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INDEPENDENT MODES IN A BOUNDARY LAYER SEPARATION REGION

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Abstract: *The method for evaluation of temporal and spatial independent modes of a dynamical system is suggested. The dynamical system is represented by time dependent vector field representing experimental data from separation of a boundary layer.*

Keywords: ***Boundary layer separation, independent component analysis.***

1. Introduction

Recently we have at our disposal time evolutions of vector fields obtained using time-resolved PIV method representing turbulent flow-field as extended dynamical system. Analysis of this data requires adequate methods. Proper Orthogonal Decomposition (POD) working on energetic principle became classical method used for this purpose. The POD provides set of orthogonal spatial modes, number of which could be easily truncated to obtain reduced model of the system optimal from the point of view energetic content. However physical interpretation of the modes is very unclear and confusing. This is price for nonphysical condition of the modes orthogonality, howsoever this is very convenient and practical for treatment. A different model is needed for study the dynamical system dynamical behavior.

Individual sources of perturbations in turbulent field could be assigned to independent time signals detected within the flow-field. Although the POD modes are orthogonal and thus uncorrelated they are not necessarily independent. To decompose the turbulent signals into independent components some other method should be used. We suggest application of Independent Component Analysis (ICA) method. The ICA has been introduced in 80‘s to treat some neurophysiological problems (muscle contraction), while from 90’s it is applied in numerous fields of mathematics and physics. ICA is a statistical method, its goal is decomposition of a given multivariate data into a sum of statistically independent components. The method works with little prior information.

2. Independent Component Analysis

The ICA finds the independent components (aka factors, latent variables or sources) by maximizing the statistical independence of the estimated components. We may choose one of many ways to define independence, and this choice governs the form of the ICA algorithms. The two broadest definitions of independence for ICA are minimization of mutual information and maximization of non-Gaussianity.

The Non-Gaussianity family of ICA algorithms, motivated by the central limit theorem, uses kurtosis and negentropy. The Minimization of Mutual Information family of ICA algorithms uses measures like Kullback-Leibler Divergence and Maximum-Entropy.

Let us define the statistical latent variables model first. We observe n linear mixtures $x_1,...,x_n$ of m signals $s_1,...s_m$:

$$x_j = a_{j1}s_1 + a_{j2}s_2 + ... + a_{jm}s_m\,,\ \ j = 1,...n \qquad (1)$$

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Note, that number of signals and mixtures could be different in general, there are no restrictions even for theirs relation, i.e. the n could be equal, larger or even smaller then m. All mixtures and independent components are random variables

The starting point for ICA is the very simple assumption that the components s_i are statistically independent. In addition we also assume that the independent components must have Non-Gaussian distributions, however we do not assume these distributions known. Then, after estimating the matrix $\mathbf{A}$, we can compute its inverse and obtain the independent components. In tensor notation we have vectors $\mathbf{x}$, $\mathbf{s}$ and matrix $\mathbf{A}$:

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{s}, \quad \mathbf{s} = \mathbf{A}^{-1} \cdot \mathbf{x} \tag{2}$$

ICA is very closely related to the method called Blind Source Separation or Blind Signal Separation (BSS). "Blind" means that we know very little, if anything, on the mixing matrix, and make little assumptions on the source signals. ICA is one of the methods, perhaps the most widely used, for performing blind source separation.

Typical algorithms for ICA use centering, whitening (usually with the eigenvalue decomposition), and dimensionality reduction as preprocessing steps in order to simplify and reduce the complexity of the problem for the actual iterative algorithm. Whitening and dimension reduction can be achieved with POD or similar method. Whitening ensures that all dimensions are treated equally a priori before the algorithm is run.

However there are some ambiguities connected with the ICA method. In general, ICA cannot identify the actual number of source signals, a uniquely correct ordering of the source signals, nor the proper scaling (including sign) of the source signals. It could not indicate the components variances as well, because it introduces components normalization.

3. Experiment

Experiments on a boundary layer separation in adverse pressure gradient have been carried out in the IT AS CR using the time-resolved PIV technique. We acquired 5000 doublesnaps in frequency 1518 Hz corresponding to 3.2 s of record. Fields of 18 x 79 2-componnent vectors result in dynamical system with 2844 degrees of freedom and the phase space same size. The setup of this experiment and results were published in Uruba (2008, 2009).

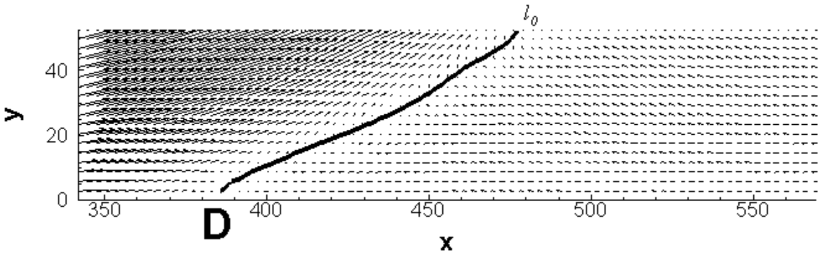


Fig. 1: Mean velocity vectors distribution.

The boundary layer shows highly dynamical behavior with many vortical structures arising within the free shear layer. However the mean vector field in Fig. 1 is very regular and smooth. The line l_0 represents positions with zero streamwise velocity component. Extrapolation of this line on the wall ($y = 0$) denoted D is the detachment point after classical definition ($x = 382$ mm).

4. Results

The POD modes have been evaluated both in spatial (topoi) and temporal (chronoses) domains. In the analysis we will consider the 10 most energetic modes covering 65 % of total fluctuating energy. Thus we treat the flow-field as a dynamical system with 10 degrees of freedom. The first 10 topoi are depicted in Fig. 2 as vector fields with vorticity isolines (negative vorticity is dashed line).

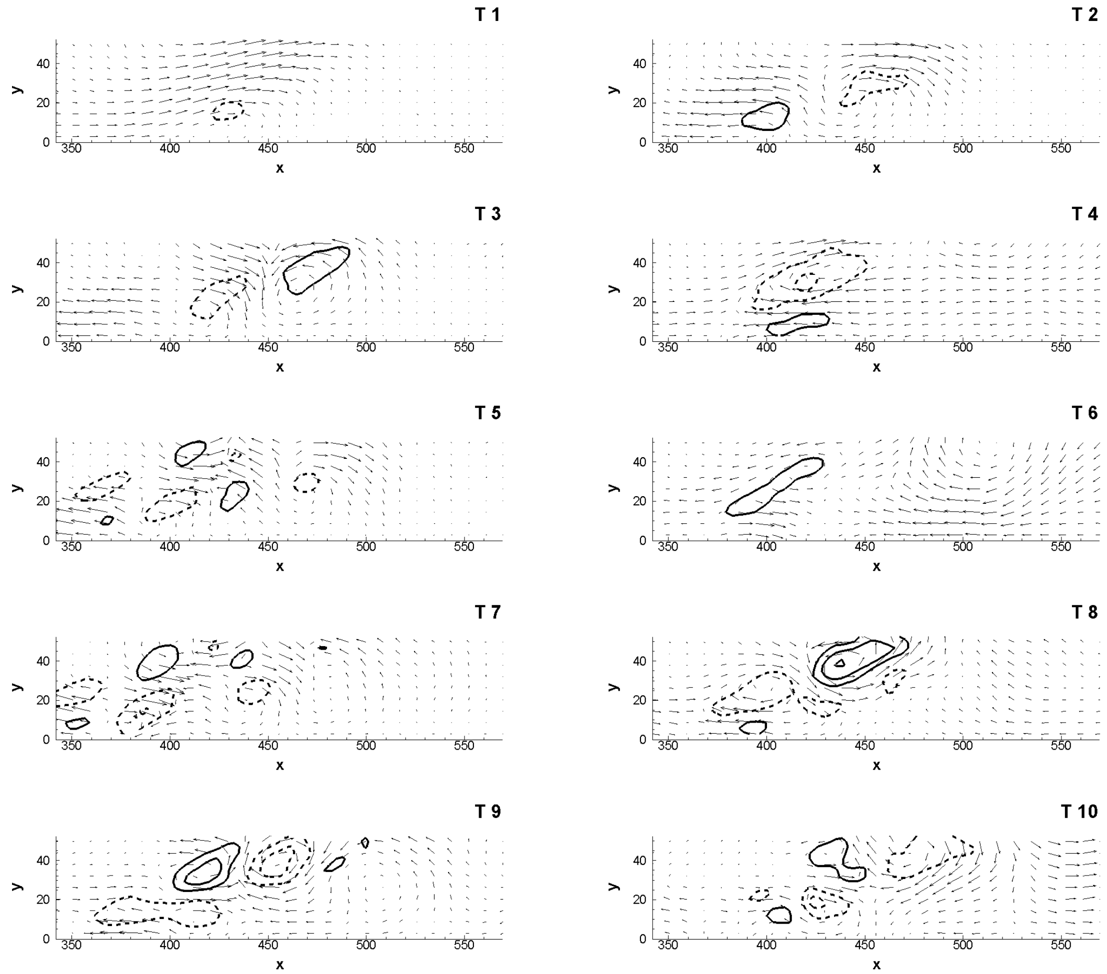
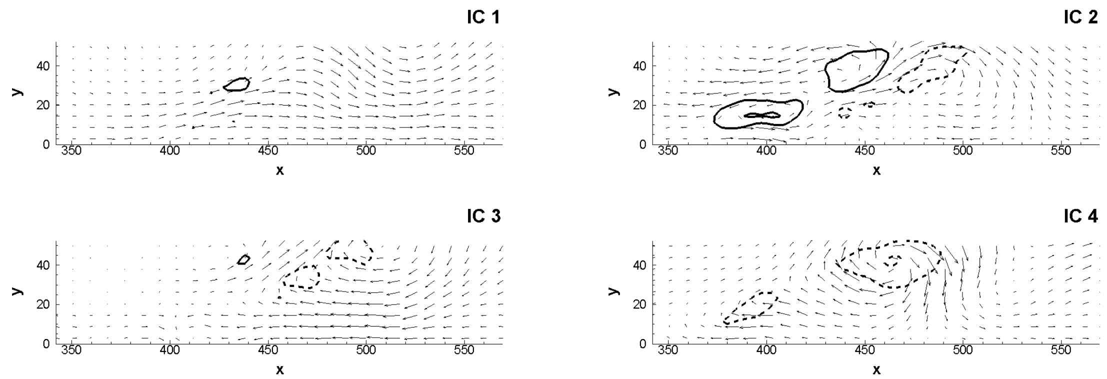


Fig. 2: The first 10 toposes.

We could observe the systems of vortical structures in proximity of the l_0 line. Than the ICA has been applied on the 10 chronoses, the implementation using matrix pencil method has been chosen. This method consists in two steps. In first step the time series is filtered by a selected FIR filter and the matrix pencil is formed. Than the general eigenvector of the matrix pencil is calculated. The details of the method are described in Chang et al. (2000). Thus the 10 ICA temporal modes have been constructed as recombination of the original POD modes. Unlike the POD modes, the ICA modes are really statistically independent. Then the corresponding topological ICA modes were evaluated – see Fig. 2.



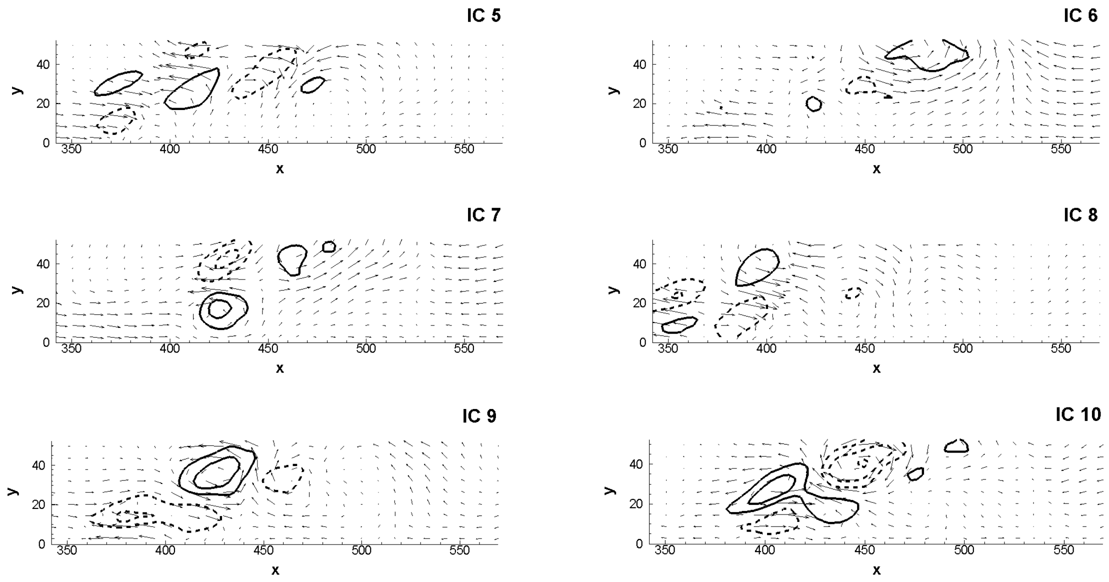


Fig. 3: The first 10 independent modes.

Comparing the Figs. 2 and 3 it is clear that the ICA modes are of a different structure than the POD modes however they consist of vortical structures close to l_0 as well. The ICA modes better represent the dynamical behavior of the underlying dynamical system. However the 10 ICA modes cover only 40 % of the total fluctuating energy (the 10 POD modes cover 65 %).

5. Conclusions

Both temporal and topological independent modes of the dynamical system represented by time dependent vector fields were evaluated by consecutive application of the POD and ICA methods. The evaluated modes could be used for detailed study of the dynamical system properties as they have clear physical meaning and interpretation.

Acknowledgement

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