A are prescribed the mechanical displacement, the electric potential and the magnetic potential, respectively.

The Neumann boundary conditions of magnetoelectroelastic body are

$$\begin{aligned} t_i &= \sigma_{ij} n_j = T_i, \quad \text{on} \quad \partial\Omega_t, \quad \partial\Omega = \partial\Omega_u \cup \partial\Omega_t \\ q &= D_i n_i = -Q, \quad \text{on} \quad \partial\Omega_q, \quad \partial\Omega = \partial\Omega_v \cup \partial\Omega_q \\ s &= B_i n_i = -S, \quad \text{on} \quad \partial\Omega_s, \quad \partial\Omega = \partial\Omega_a \cup \partial\Omega_s \end{aligned} \tag{14}$$

where T_i, Q, S and n_i are the traction vector, the normal component of the electric displacement vector, the normal component of the magnetic induction and unit outward vector components.

A general boundary value problem in a magnetoelectroelastic solid is uniquely specified by the governing equations (10)-(12), constitutive equations (7)-(9) and boundary conditions (13)-(14). The FEM is applied to solve above stated boundary value problem. For this purpose, we will utilize the COMSOL computer code for multi-physics problems.

The governing equation for a transient dynamic problem with inertial and damping term is written in COMSOL as

$$e_a u_{,tt} + d_a u_{,t} + \nabla.\Gamma = F \qquad \text{in } \Omega \tag{15}$$

where u is independent variable. The mass and damping coefficients are denoted by e_a and d_a, respectively, and will be excluded in the static analysis. The "scalar" F is the source term. The "flux vector" Γ is represented by equation

$$\Gamma = c\nabla u + \alpha u - \gamma \tag{16}$$

in which coefficients c, α and γ are the diffusion coefficient, the flux convection coefficient and the flux source term, respectively.

The symbols used in Eq. (15) can be represented for particular governing equations (10)-(12) completed with the constitutive equations (7)-(9) as

$$\Gamma = \left(-\begin{bmatrix} \sigma_{11} \\ \sigma_{33} \\ \sigma_{13} \end{bmatrix} \right) = \left(-\begin{bmatrix} c_{11}u_{1,1} + c_{13}u_{3,3} + e_{31}\psi_{,3} + d_{31}\mu_{,3} \\ c_{13}u_{1,1} + c_{33}u_{3,3} + e_{33}\psi_{,3} + d_{33}\mu_{,3} \\ c_{44}(u_{1,3} + u_{3,1}) + e_{15}\psi_{,1} + d_{15}\mu_{,1} \end{bmatrix} \right), \quad \nabla = \mathbf{D}_u^T := \begin{pmatrix} \partial_1 & 0 & \partial_3 \\ 0 & \partial_3 & \partial_1 \end{pmatrix}, \quad F = \begin{pmatrix} X_1 \\ X_3 \end{pmatrix} \tag{17}$$

$$\Gamma = \left(\begin{bmatrix} D_1 \\ D_3 \end{bmatrix} \right) = \begin{pmatrix} e_{15}(u_{1,3} + u_{3,1}) & -h_{11}\psi_{,1} & -\alpha_{11}\mu_{,1} \\ e_{31}u_{1,1} + e_{33}u_{3,3} & -h_{33}\psi_{,3} & -\alpha_{33}\mu_{,3} \end{pmatrix}, \quad \nabla = \mathbf{D}_\psi^T := \begin{pmatrix} \partial_1 & \partial_3 \end{pmatrix}, \quad F = \Pi \tag{18}$$

$$\mathbf{\Gamma}=\left(\begin{bmatrix}B_1\\B_3\end{bmatrix}\right)=\begin{pmatrix}d_{15}(u_{1,3}+u_{3,1}) & -\alpha_{11}\psi_{,1} & -\gamma_{11}\mu_{,1}\\ d_{31}u_{1,1}+d_{33}u_{3,3} & -\alpha_{33}\psi_{,3} & -\gamma_{33}\mu_{,3}\end{pmatrix},\quad \nabla=\mathbf{D}_\mu^T:=\begin{pmatrix}\partial_1 & \partial_3\end{pmatrix},\quad F=0 \tag{19}$$

3. Finite element formulation

Introducing the virtual displacement δu_i , virtual electric potential $\delta\psi$ and virtual magnetic potential $\delta\mu$, equations (10)-(12) can be rewritten as

$$\int_\Omega(\sigma_{ij,j}+X_i)\delta u_i d\Omega+\int_\Omega(D_{j,j}-\Pi)\delta\psi d\Omega+\int_\Omega B_{j,j}\delta\mu d\Omega=0 \tag{21}$$

The equation (21) as well as the boundary equations (14) and relations (4)-(6) can be represented equivalently as

$$\begin{aligned}&-\int_\Omega\sigma_{ij}\delta\varepsilon_{ij}d\Omega+\int_{\partial\Omega_u}\sigma_{ij}n_j\delta u_i d(\partial\Omega)+\int_{\partial\Omega_t}T_i\delta u_i d(\partial\Omega)+\int_\Omega X_i\delta u_i d\Omega-\int_\Omega D_i\delta E_i d\Omega\\&+\int_{\partial\Omega_v}n_iD_i\delta\psi d(\partial\Omega)-\int_{\partial\Omega_q}Q\delta\psi d(\partial\Omega)-\int_\Omega\Pi\delta\psi d\Omega-\int_\Omega B_i\delta H_i d\Omega+\int_{\partial\Omega_a}n_iB_i\delta\mu d(\partial\Omega)\\&-\int_{\partial\Omega_s}S\delta\mu d(\partial\Omega)=0\end{aligned} \tag{22}$$

The field variables u_i , ψ and μ can be approximated by the shape functions N_u , N_ψ and N_μ and the unknown nodal degrees of freedom $\mathbf{u}_k$, ψ_k and μ_k

$$\mathbf{u}_e=N_u(\xi_1,\xi_2)\mathbf{u}_k,\ \mathbf{u}_e=\begin{pmatrix}u_1\\u_3\end{pmatrix}_e,\ \mathbf{u}_k=\begin{pmatrix}u_1\\u_3\end{pmatrix}_k,\quad \psi_e=N_u(\xi_1,\xi_2)\psi_k,\quad \mu_e=N_u(\xi_1,\xi_2)\mu_k. \tag{23}$$

Usually, the matrices B_u , B_ψ and B_μ are introduced for simplification

$$\mathbf{B}_u=\mathbf{D}_uN_u,\quad \mathbf{B}_\psi=\mathbf{D}_\psi N_\psi,\quad \mathbf{B}_\mu=\mathbf{D}_\mu N_\mu \tag{24}$$

where the transposed matrices to $\mathbf{D}_u$, $\mathbf{D}_\psi$ and $\mathbf{D}_\mu$ have been defined in Eqs. (17)-(19). Recall that the matrices $\mathbf{D}_u$, $\mathbf{D}_\psi$ and $\mathbf{D}_\mu$ involve the derivatives with respect to global Cartesian coordinates x_1 and x_3, which are expressed in terms of the derivatives with respect to the local coordinates on elements as

$$\begin{pmatrix}\partial/\partial x_1\\ \partial/\partial x_3\end{pmatrix}=\left[J\right]^{-1}\begin{pmatrix}\partial/\partial\xi_1\\ \partial/\partial\xi_2\end{pmatrix},\quad \left[J\right]^{-1}=\begin{pmatrix}\partial\xi_1/\partial x_1 & \partial\xi_2/\partial x_1\\ \partial\xi_1/\partial x_3 & \partial\xi_2/\partial x_3\end{pmatrix}. \tag{25}$$

The element energy functional is resulted from equations (22), (24) and (1)-(3)

$$\begin{aligned}\delta F_u^e=&-\delta\mathbf{u}_k^T\left\{\left(\int_{\Omega^e}\mathbf{B}_u^T\mathbf{C}\mathbf{B}_u d\Omega\right)\mathbf{u}_k+\left(\int_{\Omega^e}\mathbf{B}_u^T\mathbf{L}\mathbf{B}_\psi d\Omega\right)\psi_k+\left(\int_{\Omega^e}\mathbf{B}_u^T\mathbf{K}\mathbf{B}_\mu d\Omega\right)\mu_k\right\}\\&+\delta\mathbf{u}_k^T\left\{\left(\int_{\partial\Omega_u^e}\mathbf{B}_u^T\mathbf{NCB}_u d(\partial\Omega)\right)\mathbf{u}_k+\left(\int_{\partial\Omega_u^e}\mathbf{B}_u^T\mathbf{NLB}_\psi d(\partial\Omega)\right)\psi_k\right.\\&\left.+\left(\int_{\partial\Omega_u^e}\mathbf{B}_u^T\mathbf{NKB}_\mu d(\partial\Omega)\right)\mu_k\right\}+\delta\mathbf{u}_k^T\int_{\Omega^e}N_u^T\mathbf{X}d\Omega+\delta\mathbf{u}_k^T\int_{\partial\Omega_t^e}N_u^T\mathbf{T}d(\partial\Omega)=0\end{aligned} \tag{26}$$

$$\delta F_\psi^e = -\delta\psi_k^T\left\{\left(\int_{\Omega^e}\mathbf{B}_\psi^T\mathbf{G}\mathbf{B}_u d\Omega\right)\mathbf{u}_k + \left(\int_{\Omega^e}\mathbf{B}_\psi^T\mathbf{H}\mathbf{B}_\psi d\Omega\right)\psi_k + \left(\int_{\Omega^e}\mathbf{B}_\psi^T\mathbf{A}\mathbf{B}_\mu d\Omega\right)\mu_k\right\}$$
$$+\delta\psi_k^T\left\{\left(\int_{\partial\Omega_v^e}N_\psi^T\mathbf{n}\mathbf{G}\mathbf{B}_u d(\partial\Omega)\right)\mathbf{u}_k + \left(\int_{\partial\Omega_v^e}N_\psi^T\mathbf{n}\mathbf{H}\mathbf{B}_\psi d(\partial\Omega)\right)\psi_k \right. \tag{27}$$
$$\left. +\left(\int_{\partial\Omega_v^e}N_\psi^T\mathbf{n}\mathbf{A}\mathbf{B}_\mu d(\partial\Omega)\right)\mu_k - \left(\int_{\Omega^e}N_\psi^T\Pi d\Omega\right)\right\} - \delta\psi_k^T\left(\int_{\partial\Omega_q^e}N_\psi^T Q d(\partial\Omega)\right) = 0$$

$$\delta F_\mu^e = -\delta\mu_k^T\left\{\left(\int_{\Omega^e}\mathbf{B}_\mu^T\mathbf{R}\mathbf{B}_u d\Omega\right)\mathbf{u}_k + \left(\int_{\Omega^e}\mathbf{B}_\mu^T\mathbf{A}\mathbf{B}_\psi d\Omega\right)\psi_k + \left(\int_{\Omega^e}\mathbf{B}_\mu^T\mathbf{M}\mathbf{B}_\mu d\Omega\right)\mu_k\right\}$$
$$+\delta\mu_k^T\left\{\left(\int_{\partial\Omega_a^e}N_\mu^T\mathbf{n}\mathbf{R}\mathbf{B}_u d(\partial\Omega)\right)\mathbf{u}_k + \left(\int_{\partial\Omega_a^e}N_\mu^T\mathbf{n}\mathbf{A}\mathbf{B}_\psi d(\partial\Omega)\right)\psi_k \right. \tag{28}$$
$$\left. +\left(\int_{\partial\Omega_a^e}N_\mu^T\mathbf{n}\mathbf{M}\mathbf{B}_\mu d(\partial\Omega)\right)\mu_k\right\} - \delta\mu_k^T\left(\int_{\partial\Omega_s^e}N_\mu^T S d(\partial\Omega)\right) = 0$$

where $\mathbf{n} = \begin{pmatrix} n_1 & n_3 \end{pmatrix}$, $\mathbf{N} = \begin{pmatrix} n_1 & 0 & n_3 \\ 0 & n_3 & n_1 \end{pmatrix}$.

Assuming that $\delta F_u = \sum \delta F_u^e$, $\delta F_\psi = \sum \delta F_\psi^e$ and $\delta F_\mu = \sum \delta F_\mu^e$ with using the abbreviations

$$\mathbf{K}_{uu}^e = \left(\int_{\Omega^e}\mathbf{B}_u^T\mathbf{C}\mathbf{B}_u d\Omega\right) - \left(\int_{\partial\Omega_u^e}\mathbf{B}_u^T\mathbf{N}\mathbf{C}\mathbf{B}_u d(\partial\Omega)\right),$$

$$\mathbf{K}_{u\psi}^e = \left(\int_{\Omega^e}\mathbf{B}_u^T\mathbf{L}\mathbf{B}_\psi d\Omega\right) - \left(\int_{\partial\Omega_u^e}\mathbf{B}_u^T\mathbf{N}\mathbf{L}\mathbf{B}_\psi d(\partial\Omega)\right),$$

$$\mathbf{K}_{u\mu}^e = \left(\int_{\Omega^e}\mathbf{B}_u^T\mathbf{K}\mathbf{B}_\mu d\Omega\right) - \left(\int_{\partial\Omega_u^e}\mathbf{B}_u^T\mathbf{N}\mathbf{K}\mathbf{B}_\mu d(\partial\Omega)\right), \quad \mathbf{f}_u^e = \int_{\Omega^e}N_u^T\mathbf{X}d\Omega + \int_{\partial\Omega_t^e}N_u^T\mathbf{T}d(\partial\Omega)$$

$$\mathbf{K}_{\psi u}^e = -\left(\int_{\Omega^e}\mathbf{B}_\psi^T\mathbf{G}\mathbf{B}_u d\Omega\right) + \left(\int_{\partial\Omega_v^e}N_\psi^T\mathbf{n}\mathbf{G}\mathbf{B}_u d(\partial\Omega)\right),$$

$$K_{\psi\psi}^e = -\left(\int_{\Omega^e}\mathbf{B}_\psi^T\mathbf{H}\mathbf{B}_\psi d\Omega\right) + \left(\int_{\partial\Omega_v^e}N_\psi^T\mathbf{n}\mathbf{H}\mathbf{B}_\psi d(\partial\Omega)\right),$$

$$K_{\psi\mu}^e = -\left(\int_{\Omega^e}\mathbf{B}_\psi^T\mathbf{A}\mathbf{B}_\mu d\Omega\right) + \left(\int_{\partial\Omega_v^e}N_\psi^T\mathbf{n}\mathbf{A}\mathbf{B}_\mu d(\partial\Omega)\right), \quad f_\psi^e = \int_{\Omega^e}N_\psi^T\Pi d\Omega + \int_{\partial\Omega_q^e}N_\psi^T Q d(\partial\Omega)$$

$$\mathbf{K}_{\mu u}^e = -\left(\int_{\Omega^e}\mathbf{B}_\mu^T\mathbf{R}\mathbf{B}_u d\Omega\right) + \left(\int_{\partial\Omega_a^e}N_\mu^T\mathbf{n}\mathbf{R}\mathbf{B}_u d(\partial\Omega)\right),$$

$$K_{\mu\psi}^e = -\left(\int_{\Omega^e}\mathbf{B}_\mu^T\mathbf{A}\mathbf{B}_\psi d\Omega\right) + \left(\int_{\partial\Omega_a^e}N_\mu^T\mathbf{n}\mathbf{A}\mathbf{B}_\psi d(\partial\Omega)\right),$$

$$K_{\mu\mu}^e = -\left(\int_{\Omega^e}\mathbf{B}_\mu^T\mathbf{M}\mathbf{B}_\mu d\Omega\right) + \left(\int_{\partial\Omega_a^e}N_\mu^T\mathbf{n}\mathbf{M}\mathbf{B}_\mu d(\partial\Omega)\right), \quad f_\mu^e = \int_{\partial\Omega_s^e}N_\mu^T S d(\partial\Omega) \tag{29}$$

and assembling the matrices into global matrix, the discretized equations can be written as

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\psi} & \mathbf{K}_{u\mu} \\ \mathbf{K}_{\psi u} & K_{\psi\psi} & K_{\psi\mu} \\ \mathbf{K}_{\mu u} & K_{\mu\psi} & K_{\mu\mu} \end{bmatrix}\begin{bmatrix} \mathbf{u} \\ \psi \\ \mu \end{bmatrix} = \begin{bmatrix} \mathbf{f}_u \\ f_\psi \\ f_\mu \end{bmatrix} \tag{30}$$

or

$$\mathbf{Kx} = \mathbf{F} \tag{31}$$

where $\mathbf{K}$ is the stiffness matrix of the structure, $\mathbf{x}$ is the vector of unknown quantities and $\mathbf{F}$ is the loading vector.

4. Boundary conditions for crack-face

In this paper, two extreme boundary conditions on crack faces are considered, i.e., the fully impermeable and the fully permeable conditions. The electrical and magnetic boundary conditions on crack faces along x_1 for fully impermeable example are given as

$$D_3(\mathbf{x} \in \partial\Omega_c^+) = D_3(\mathbf{x} \in \partial\Omega_c^-) = 0$$

$$B_3(\mathbf{x} \in \partial\Omega_c^+) = B_3(\mathbf{x} \in \partial\Omega_c^-) = 0 \tag{32}$$

where $\partial\Omega_c^+$ and $\partial\Omega_c^-$ are the upper and the lower crack faces, respectively.

The fully permeable crack face boundary conditions along x_1 are given by

$$D_3(\mathbf{x} \in \partial\Omega_c^+) = D_3(\mathbf{x} \in \partial\Omega_c^-),\ \psi(\mathbf{x} \in \partial\Omega_c^+) - \psi(\mathbf{x} \in \partial\Omega_c^-) = 0$$

$$B_3(\mathbf{x} \in \partial\Omega_c^+) = B_3(\mathbf{x} \in \partial\Omega_c^-),\ \mu(\mathbf{x} \in \partial\Omega_c^+) - \mu(\mathbf{x} \in \partial\Omega_c^-) = 0 \tag{33}$$

5. Numerical examples

It is considered a magnetoelectroelastic straight strip. It can be solved as a 2-D problem under plane deformation conditions with the width of the strip $w = 2.5m$ and height $h = 3m$. The central crack with length $a = 1m$ along the axis x_1 is assumed. On the top and bottom surfaces of the strip, we consider either single pure loadings (a pure electrical load D_0, a pure magnetic load B_0, a pure mechanical load σ_0) or combinations of such pure loadings, while the lateral sides are traction free and with vanishing normal components of the electric displacement and magnetic induction vectors. A quarter of the strip is analyzed because of bi-axial symmetry. On the symmetry cuts, the normal displacements and tangential traction vector components are vanishing as well as the normal components of the electric displacement and magnetic induction vectors disappear. Thus, on the bottom of the quarter of the strip except the crack face we have $u_3 = 0$, $t_1 = 0$, $Q = 0$, $S = 0$, while on the right lateral $u_1 = 0$, $t_3 = 0$, $Q = 0$, $S = 0$. As long as the impermeable crack is assumed, Q and S are vanishing on the surface of the crack, while in the case of permeable crack the potentials ψ and μ are vanishing on both crack faces as long as no potentials are applied on the crack surface.

For the magnetoelectroelastic material, we chosen the $BaTiO_3$-$CoFe_2O_4$ composite (Li, 2000):

$$c_{11} = 22.6 \cdot 10^{10}\, Nm^{-2}\ ,\ c_{13} = 12.4 \cdot 10^{10}\, Nm^{-2}\ ,\ c_{33} = 21.6 \cdot 10^{10}\, Nm^{-2}\ ,\ c_{44} = 4.4 \cdot 10^{10}\, Nm^{-2}\ ,$$

$$e_{15} = 5.8\, Cm^{-2}\ ,\ e_{31} = -2.2\, Cm^{-2}\ ,\ e_{33} = 9.3\, Cm^{-2}\ ,$$

$$h_{11} = 5.64 \cdot 10^{-9}\, C(Vm)^{-1}\ ,\ h_{33} = 6.35 \cdot 10^{-9}\, C(Vm)^{-1}\ .$$

$$d_{15} = 275.0\, N(Am)^{-1}\ ,\ d_{31} = 290.2\, N(Am)^{-1}\ ,\ d_{33} = 350.0\, N(Am)^{-1}\ ,$$

$$\alpha_{11} = 5.367 \cdot 10^{-12}\, Ns/VC,\ \alpha_{33} = 2737.5 \cdot 10^{-12}\, Ns/VC.$$

$$\gamma_{11} = 297.0 \cdot 10^{-6} Wb(Am)^{-1} \;,\; \gamma_{33} = 83.5 \cdot 10^{-6} Wb(Am)^{-1}$$

The quarter of the strip is covered by 8400 linear quadrate finite elements.

In Figs. 1-4, the numerical results are given for all considered cases of single loadings: (i) pure mechanical load $\sigma_0 = 1Pa$; (ii) pure electrical load $D_0 = 1C/m^2$; (iii) pure magnetic load $B_0 = 1Vs/m^2$. Fig. 1 shows the distribution of the electric potential along the crack face for the fully impermeable crack condition. It can be seen that the electrical potential is negative, if a pure electrical load is applied. A similar behaviour can be observed for the magnetic potential (Fig.2) on the crack face under fully impermeable crack conditions, but the negative magnetic potential is appears if a pure magnetic load is applied and its magnitude is smaller than the value corresponding to a pure electrical load.

More interesting is the crack opening displacement shown in Fig. 3. and Fig.4. For the fully impermeable crack faces (Fig. 3.), a considerable opening of crack is caused by a pure magnetic loading and a pure electrical loading, though it is still slightly lower than in the case of a pure mechanical loading. For the fully permeable crack faces, however, the inverse behaviour is observed, when the main opening is caused by a pure mechanical loading, while the crack opening displacement is negative (crack closure) under a pure magnetic loading.

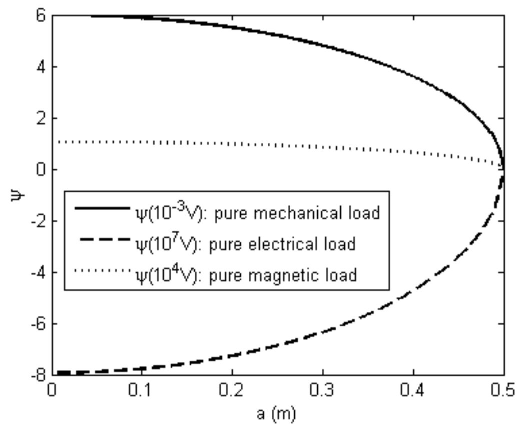

Fig. 1. Variation of the electrical potential on the crack face for the fully impermeable crack condition under a various loads.

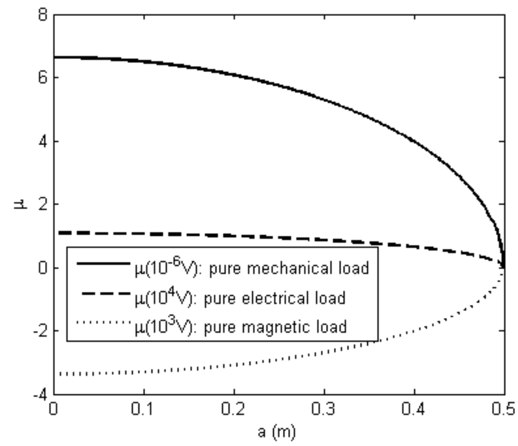

Fig. 2. Variation of the magnetic potential on the crack face for the fully impermeable crack condition under a various loads.

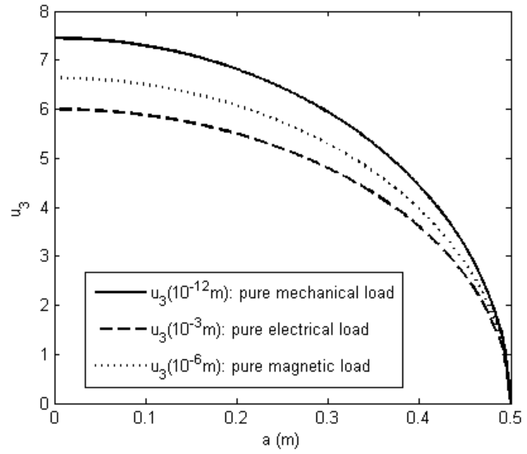

Fig. 3. Variation of the crack displacement for the fully impermeable crack condition under a various loads.

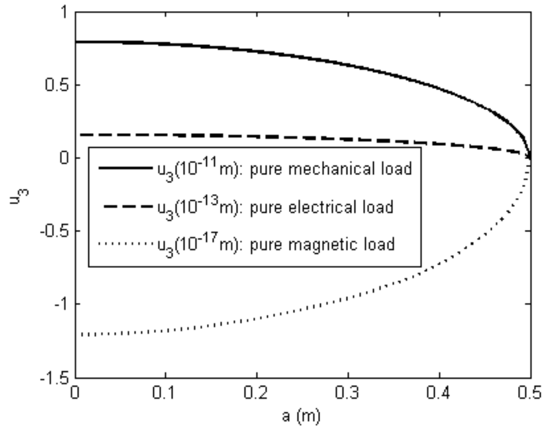

Fig. 4. Variation of the crack displacement for the fully permeable crack condition under a various loads.

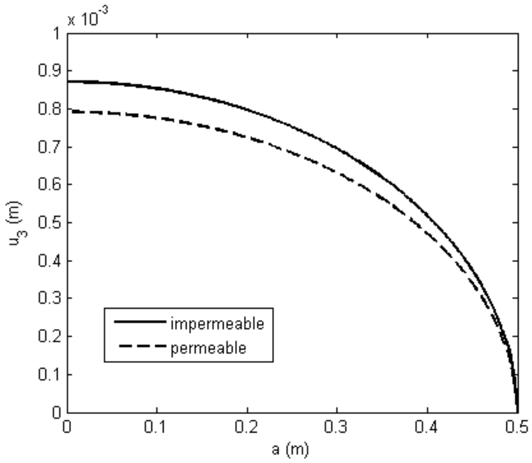

Fig. 5. Influence of the electromagnetic conditions on the crack displacement under a combined load.

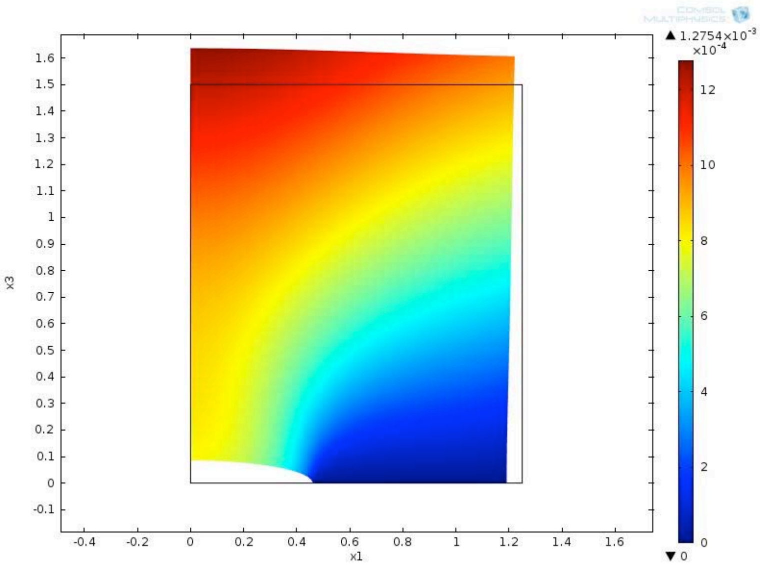

Fig. 6. Deformation of the quarter of the strip in direction x_3 for the fully permeable crack condition under combined load.

In the last example (Fig.5. and Fig. 6.), combined loading by mechanical load $\sigma_0 = 10^8 Pa$ with electrical load $D_0 = 10^{-2} C/m^2$ and magnetic load $B_0 = 10 Vs/m^2$ are applied. The influence of the crack electromagnetic boundary conditions on the crack opening displacement u_3 along the crack surface is shown in Fig. 5, when a combined loading is applied to the strip. A larger crack opening displacement appears in the case of fully impermeable electro-magnetic boundary conditions on the crack faces. Fig. 6. illustrates the displacements u_3 in a quarter of the strip under combined loading for the fully permeable crack conditions.

6. Conclusions

2-D crack problems in homogeneous magnetoelectroelastic composites are analyzed by finite element method using the commercial code COMSOL. The relevant governing equations are realized within the general framework for the partial differential equations. The mechanical and electromagnetic responses are studied under pure and/or combined mechanical and electro-magnetic loadings with assuming either permeable or impermeable electro-magnetic boundary conditions on the crack surface.

The coupling among the mechanical and electro-magnetic fields is confirmed by induced mechanical and electro-magnetic responses induced by pure single (pure mechanical, or electrical, or magnetic) loadings as well as by combined loadings. The crack opening displacements are dependent on the electro-magnetic boundary conditions applied on the crack surface. It has been found that the magnetic loading leads to crack closure.

References

Song, G., Sethi, V., Li H.-N. (2006) Vibration control of civil structures using piezoceramic smart materials: A review. *Engineering Structures* 28, , 1513-1524.

Nan, C.W. (1994) Magnetoelectric effect in composites of piezoelectric and piezomagnetic phases, *Phys. Rev. B* 50 6082-6088.

Enderlein, M., Ricoeur, A., Kuna, M. (2005) Finite element techniques for dynamic crack analysis in piezoelectrics, *International Journal of Fracture* 134, 191-208.

Parton, V.Z., Kudryavtsev, B.A. (1988) *Electromagnetoelasticity, Piezoelectrics and Electrically Conductive Solids.* Gordon and Breach Science Publishers, New York,.

García-Sánchez, F. , Rojas-Díaz, R., Sáez, A., Zhang, Ch. (2007) Fracture of magnetoelectroelastic composite materials using boundary element method (BEM), *Theoretical and Applied Fracture Mechanics* 47 192-204.

Sladek, J., Sladek, V., Solek, P., Pan, E. (2008)Fracture analysis of cracks in magneto-electro-elastic solids by the MLPG, *Computational Mechanics* 42 697-714.

Li, J.Y. (2000) Magnetoelectroelastic multi-inclusion and inhomogeneity problems and their applications in composite materials. *International Journal of Engineering Science* 38, 1993-2011.