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KINEMATICAL EXCITED VIBRATION OF THE NUCLEAR FUEL ASSEMBLY

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Abstract: *The paper deals with modelling of the hexagonal type nuclear fuel assembly vibration caused by kinematical excitation determined by motion of the support plates in the reactor core. The support plate motion is excited by pressure pulsations generated by circulation pumps in the main circulation loops. The cyclic and central symmetry of the system is advantageous for the fuel assembly decomposition into six identical revolved fuel rod segments, central tube and skeleton linked by several spacer grids in horizontal planes. The modal synthesis method with condensation of the fuel rod segments is used for determination of steady vibration.*

Keywords: ***Vibrations, nuclear fuel assembly, kinematic excitation, modal synthesis method, DOF number reduction.***

1. Introduction

The hexagonal nuclear fuel assembly TVSA/T (further FA) (Fig.1) is in term of mechanics very complicated system of beam type (Sýkora (2009)). Because of the cyclic and central symmetry of the whole system (Fig.2) the FA decomposition into six rod segments (S), centre tube (CT) and load-bearing skeleton (LS) shall be applied (Zeman & Hlaváč (2011b)). Each rod segment (on Fig.2 circumscribed by triangles) is composed of 52 fuel rods with fixed bottom ends in the lower support plate and 3 guide thimbles fully restrained in the lower and upper support plates in reactor core. The centre tube is fully restrained. The skeleton is created of 6 angle pieces (AP) coupled by divided grid rims (GR) at all levels

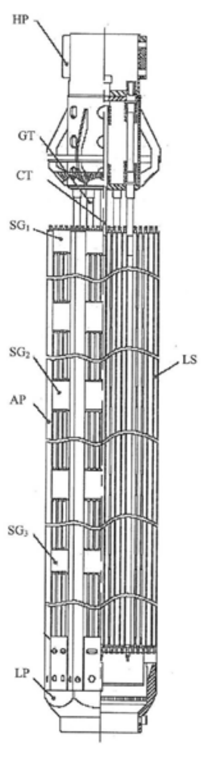

Fig. 1: Scheme of the fuel assembly

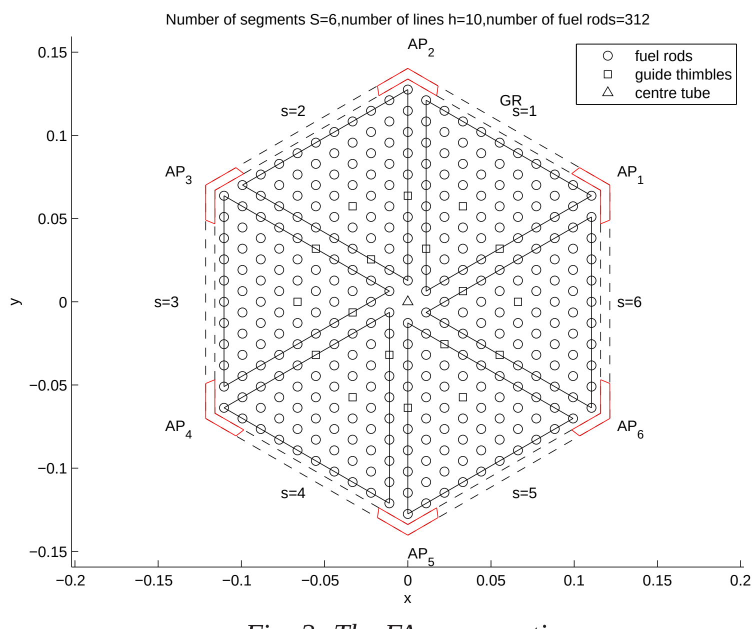

Fig. 2: The FA cross-section

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of spacer grigs (SG). All FA components are linked by transverse spacer grids which elastic properties are expressed by linear springs with stiffness k_g placed on 8 different horizontal planes $g = 1, \dots, 8$.

2. Mathematical model of the system

The vectors of generalized coordinates of the fully restrained subsystems (rod segments and centre tube) loosed in kinematical excited nodes can be partitioned in the form

$$\boldsymbol{q}_s = [(\boldsymbol{q}_L^{(s)})^T, (\boldsymbol{q}_F^{(s)})^T, (\boldsymbol{q}_U^{(s)})^T]^T,\ s = 1, \dots, 6, CT\,, \tag{1}$$

and the skeleton $s =$LS fixed only in bottom ends in the form

$$\boldsymbol{q}_{LS} = [(\boldsymbol{q}_L^{(LS)})^T, (\boldsymbol{q}_F^{(LS)})^T]^T\,. \tag{2}$$

The coordinates of subvectors $\boldsymbol{q}_L^{(s)}$ and $\boldsymbol{q}_U^{(s)}$ are displacements of end-nods of fuel assembly components coupled with moving rigid support plates and displacements of free system nodes are integrated in vectors $\boldsymbol{q}_F^{(s)} \in R^{n_s}$. The conservative mathematical models of the loosed subsystems in the decomposed block form corresponding to partitioned vectors can be written as

$$\begin{bmatrix} \boldsymbol{M}_L^{(s)} & \boldsymbol{M}_{L,F}^{(s)} & \boldsymbol{0} \\ \boldsymbol{M}_{F,L}^{(s)} & \boldsymbol{M}_F^{(s)} & \boldsymbol{M}_{F,U}^{(s)} \\ \boldsymbol{0} & \boldsymbol{M}_{U,F}^{(s)} & \boldsymbol{M}_U^{(s)} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_L^{(s)} \\ \ddot{\boldsymbol{q}}_F^{(s)} \\ \ddot{\boldsymbol{q}}_U^{(s)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_L^{(s)} & \boldsymbol{K}_{L,F}^{(s)} & \boldsymbol{0} \\ \boldsymbol{K}_{F,L}^{(s)} & \boldsymbol{K}_F^{(s)} & \boldsymbol{K}_{F,U}^{(s)} \\ \boldsymbol{0} & \boldsymbol{K}_{U,F}^{(s)} & \boldsymbol{K}_U^{(s)} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_L^{(s)} \\ \boldsymbol{q}_F^{(s)} \\ \boldsymbol{q}_U^{(s)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_L^{(s)} \\ \boldsymbol{f}_C^{(s)} \\ \boldsymbol{f}_U^{(s)} \end{bmatrix} \tag{3}$$

for the $s = 1, \dots, 6, CT$ and for the skeleton as

$$\begin{bmatrix} \boldsymbol{M}_L^{(LS)} & \boldsymbol{M}_{L,F}^{(LS)} \\ \boldsymbol{M}_{F,L}^{(LS)} & \boldsymbol{M}_F^{(LS)} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_L^{(LS)} \\ \ddot{\boldsymbol{q}}_F^{(LS)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_L^{(LS)} & \boldsymbol{K}_{L,F}^{(LS)} \\ \boldsymbol{K}_{F,L}^{(LS)} & \boldsymbol{K}_F^{(LS)} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_L^{(LS)} \\ \boldsymbol{q}_F^{(LS)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_L^{(LS)} \\ \boldsymbol{f}_C^{(LS)} \end{bmatrix}\,. \tag{4}$$

The force subvectors $\boldsymbol{f}_C^{(s)}$ express the coupling forces between subsystem s and adjacent subsystems transmitted by spacer grids. The displacements of the end-nodes of the subsystem components coupled with support plates can be expressed by the displacements of the lower and upper plates in the form

$$\boldsymbol{q}_L^{(s)} = \boldsymbol{T}_L^{(s)} \boldsymbol{q}_L,\ s = 1, \dots, 6, CT, NS\,;\ \boldsymbol{q}_U^{(s)} = \boldsymbol{T}_U^{(s)} \boldsymbol{q}_U,\ s = 1, \dots, 6, CT\,. \tag{5}$$

The second set of equations extracted from (3) and (4) for each subsystem is

$$\boldsymbol{M}_F^{(s)} \ddot{\boldsymbol{q}}_F^{(s)} + \boldsymbol{K}_F^{(s)} \boldsymbol{q}_F^{(s)} = -\boldsymbol{M}_{F,L}^{(s)} \boldsymbol{T}_L^{(s)} \ddot{\boldsymbol{q}}_L - \boldsymbol{M}_{F,U}^{(s)} \boldsymbol{T}_U^{(s)} \ddot{\boldsymbol{q}}_U - \boldsymbol{K}_{F,L}^{(s)} \boldsymbol{T}_L^{(s)} \boldsymbol{q}_L - \boldsymbol{K}_{F,U}^{(s)} \boldsymbol{T}_U^{(s)} \boldsymbol{q}_U + \boldsymbol{f}_C^{(s)}\,, \tag{6}$$

where $\boldsymbol{M}_{F,U}^{(LS)} = \boldsymbol{0}$, $\boldsymbol{K}_{F,U}^{(LS)} = \boldsymbol{0}$. The global model of the FA has to large DOF number for calculation of dynamic response excited by support plate motion. Therefore we assemble the condensed model using the modal synthesis method presented in the paper Zeman & Hlaváč (2011a). Let the modal properties of the conservative models of the mutually uncoupled subsystems with the strengthened end-nodes coupled with immovable support plates be characterized by spectral $\boldsymbol{\Lambda}_s$ and modal $\boldsymbol{V}_s$ matrices of order n_s, suitable to orthonormality conditions

$$\boldsymbol{V}_s^T \boldsymbol{M}_F^{(s)} \boldsymbol{V}_s = \boldsymbol{E}\,,\ \boldsymbol{V}_s^T \boldsymbol{K}_F^{(s)} \boldsymbol{V}_s = \boldsymbol{\Lambda}_s,\ s = 1, \dots, 6, CT, LS\,. \tag{7}$$

The vectors $\boldsymbol{q}_F^{(s)}$ of dimension n_s, corresponding to free nodes of subsystems, can be approximately transformed in the form

$$\boldsymbol{q}_F^{(s)} = {}^m\boldsymbol{V}_s \boldsymbol{x}_s\,,\ \boldsymbol{x}_s \in R^{m_s}\,,\ s = 1, \dots, 6, CT, LS\,, \tag{8}$$

where ${}^m\boldsymbol{V}_s \in R^{n_s, m_s}$ are modal submatrices of chosen m_s master eigenvectors of fixed subsystems. The equations (6) can be rewritten using (7) and (8) in the form

$$\ddot{\boldsymbol{x}}_s + {}^m\boldsymbol{\Lambda}_s \boldsymbol{x}_s = -{}^m\boldsymbol{V}_s^T (\boldsymbol{M}_{F,L}^{(s)} \boldsymbol{T}_L^{(s)} \ddot{\boldsymbol{q}}_L + \boldsymbol{M}_{F,U}^{(s)} \boldsymbol{T}_U^{(s)} \ddot{\boldsymbol{q}}_U + \boldsymbol{K}_{F,L}^{(s)} \boldsymbol{T}_L^{(s)} \boldsymbol{q}_L + \boldsymbol{K}_{F,U}^{(s)} \boldsymbol{T}_U^{(s)} \boldsymbol{q}_U) + {}^m\boldsymbol{V}_s^T \boldsymbol{f}_C^{(s)}\,, \tag{9}$$

$$s = 1, \ldots, 6, CT, LS\,,$$

where spectral submatrices ${}^m\boldsymbol{\Lambda}_s \in R^{m_s,m_s}$ correspond to chosen master eigenvectors in ${}^m\boldsymbol{V}_s$. The models (9) of all subsystems can be written in the configuration space $\boldsymbol{x} = [\boldsymbol{x}_s],\ s = 1, \ldots, 6, CT, LS$ of dimension $m = \sum_s m_s$ as

$$\ddot{\boldsymbol{x}}(t) + \boldsymbol{\Lambda}\boldsymbol{x}(t) = -\boldsymbol{V}^T(\boldsymbol{M}_L\ddot{\boldsymbol{Q}}_L + \boldsymbol{M}_U\ddot{\boldsymbol{Q}}_U + \boldsymbol{K}_L\boldsymbol{Q}_L + \boldsymbol{K}_U\boldsymbol{Q}_U) + \boldsymbol{V}^T\boldsymbol{f}_C\,, \tag{10}$$

where $\boldsymbol{f}_C = [\boldsymbol{f}_C^{(s)}] \in R^n,\ n = \sum_s n_s$ is global vector of coupling forces between subsystems and matrices

$$\boldsymbol{\Lambda} = \mathrm{diag}[{}^m\boldsymbol{\Lambda}_s] \in R^{m,m};\ \boldsymbol{V} = \mathrm{diag}[{}^m\boldsymbol{V}_s] \in R^{n,m};\ \boldsymbol{X}_X = \mathrm{diag}[\boldsymbol{X}_{F,X}^{(s)}\boldsymbol{T}_X^{(s)}] \in R^{n,48};$$

$$\boldsymbol{Q}_X = [\boldsymbol{q}_X^T, \ldots, \boldsymbol{q}_X^T]^T \in R^{48};\ \boldsymbol{X} = \boldsymbol{M}, \boldsymbol{K};\ X = L, U;\ s = 1, \ldots, 6, CT, LS$$

are block diagonal, composed from corresponding matrices of subsystems. The global vector of coupling forces between subsystems can be calculated from identity

$$\boldsymbol{f}_C = -\frac{\partial E_p}{\partial \boldsymbol{q}_F} = -\boldsymbol{K}_C\boldsymbol{q}_F,\ \boldsymbol{q}_F = [\boldsymbol{q}_F^{(s)}]\,, \tag{11}$$

where E_p is potential (deformation) energy of the all spacer grids (springs) between subsystems. The expressions (11) can be substituted in (10) and then we get the condensed model of the nuclear fuel assembly of order m

$$\ddot{\boldsymbol{x}}(t) + (\boldsymbol{\Lambda} + \boldsymbol{V}^T\boldsymbol{K}_C\boldsymbol{V})\boldsymbol{x}(t) = -\boldsymbol{V}^T(\boldsymbol{M}_L\ddot{\boldsymbol{Q}}_L(t) + \boldsymbol{M}_U\ddot{\boldsymbol{Q}}_U(t) + \boldsymbol{K}_L\boldsymbol{Q}_L(t) + \boldsymbol{K}_U\boldsymbol{Q}_U(t))\,. \tag{12}$$

3. Application

The VVER 1000 reactor core is formed from 163 nuclear fuel assemblies. Each fuel assembly is placed in the core basket between core support plate and lower supporting plate of the block of protection tubes by means of lower support tailpiece (LP) and headpiece (HP) (see Fig.1). These support plates and pieces can be considered in transverse direction as rigid bodies.

Let us consider the steady vibration of both mentioned support plates excited by pressure pulsations generated by circulation pumps in the main circulation loops (Pečínka - Krupa & Klátil (1997)). The force effect of pressure pulsations in the gap between core barrel and pressure vessel walls can be expressed in the global model of the reactor by excitation vector in the complex form (Zeman & Hlaváč (2008))

$$\boldsymbol{f}(t) = \sum_j \sum_k \boldsymbol{f}_j^{(k)} \mathrm{e}^{\mathrm{i}k\omega_j t}\,, \tag{13}$$

where $\boldsymbol{f}_j^{(k)}$ is vector of complex amplitudes of $k-$th excitation harmonic component caused by hydrodynamic forces generated in one $j-$th circulation pump. Corresponding angular rotational frequency of the $j-$th pump $\omega_j = 2\pi f_j$ is defined by pump revolutions per minute n_j [rpm], where can be for particular pumps slightly different. Steady dynamic response of the reactor in generalized coordinates is given by identical form

$$\boldsymbol{q}(t) = \sum_j \sum_k \boldsymbol{q}_j^{(k)} \mathrm{e}^{\mathrm{i}k\omega_j t}\,. \tag{14}$$

The vectors of complex amplitudes $\boldsymbol{q}_j^{(k)}$ must be transformed into vectors of $\boldsymbol{Q}_{X,j}^{(k)}$ $(X = L, U)$ describing steady vibration of the support plates caused by $k-$th harmonic of $j-$th pump.

In consequence of lightly damped fuel assembly components we consider modal damping of the subsystems characterized in the space of modal coordinates $\boldsymbol{x}_s$ by diagonal matrices $\boldsymbol{D}_s = \mathrm{diag}[2D_\nu^{(s)}\Omega_\nu^{(s)}]$, where $D_\nu^{(s)}$ are damping factors of natural modes and $\Omega_\nu^{(s)}$ are eigenfrequencies of the mutually uncoupled subsystems. The damping of spacer grids can be approximative expressed by damping matrix $\boldsymbol{B}_C = \beta\boldsymbol{K}_C$ proportional to stiffness matrix $\boldsymbol{K}_C$ by coefficient β.

That being simplifying supposed and the polyharmonic excitation (13) the conservative condensed model (12) will be completed in the complex form

$$\ddot{\boldsymbol{x}}(t) + (\boldsymbol{D} + \beta\boldsymbol{V}^T\boldsymbol{K}_C\boldsymbol{V})\dot{\boldsymbol{x}}(t) + (\boldsymbol{\Lambda} + \boldsymbol{V}^T\boldsymbol{K}_C\boldsymbol{V})\boldsymbol{x}(t) = \\ = -\boldsymbol{V}^T\sum_j\sum_k\left[(\boldsymbol{K}_L - k^2\omega_j^2\boldsymbol{M}_L)\boldsymbol{Q}_{L,j}^{(k)} + (\boldsymbol{K}_U - k^2\omega_j^2\boldsymbol{M}_U)\boldsymbol{Q}_{U,j}^{(k)}\right]\mathrm{e}^{\mathrm{i}k\omega_j t}\,. \tag{15}$$

Steady response of the fuel assembly subsystems according to (8) is

$$\boldsymbol{q}_F^{(s)}(t) = \sum_j\sum_k {}^m\boldsymbol{V}_s\boldsymbol{x}_{s,j}^{(k)}\mathrm{e}^{\mathrm{i}k\omega_j t}\,,\ \ s = 1,\dots,6,CT,LS\,, \tag{16}$$

where $\boldsymbol{x}_{s,j}^{(k)}$ are subvectors of the global vector $\boldsymbol{x}_j^{(k)}$ of the complex amplitudes

$$\boldsymbol{x}_j^{(k)} = -[\boldsymbol{\Lambda} + (1 + \mathrm{i}\beta k\omega_j)\boldsymbol{V}^T\boldsymbol{K}_C\boldsymbol{V} + \mathrm{i}k\omega_j\boldsymbol{D}]^{-1}\cdot \\ \cdot\boldsymbol{V}^T\sum_j\sum_k\left[(\boldsymbol{K}_L - k^2\omega_j^2\boldsymbol{M}_L)\boldsymbol{Q}_{L,j}^{(k)} + (\boldsymbol{K}_U - k^2\omega_j^2\boldsymbol{M}_U)\boldsymbol{Q}_{U,j}^{(k)}\right] \tag{17}$$

corresponding to subsystem s. Subscript $j \in \{1,2,3,4\}$ is assigned to the operating circulation pump and subscript k to the harmonic component of pressure pulsations.

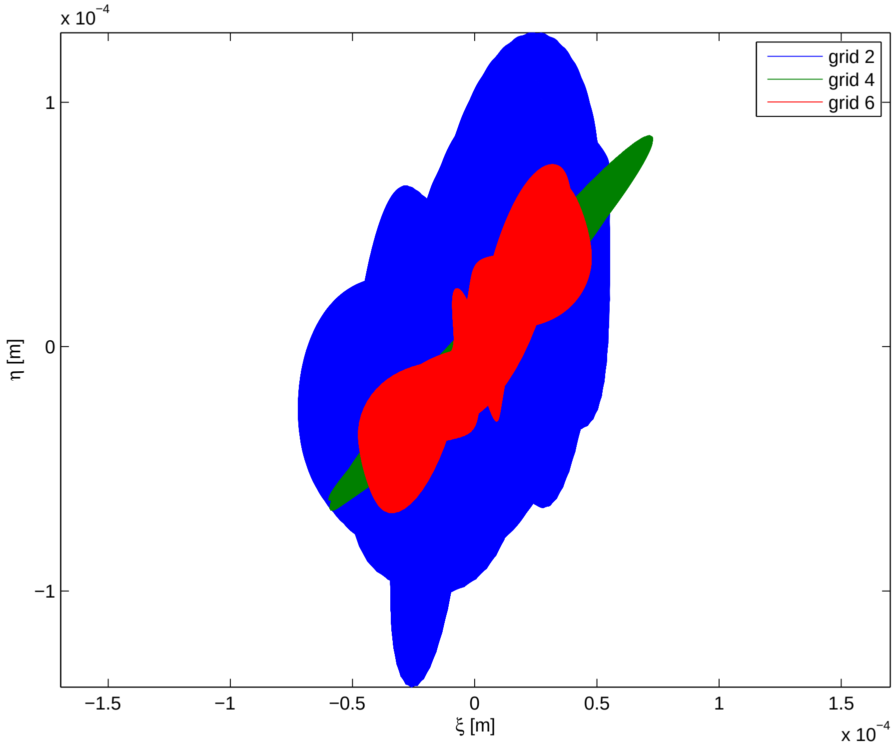

Fig. 3: Orbits of the fuel rod centre r =14 in the first fuel rod segment on the level spacer grids 2,4,6

The methodology was applied for steady polyharmonic response of the Russian TVSA-T fuel assembly in the VVER 1000 reactor core in NPP Temelín. As an illustration, the orbits in transverse planes of the random selected (r =14) fuel rod centre in the first fuel rod segment (s = 1) on the level spacer grids g =2,4,6 caused by pressure pulsations generated by all circulation pumps (Zeman & Hlaváč (2008)) are shown in Fig.3 and separately in Fig.4. The rotational frequencies of the particular pumps were $f_1 = f_2$ =16,635 Hz and $f_3 = f_4 = 16,645$ Hz. The condensed model (15) with 3272 DOF ($m_s = 500$, $m_{CT} = n_{CT} = 32$, $m_{LS} = n_{LS} = 240$) was used for the calculation of the orbits. The accuracy of condensed model was tested in terms of relative errors of 125 lowest fuel assembly eigenfrequencies defined in the form

$$\varepsilon_\nu = \frac{|f_\nu(m_s) - f_\nu|}{f_\nu},\ \nu = 1,\dots,125\,, \tag{18}$$

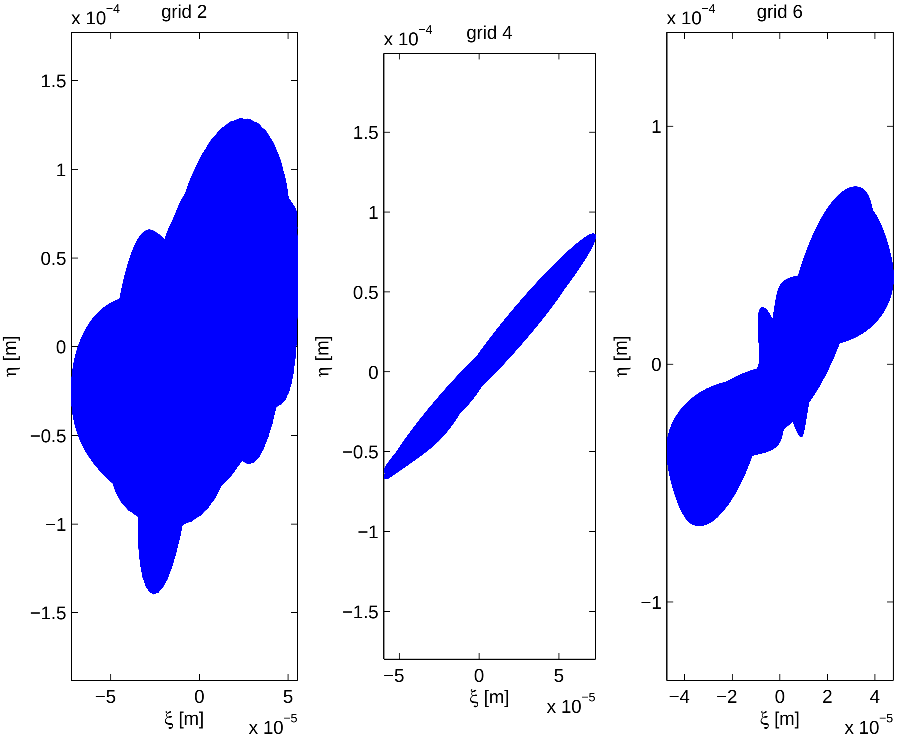

Fig. 4: Orbits from the figure 3 depicted separately

where f_ν are eigenfrequencies of the full (noncondensed) model with 10832 DOF. The relative errors ε_ν for different condensation level of the rod segments expressed by number of the rod segment master eigenvectors m_s =100,300,500 is shown in Fig.5. Relative errors decrease with decreasing condensation level (m_s increases) in all FA eigenfrequencies. The orbits of these particular models distinguish only little.

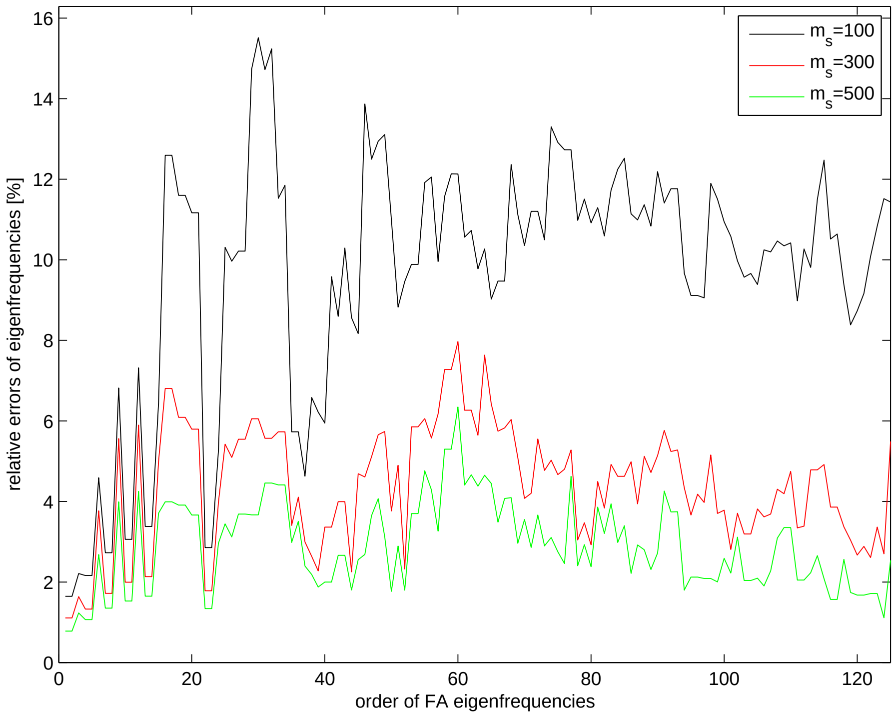

Fig. 5: Relative errors of the fuel assembly eigenfrequencies for different number m_s of rod segment master eigenvectors

4. Conclusion

The described method enables to investigate effectively the combined flexural and torsional kinematic excited vibrations of the nuclear fuel assembly. The fuel assembly vibrations are caused by motion of the two support horizontal plates in the reactor core. The special coordinate system of radial and orthogonal lateral axes for each fuel rod and guide thimbles on the all spacer grid levels makes possible to separate the central symmetrical fuel assembly into several identical revolved rod segments characterized by identical mass, damping and stiffness matrices.

The presented new approach based on the system decomposition into subsystems linked by spacer grids and modal synthesis method with reduction of DOF number was applied to hexagonal type nuclear fuel assembly vibration. The developed methodology, mathematical model and software in MATLAB was used for modelling and dynamic deformation analysis of the Russian type nuclear fuel assembly components caused by motion of the support plates excited by pressure pulsations generated by circulation pumps in the main reactor circulation loops.

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