17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

DEFLECTION OF REINFORCED CONCRETE STRUCTURES ACCORDING TO EC2: COMPARISON OF METHODS

R. Zídek[*], L. Brdečko[*]

Abstract: *This paper compares two approaches to the analysis of deflections of reinforced concrete beam structures with consideration of creep. The first is based on simplified methods recommended in Eurocode 2. The second approach is more general and uses the incremental solution, the smeared crack model for modeling of material non-linearity and time discretization method for the assessment of rheology. The results of both analyses are similar. The general method also allows the influence of the erection process or stepwise loading to be taken into account.*

Keywords: ***Creep, reinforced concrete, deflection, time discretization method, Eurocode.***

1. Introduction

Eurocode 2 for the analysis of reinforced concrete structures allows the use of a simplified approach to consider creep in the computation of displacements via the effective modulus of elasticity for concrete. The code also defines rules for the calculation of stiffness for a cross-section with cracks and states the possibility of using more accurate methods. The authors have assembled a program for the analysis of creep of reinforced concrete structures which is based on more general assumptions. The paper gives a short summary of the theoretical assumptions on which the Asteres computer program is built and the basic features of the method in (Eurocode 2). Consequently, a study of both approaches applied to practical structures is presented.

2. Analysis methods

Asteres software is designed for the analysis of planar beam structures. It is based on the finite element method. The program uses a special beam element which allows the behavior of concrete and groups of steel to be modeled separately. It is useful especially for the effective expression of concrete volume changes.

An iterative computation process with adaptive control of load level is implemented to consider the nonlinear behavior of reinforced concrete. For concrete in compression the nonlinear stress-strain diagram according to (Eurocode 2) is considered. The smeared cohesive crack model is used for concrete in tension. After the tension strength is reached the residual stiffness of the crack is considered. This stiffness depends on crack width and fracture energy G_f. The crack is smeared into a certain zone which for planar concrete structures is usually in relation to the size of finite elements. For reinforced structures in bending this model works only after reinforcement yielding occurs and one dominant crack has developed (Zídek, 2008). Before this limit is reached, the zone is considered as the real crack spacing calculated approximately according to (Eurocode 2). The redistribution of internal forces in the case of statically indeterminate structures and the redistributions of stresses between parts of the cross-section (especially between concrete and steel in compression) is ensured using incremental load solution and using the time discretization method.

Creep of concrete depends on the whole history of the stresses in the structure. Therefore, the method of time discretization was implemented. This method allows the observation of changes in the stresses in particular elements of the structure. The accuracy depends on the density of division of the considered time. Details about the theoretical background of the Asteres program are in (Zídek, 2008).

[*] Ing. Rostislav Zídek, Ph.D. and Ing. Luděk Brdečko, Ph.D.: Institute of Structural Mechanics, Brno University of Technology, Veveří 97; 602 00, Brno; CZ, e-mails: zidek.r@fce.vutbr.cz, brdecko.l@fce.vutbr.cz

The method according to Eurocode 2, chapter 7.4, uses the stiffness of the not-fully-cracked cross-section based on another principle. The stress in the concrete crack is considered zero, but the stiffness of the steel is raised due to the concrete between the two cracks and its interaction with the steel. The change in stiffness is considered at one time and the redistribution of stresses is not allowed.

For the expression of creep the effective modulus of elasticity method is used. This method of analysis gives exact solutions only for structures where the redistribution of stresses is not assumed. In real structures, the redistribution is caused by a change in stiffness due to cracks in the case of statically indeterminate structures or by redistribution stresses between compressed concrete and steel.

According to the theory defined in (Eurocode 2) an Asteres software module was developed. The solution of linear FEM is common for both variants. Therefore, all examples mentioned below were analyzed with the same precision and with the same finite element division. The details of this implementation are in (Zídek & Brdečko, 2010).

3. Comparative study

The goal of this study was to compare the methods used in the Asteres program with the simplified approach of the solution of displacements according to (Eurocode 2). Asteres also allows the observation of other quantities, for example stresses in concrete or reinforcement. The process of loading of the structures was considered simply to make it possible to analyze structures also using the simplified method according to (Eurocode 2). Because of the possibility of assessing a particular influence separately, shrinkage was not considered in any example. From all performed analyses three examples were chosen and presented: a continuous reinforced concrete beam, a one-storey one-span frame and a simply supported beam with reinforcement on one side. These examples best satisfied the conditions for the use of simplified methods.

Concrete C25/30 and reinforced steel (E = 200 GPa, f_y = 500 MPa) were considered for all examples. During the whole time of loading the yield strength was not reached for any of the examples. The erection progress proceeded as follows: After laying, the concrete structures were cured for 3 days and on the 28th day the temporary supports were removed and load was applied. The load didn't change for the whole observed time (10028 days from laying the concrete). The relative humidity of the ambient environment was 60%. The length of the finite elements was approx. 0.25 m and special elements were used for the concrete part of the cross-section and for top and bottom reinforcement.

The analysis of 40 time intervals with constant load was accomplished in Asteres. Particular time intervals were designed in line with the rule that the length of each following interval was to be 1.5 times larger than the previous one. The first, shortest interval was 39 seconds in length and the last had a length of 3333 days. For the analysis according to EC2 the considered time was divided into 13 time intervals to obtain a relatively smooth curve of deflection. The quality of the solution obtained by the time discretization method depends on the length of time intervals because the uniform distribution of stresses during the interval is assumed. Computation according to (Eurocode 2) - simplified method - is designed for structures where the stress does not significantly change. That criterion is approximately satisfied for these structures, and therefore the creep can be expressed en bloc for the whole considered time. The history of stress changes due to creep is not taken into account.

Note; The time 0 in all graphs means 28 days after placing the laying of concrete (removal of temporary supports).

3.1. Continuous beam

The first presented example is the two-span continuous beam. A scheme of the beam and reinforcement is shown in Fig. 1. The intensity of the transversal uniform load is 36.5 kN/m. This load continues for the whole considered time, which is 10028 days from the laying of the concrete.

3.2. Frame

The next solved structure was a one-storey one-span frame (Fig. 4) designed as a part of a building with a loading width of 6 m. The load is thus relatively high – 80.41 kN/m. The scheme of the structure – see Fig. 4; the deflection in the middle of the cross-beam – see Fig. 5.

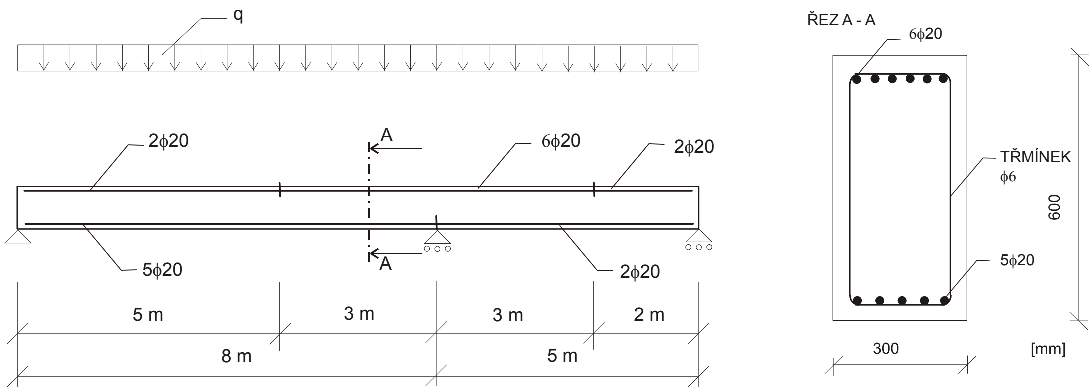

Fig. 1: Continuous beam – scheme of the structure.

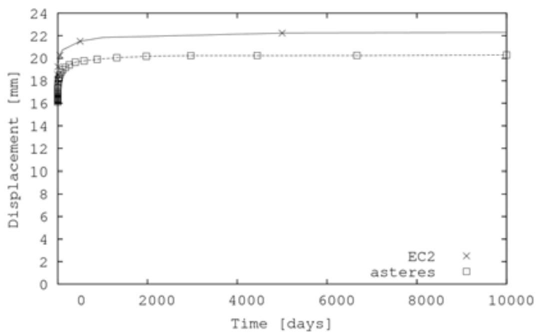

Fig. 2: Continuous beam – deflection in the middle of the span in time.

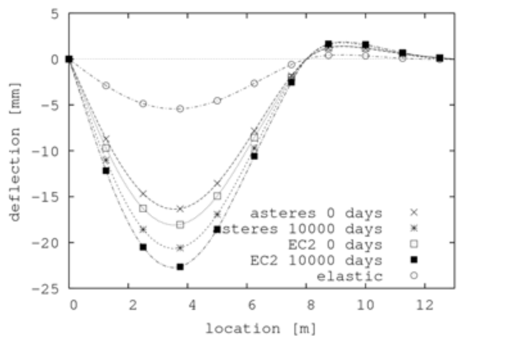

Fig. 3: Continuous beam – deflection along the length of the beam.

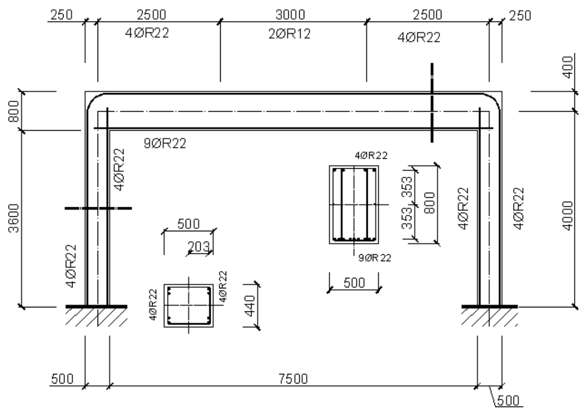

Fig. 4: Frame – scheme of the structure.

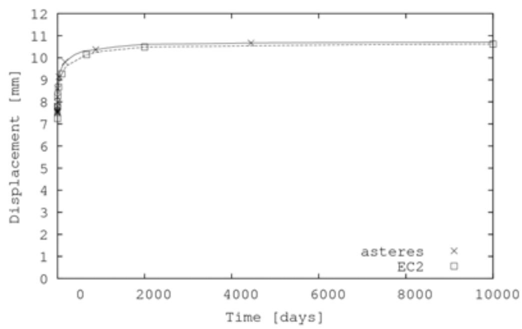

Fig. 5: Frame – deflection in the middle of the span of the cross-beam.

3.3. Simply supported beam

The third presented structure is a simply supported beam subjected only to bending moment and shear force. Internal force can not be redistributed because of its static determination. Reinforcement is only on one side, and because of the equilibrium between the forces in tension in the reinforcement and in compression in the concrete there is no redistribution of stresses between concrete and reinforcement (the tension force in cracked concrete is negligible).

The cross-section is shown in Fig. 6. Span is 8 m and intensity of uniformly distributed load is 26kN/m. Fig. 7 shows the dependency between time and the deflection in the middle of the span.

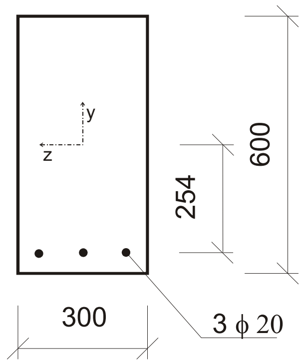

Fig. 6: Simply supported beam – scheme of the structure.

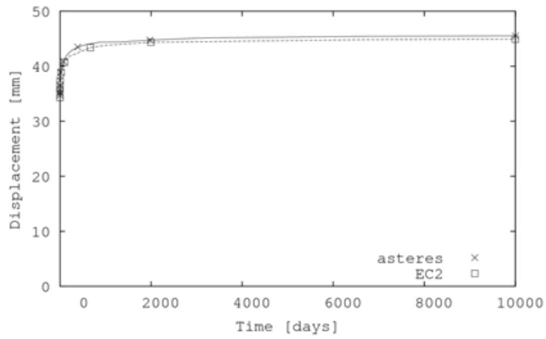

Fig. 7: Simply supported beam – deflection in the middle of span in time.

4. Conclusions

The presented study compares the deflections of reinforced concrete structures solved using a method implemented in Asteres software and using the method recommended in (Eurocode 2), chapter 7.4. The influence of fracture energy (considered as 65 N/m) and crack spacing are not analyzed (Asteres input data). Earlier computations have shown the low influence of these parameters. Shrinkage was not considered in any of the models in order to achieve better separation of the particular influences.

Both approaches have shown good agreement in their results. Certain differences could be explained by the redistribution of internal forces and stresses, as the simplified approach is not able to deal with them.

The presented examples show acceptable agreement between the approach described in chapter 7.4 in (Eurocode 2) and Asteres. The theoretical background of Asteres satisfies general code requirements (Eurocode 2). This allows the use of approaches implemented in Asteres software for the assessment of the serviceability limit state of reinforced concrete beam structures. The main field of application of these methods is not only the improvement of deflection analysis of reinforced concrete structures but also the analysis of structures with a complicated history of building and loading. In these cases the use of simplified methods is problematic. If an appropriate erection process and loading is designed, it is possible to lower the deflections caused by rheology and design more slender or less reinforced structures.

Acknowledgement

This research has been sponsored by Grant Agency of the Czech Republic project P104/10/2359, and by the MSM0021630519 research fund of the Ministry of Education Youth and Sports.

References

EN 1992-1-1 (Eurocode 2): Design of Concrete Structures, Part 1-1: General Rules and Rules for Buildings, (2005) ČNI, Prague, pp. 126-130.

Zídek, R. - Brdečko, L. (2010) Alternativní řešení deformací železobetonových konstrukcí, Sborník abstraktů a CD příspěvků Mezinárodní konference Modelování v mechanice, Ostrava , pp. 51- 52.

Zídek, R. (2008) Prediction of Rheology in Material Nonlinear Analysis of Concrete Reinforced Structures, Proceedings of Conference Life Cycle Assessment, Optimisation, Behaviour and Properties of Concrete and Concrete Structures, November 12-14, Brno, pp. 297-301.