

A COMPUTATIONAL ANALYSIS OF THE STEADY STATE LATERAL VIBRATION OF A FLEXIBLY SUPPORTED RIGID UNBALANCED ROTOR ATTENUATED BY SQUEEZING THE THIN FILMS OF CLASSICAL AND MAGNETORHEOLOGICAL LIQUIDS

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Abstract: The vibration amplitude of rotors excited by imbalance can be significantly reduced if damping devices are inserted between the rotor and its stationary part. To achieve their optimum performance, the damping effect must be controllable. For this purpose a new semiactive damping element has been proposed. It works on the principle of squeezing two concentric lubricating films formed by classical and magnetorheological liquids. In the mathematical model they are represented by newtonian and Bingham materials respectively. The damping effect is controlled by the change of intensity of the magnetic field generated by electric current. The rotor is assumed to be absolutely rigid and is coupled with the stationary part by spring elements. Its vibration is governed by nonlinear equations of motion whose steady state solution is obtained by application of a collocation method. The newly proposed semiactive damping element minimizes amplitude of the rotor vibration by adapting the damping effect to the current operating conditions.

Keywords: Rotors, semiactive damping elements, magnetorheological fluid, steady state response.

1. Introduction

Amplitude of the lateral vibration of rotors can be significantly reduced if damping devices are inserted between their rotating and stationary parts. To achieve their optimum performance, the damping effect must be controllable. For this purpose a new semiactive coupling element working on the principle of squeezing two concentric films of classical and magnetorheological liquids has been proposed. The damping effect is controlled by changing induction of the magnetic field passing through the film of magnetorheological fluid.

In this paper there is investigated the steady state vibration of a flexibly supported rigid rotor that is excited by the unbalance forces and attenuated by the new damping element. The attention is focused on minimizing amplitude of the rotor vibration by means of adapting the damping effect to the current operating conditions.

2. Modelling of the new controllable damping element

The rotor is supported by rolling-element bearings whose outer races are flexibly coupled through the squirrel springs with the stationary part. The proposed damping element consists of four rigid rings. Two rings are mounted with the stationary part directly and two ones with the squirrel springs carrying the bearings. The springs enable oscillation of the movable rings in the radial direction and prevent their rotation together with the shaft (Fig.1). The clearances between the fixed and flexibly supported rings are filled with classical (the inner clearances) and magnetorheological oils (the outer clearances). In the damper body there is imbedded an electric coil generating magnetic filed. The magnetic flux passes through the layer of magnetorheological liquid and as resistence against its flow depends on magnetic induction, magnitude of the current can be used to control the damping effect.

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Fig. 1: Schema of the proposed damping element.

Fig. 2: The coordinate system.

The mathematical model of the proposed damping element originates from the assumptions of the classical theory of lubrication. The classical and magnetorheological oils are represented by newtonian and Bingham materials and the yield shear stress of the magnetorheological liquid depends on magnetic induction. The further attention is focused only on the dampers whose geometry and design make possible to consider them as short and on the dampers symmetric relative to their middle plane.

The thickness of the oil films depends on position of the rotor journal centre relative to the stationary part of the rotor system

$$h_{co} = c_{co} - e_H \cos(\varphi - \gamma), \qquad h_{MR} = c_{MR} - e_H \cos(\varphi - \gamma).$$
 (1)

 h_{CO} , h_{MR} denote the thickness of the films of classical and magnetorheological oils, c_{CO} , c_{MR} represent the width of the gaps between the rings filled with classical and magnetorheological lubricants, e_H is the rotor journal eccentricity, φ is the circumferential coordinate and γ denotes the position angle of the line of centres (Fig.2).

The pressure distribution in noncavitated parts of the lubricating films is governed by equations derived in Krämer (1993) and Zapoměl & Ferfecki (2009),

$$\frac{\partial^2 p_{co}}{\partial Z^2} = \frac{12\eta}{h_{co}^3} \dot{h}_{co}, \qquad (2)$$

$$h_{MR}^{3} p_{MR}^{\prime 3} + 3 \left(h_{MR}^{2} \tau_{y} - 4 \eta_{B} \dot{h}_{MR} Z \right) p_{MR}^{\prime 2} - 4 \tau_{y}^{3} = 0.$$
(3)

 p_{CO} , p_{MR} , p'_{MR} denote the pressure and the pressure gradient in the axial direction in the layers of the classical and magnetorheological liquids, η is the dynamical viscosity of the classical oil, η_B is the Bingham viscosity, Z is the axial coordinate, τ_y is the yield shear stress and () denotes the first derivative with respect to time. Equation (3) holds for the case when the pressure gradient in the axial direction is real (not complex), negative and satisfies the relation

$$p'_{MR} < -\frac{2\tau_{y}}{h_{MR}}$$
 (4)

The boundary conditions needed for solving equations (2) and (3) express that the pressure at the damper faces is equal to the pressure in the ambient space p_A . *L* is the length of the damping element

$$p_{co} = p_A, \quad p_{MR} = p_A \quad \text{for} \quad Z = \frac{L}{2}.$$
 (5)

In the simplest design case of the damping element, the rings, between which there is situated a layer of the magnetorheological liquid, can be considered as a divided core of an electromagnet. Then the dependence of the yield shear stress on magnetic induction can be approximately expressed

$$\tau_{y} = k_{y} \left(\frac{NI}{2h_{MR}}\right)^{n_{y}}.$$
(6)

 k_y and n_y are the material constants of the magnetorheological liquid, N is the number the coil turns and I is the electric current.

In the areas where the thickness of the lubricating films rises with time ($\dot{h}_{CO} > 0$, $\dot{h}_{MR} > 0$) a cavitation is assumed. The pressure in these areas remains constant and is equal to the pressure in the ambient space. The damping force is given by integration of the pressure distributions p_{DCO} , p_{DMR} in the cavitated and noncavitated regions around the circumference and along the length of the damping element

$$F_{dy} = -2R_{CO} \int_{0}^{2\pi^{\frac{L}{2}}} \int_{0}^{p} p_{DCO} \cos\varphi \, dZ \, d\varphi - 2R_{MR} \int_{0}^{2\pi^{\frac{L}{2}}} \int_{0}^{2\pi^{\frac{L}{2}}} p_{DMR} \cos\varphi \, dZ \, d\varphi \,, \tag{7}$$

$$F_{dz} = -2R_{CO} \int_{0}^{2\pi\frac{L}{2}} p_{DCO} \sin \varphi \, dZ \, d\varphi - 2R_{MR} \int_{0}^{2\pi\frac{L}{2}} p_{DMR} \sin \varphi \, dZ \, d\varphi \tag{8}$$

 R_{CO} and R_{MR} are the radii of the layers of the classical and magnetorheological oils.

3. The equations of motion of the investigated rotor



Fig. 3: Investigated rotor system.

The investigated rotor consists of a shaft carrying one disc (Fig.3). With the stationary part it is coupled by the squirrel springs and the new damping elements. The system is symmetric relative to the disc middle plane perpendicular to the shaft centreline. The rotor is loaded by its weight and is excited by the disc unbalance. The squirrel springs are prestressed to eliminate their deflection caused by the rotor weight.

Taking into account the system symmetry, the rotor lateral vibration is governed by two equations of motion that are nonlinear due to the damping forces

$$m_{R}\ddot{y} + b_{P}\dot{y} + 2k_{D}y = 2F_{dy}(y, z, \dot{y}, \dot{z}) + m_{R}e_{T}\omega^{2}\cos(\omega t + \psi_{o}), \qquad (9)$$

$$m_{R}\ddot{z} + b_{P}\dot{z} + 2k_{D}z = 2F_{dz}(y, z, \dot{y}, \dot{z}) + m_{R}e_{T}\omega^{2}\sin(\omega t + \psi_{o}).$$
(10)

 m_R is the rotor mass, b_P is the external damping coefficient, k_D is the squirrel spring stiffness, ω is the angular rotation speed, e_T is the eccentricity of the rotor unbalance, t is the time, ψ_o is the phase shift, y, z are displacements of the rotor centre and (") denote the second derivatives with respect to time.

Because of prestressing the spring elements, the steady state trajectory of the rotor centre is circular and therefore the steady state solution of (9) and (10) can be estimated as

$$y = r_c \cos \omega t - r_s \sin \omega t$$
, $z = r_c \sin \omega t + r_s \cos \omega t$. (11)

To determine the unknown parameters r_c and r_s a collocation method is applied. Only one collocation moment of time is needed. Then substitution of relations (11) and their first and second derivatives with respect to time for *t* equal to zero yields a set of two nonlinear algebraic equations

$$(2k_D - m_R\omega^2)r_C - \omega b_P r_S - m_R e_T\omega^2 - 2F_{dy}(r_C, r_S) = 0, \qquad (12)$$

$$\omega b_P r_C + \left(2 k_D - m_R \omega^2\right) r_S - 2 F_{dz}(r_C, r_S) = 0.$$
(13)

4. Results of the computational simulations

The task was to analyze amplitude of the excited vibration and magnitude of the force transmitted into the rotor stationary part.

The orbits of the rotor centre and time histories of the transmitted force in the vertical direction are drawn in Fig.4 and 5. They show that for the lower rotor revolutions the increase of the current rises the damping effect and reduces both the vibration and force amplitude. For higher rotor revolutions increase of the current decreases amplitude of the vibration only negligibly but magnitude of the transmitted force significantly rises. This confirms that the controllable damping effect makes possible to achieve the optimum rotor running by adapting the damping to the current operation conditions.



Fig. 4: The rotor centre orbits and the time history of the transmitted force (100 rad/s).



Fig. 5: The rotor centre orbits and the time history of the transmitted force (500 rad/s).

5. Conclusions

The developed procedure represents a tool for analysis of the steady state vibration of rigid rotors attenuated by the newly proposed semiactive damping elements working on the principle of squeezing two thin films of classical and magnetorheological oils. Advantage of the proposed damping device is that it does not require a complicated and expensive control system and makes possible to achieve the optimum performance of the rotating machine by means of adapting the damping to the current operating conditions.

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