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OPTIMIZATION OF THE HOLE DRILLING METHOD FOR THE STRESS STATE IDENTIFICATION

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Abstract: *The idea of the presented numerical simulation technique corresponds to the E 837 standard concepts but is more universal. It transforms the strains, arising during the hole drilling experiment, in a way similar to that of the E 837 standard but, unlike the E 837 standard, it executes the transformation completely. This theory enlargement enables the drilling method to be applied for a wider spectrum of further measuring appliances. Moreover, the hole drilling process do not have to be extremely precise, which the whole procedure simplifies, since the new method principle includes an objective stress state identification, when evaluating drilling experiments, with respect to the drilled hole eccentricity.*

Keywords: ***Stress state identification, hole drilling method, sensitivity gain.***

1. Introduction

The hole-drilling experimental method for stress state identification results in a small cylindrical hole drilled into an examined component surface. The hole drilling method for the stress state identification is based on the assumption that the free surface is one of the principal planes. The stress state in the surface layer thus can be only a uniaxial or a plane one. As such, it should be identifiable by measuring strains relieved on the free surface of the pre-strained structure during the drilling of a hole perpendicular to the surface. The semi-destructive hole drilling principle is based on impairing of the inner force equilibrium of a strained structure by drilling a relatively small circular hole perpendicularly to the surface. A drilled hole induces a change of the strain state in its close vicinity. These changes can be adjusted to define the strains arisen by drilling and thus used later for an identification of the original strain state after the strains relieved by the drilled hole are measured.

$$\left\{\begin{aligned} \sigma'_r &= \frac{\sigma_x}{2}(1+\cos 2\alpha) \\ \sigma'_\theta &= \frac{\sigma_x}{2}(1-\cos 2\alpha) \\ \tau'_{r\theta} &= \frac{\sigma_x}{2}\sin 2\alpha \end{aligned}\right\} \quad (1)$$

$$\left\{\begin{aligned} \sigma''_r &= \frac{\sigma_x}{2}(1-\frac{1}{r^2}) + \frac{\sigma_x}{2}(1+\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \sigma''_\theta &= \frac{\sigma_x}{2}(1+\frac{1}{r^2}) - \frac{\sigma_x}{2}(1+\frac{3}{r^4})\cos 2\alpha \\ \tau''_{r\theta} &= \frac{\sigma_x}{2}(1-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{aligned}\right\} \quad (2)$$

If the hole of the radius R_0 has not been drilled yet, the thin plate depicted in Fig. 1, which is loaded uni-axially by principal stress σ_x, is loaded by stresses σ'_r, σ'_Θ, $\tau'_{r\Theta}$ in planes defined by r and α polar coordinates and marked by indices of their normal lines r, Θ, which are determined in Eq. 1. The theory of the hole drilling principle is based on the analytical Kirsch's stress-state solution of a plate with a hole drilled through perpendicularly and loaded on its x-borders by principal stress σ_x (Timoshenko, 1934). The Kirsch's equations (Eq. 2) describe the state of plane strain in the vicinity of the hole of radius R_0 (Fig. 1). In comparison with Eq. (1), Eq. (2) include terms dependent on the drilled hole, which are left in Eq. (3) that are otherwise of a character similar to Eq. (1) and (2). If E stands for Young's modulus and ν for Poisson's ratio, the changes of plane stresses σ_r, σ_Θ, $\tau_{r\Theta}$ can be used for any isotropic material for a calculation of changes related to strains ε_r, ε_Θ, $\gamma_{r\Theta}$ and ε_z (see Fig. 1) in a point on the plate using Hooke's law.

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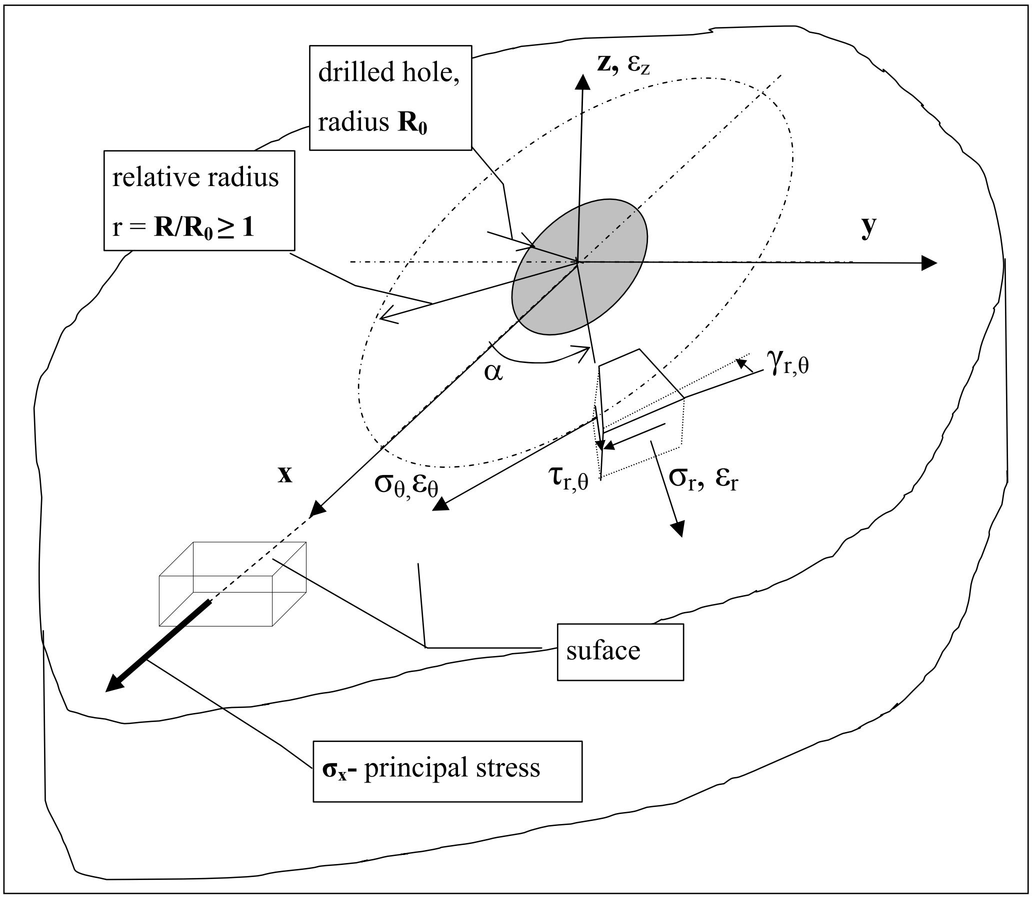

Fig. 1: State of stress and strains around the drill hole.

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\Theta \\ \gamma_{r\Theta} \\ \varepsilon_z \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \\ -\nu & -\nu & 0 \end{bmatrix} \cdot \begin{bmatrix} \sigma''_r - \sigma'_r \\ \sigma''_\Theta - \sigma'_\Theta \\ \tau''_{r\Theta} - \tau'_{r\Theta} \end{bmatrix} = \frac{\sigma_x}{2E}\begin{bmatrix} \left\{\left[-\frac{1}{r^2} - \frac{1}{r^2}\nu\right] + \left[\frac{1}{r^4}3 + \frac{1}{r^4}3\nu - \frac{1}{r^2}4\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[+\frac{1}{r^2} + \frac{1}{r^2}\nu\right] - \left[\frac{1}{r^4}3 + \frac{1}{r^4}3\nu - \frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[-\frac{1}{r^4}3 + \frac{1}{r^2}2 - \frac{1}{r^4}3\nu + \frac{1}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\} \\ \left\{\left[\frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \end{bmatrix} \tag{3}$$

The use of the hole drilling method for identification of residual stresses (Vishay Micro-Measurements, 2007) is supported by E 837 standard (ASTM, 2002). The response is measured by strain gauges assembled to a drilling rosette. The response function is similar to radial ε_r or tangential ε_Θ strains identified in the Kirsch's solution of the thin plate with a hole as described in Eq. (3). A simplification of the goniometric function Eq. (3) describing a response of an ideal strain gauge placed with a deviation of angle α from a direction related to the principal stress σ_x is used (see Eq. (4)). Standard constant variables $\overline{A} = -\overline{a}(1+\nu)/2E$ and $\overline{B} = -\overline{b}/2E$ related to the particular design of the drilling rosette are used within the superposition of principal stresses σ_x and σ_y looked for. The $\overline{a}$ constant is objectively independent of the material drilled, while constant $\overline{b}$ is here simplified because it is mildly dependent on Poisson's ratio ν of Hooke's material (see Eq. (3)). The two constants $\overline{a}, \overline{b}$ are tabulated in E 837 standard for particular types of drilling rosettes, the ratio of diameters $1/r = 2R_0/2R$ given and the relative depth of the drilled hole $z/2R$, where z is the depth of the hole.

$$\overline{\varepsilon}_r = \sigma_x(\overline{A} + \overline{B}\cos 2\alpha) + \sigma_y(\overline{A} + \overline{B}\cos(2\alpha + \pi)) = \sigma_x(\overline{A} + \overline{B}\cos 2\alpha) + \sigma_y(\overline{A} - \overline{B}\cos 2\alpha) \tag{4}$$

Strain gauge rosette sizes are comparable with those of the drilled hole diameters $2R_0$ or middle radii R, at which the strain gauges of the rosettes are placed. The measuring properties of the rosettes during the hole drilling according to E 837 standard are considerably dependent on the accuracy of compliance with standardized conditions of the experiment. If the hole is drilled eccentrically, then the hole drilling experiment, as formulated by E 837 standard, cannot be used for any more complex determination of the strain state in the vicinity of the drilled hole, which would be necessary for any eventual improving corrections. This simple standard drilled theory is not probably reliable for imperfections occurring in drilled holes.

2. Optimization principles applied to the Hole drilling method

In (Vítek, 2008, Vítek, 2008a), we describe the principle used for an objective experimental evaluation of surface strains. There we do not have to use the completely rosette strain gages around the drilling hole but only short winding segments of this strain gages. We expect that the stress state components in the surroundings of the blind drilled hole, as written in Eq. (6), are analogous to those by Eq. (3) used for a straight-through hole. Let we also modify all the seven polytropic terms of the complete Kirsch's theory by constants c_k (r, z), which are dependent on the distance from the center of the drilled hole, to be used for the blind hole. The distance is described by the relative radius r and the depth z of the drilled hole. By the way, a similar approach is also used by E 837 standard for strain - gage strains. These complete components of the stress state change, induced by drilling the hole, can be transformed to the strain components. A strain state on planes perpendicular to the surface can be set by an angular transformation, where the use of the first three components (see Fig. 1) ε_r, ε_Θ, $\gamma_{r\Theta}$ in Eq. (7) is sufficient, because the principal strain ε_z does not have any effect on it. Fig. 2 defines the position of g axis towards Θ axis for an acute angle φ. The strain in the g direction is derived from ε_r, ε_Θ, $\gamma_{r\Theta}$ strains according to the Mohr's transformation Eq. (5) by the use of goniometric functions of a double angle 2φ.

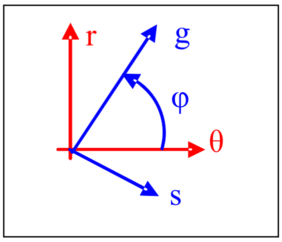

Fig. 2: Winding angle.

$$\varepsilon_g = \frac{\varepsilon_\Theta + \varepsilon_r}{2} + \frac{\varepsilon_\Theta - \varepsilon_r}{2}\cos 2\varphi + \frac{\gamma_{\Theta,r}}{2}\sin 2\varphi \tag{5}$$

$$\left\{\begin{aligned} \sigma_r'' - \sigma_r' &= \frac{\sigma_x}{2}\left(-\frac{1\cdot c_1(r,z)}{r^2}\right) + \frac{\sigma_x}{2}\left(\frac{3\cdot c_2(r,z)}{r^4} - \frac{4\cdot c_3(r,z)}{r^2}\right)\cos 2\alpha \\ \sigma_\Theta'' - \sigma_\Theta' &= \frac{\sigma_x}{2}\left(\frac{1\cdot c_4(r,z)}{r^2}\right) - \frac{\sigma_x}{2}\left(\frac{3\cdot c_5(r,z)}{r^4}\right)\cos 2\alpha \\ \tau_{r\Theta}'' - \tau_{r\Theta}' &= \frac{\sigma_x}{2}\left(-\frac{3\cdot c_6(r,z)}{r^4} + \frac{2\cdot c_7(r,z)}{r^2}\right)\sin 2\alpha \end{aligned}\right\} \tag{6}$$

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\Theta \\ \gamma_{r\Theta} \\ \varepsilon_z \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \\ -\nu & -\nu & 0 \end{bmatrix}\cdot\begin{bmatrix} \sigma_r'' - \sigma_r' \\ \sigma_\Theta'' - \sigma_\Theta' \\ \tau_{r\Theta}'' - \tau_{r\Theta}' \end{bmatrix} = \frac{\sigma_x}{2E}\begin{bmatrix} \left\{\left[-\frac{c_1}{r^2} - \frac{c_4}{r^2}\nu\right] + \left[\frac{c_2}{r^4}3 + \frac{c_5}{r^4}3\nu - \frac{c_3}{r^2}4\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[+\frac{c_4}{r^2} + \frac{c_1}{r^2}\nu\right] - \left[\frac{c_5}{r^4}3 + \frac{c_2}{r^4}3\nu - \frac{c_3}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[-\frac{c_6}{r^4}3 + \frac{c_7}{r^2}2 - \frac{c_6}{r^4}3\nu + \frac{c_7}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\} \\ \left\{\left[\frac{c_1}{r^2}\nu - \frac{c_4}{r^2}\nu\right] + \left[-\frac{c_2}{r^4}3\nu + \frac{c_5}{r^4}3\nu + \frac{c_3}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \end{bmatrix} \tag{7}$$

In (Vítek, 2010), we describe the principle used for increasing the drilling hole method sensitivity. The measurement sensitivity during the hole drilling thus can be increased by putting the strain gauges closer to the hole edge or by relative augmenting of the drill diameter to the diameter, at which the strain gauges are placed in the rosette. The experiment also can be run repeatedly with a gradual increase of the drilled hole diameter. If a minor drill diameter is chosen in the experiment first phase and the rosette strain gauges are installed in a relative distance $r = 2 \div 4$, the measurement of relaxed strain depletes about 40% of its potential, approximately. The potential of relaxed strains thus can be better exploited by increasing the drill diameter, which results in a relative shift of the strain gauges to the edge of the hole, or by the second measurement using the same drilling rosette and the drill of a bigger diameter. The stress state calibration around the hole is realized by applying seven constants $c_{11}, c_{12},..., c_{17}$, in Eq. (6) and Eq. (7), that modify the polytrope terms analogous to constants $c_1,...c_7$.

These seven constants are dependent on relative distance from the drilled hole and on relation between the two drilled holes depths.

In (Vítek, 2010a), we describe the theory for the stress state identification in the surface at the place of already drilled holes with a complete drilling rosette equipment already installed either centrically or even eccentrically. The method thus allows a further reusing of already installed measuring items, which were originally placed there for the residual stress state identification, for measurements of the stress states induced by any following external loading as if the hole had not been drilled at all. Nevertheless, the individual components reported in Eq. (2) have to be modified by multiplication by twelve different constants $c_{21}, c_{22},..., c_{32}$ according to Eq. (2), analogously to the Eq. (6) and Eq. (7). The multipliers rectify the stress state for the real conditions of the bottom hole with a perpendicular direction to the free surface. In the case of the bottom hole, the $c_{21}, c_{22},..., c_{32}$ constants depend first on the distance from the hole center described by the r radius and, second, on the h depth of the hole.

3. Conclusions

We expect the direction of the principal stress σ_x given by the angular parameter α and the second principal stress σ_y (see Fig. 1). The bonded strain gauge reads the strain field of the contact surface and we suppose that the strains in points and direction of the conductive winding, correspond to the strain values measured by the strain gauge. The strains ε_r, ε_Θ, $\gamma_{r\Theta}$ are standardized by a unit load vector introduced in the direction of principal stress and transformed by Eq. 5 to the winding direction of the strain gages. An analogy to Eq. (4) allows assembling a system of three independent equations for three unknown - principal stresses σ_x, σ_y and the angle of their position α, where strain gauges signals are simultaneously experimentally examined.

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