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SHEAR LAG EFFECT IN RESPONSE OF BOX BRIDGES

A. Tesár*

Abstract: *The paper describes the application of technical theory of thin-walled box structures for the assessment of shear lag effects in thin-walled box bridges subjected to dynamic loads. The approach takes into account multiple functions in assessment of slender box bridges constructed of thin-walled members. Theoretical and numerical assessments of the problem are presented.*

Keywords: ***Bridges, dynamic loads, Fourier integral transformation, shear lag, technical theory of thin-walled structures.***

1. Introduction

Due to economy and weight restrictions slender thin-walled box members are often utilized in advanced bridge engineering. Even for structures that behave in linear fashion under service loads the safety considerations require that an analysis is to be carried out for the nonlinear range of behavior in order to determine the reliability with respect to collapse or possible damage due to overloading.

2. Methods adopted

The stress distribution in thin-walled members of box bridges is given by combination of sectorial influences stated in scope of technical torsion-bending theory and additional sectorial influences due to distortion and shear. The decrease of rigidity in thin-walled box members is specified by the shear lag appearing in additional degrees of freedom in mechanics of cross-sectional distortion. In order to take into account the decrease of normal rigidity, there are introduced additional sectorial functions w_s due to distortional kinematics of thin-walled box cross-section studied (Bornscheuer, F.W., 1952 and Sedlacek, G., 1967). Such functions are dependent on additional cross-sectional deformations v_s given by

$$| w |=| w_t , w_s |^T=| x, y, \omega ; w_1 , w_2 , ... |^T , \tag{1}$$

and

$$| v |=| v_t , v_s |^T=| \xi, \eta, \upsilon ; v_1 , v_2 , ... |^T, \tag{2}$$

The warping u is given by

$$u= -w^T.v' , \tag{3}$$

with strain

$$\varepsilon= -w^T.v'' , \tag{4}$$

and with normal and shear stresses given by

$$\sigma =E.\varepsilon = -E.w^T.v'' \tag{5}$$

and

$$\tau= -G.w_s.\upsilon_s' . \tag{6}$$

The choice of additional sectorial functions w_s appearing as linear functions of shear stress τ along the cross-section is made in accordance with rules of technical theory of thin-walled beams. Used are the

* Alexander Tesár, Civ. Eng., PhD, DrSc, visiting prof.: Institute of Construction and Architecture, Slovak Academy of Sciences, Dúbravská cesta 9, 845 03 Bratislava 45, Slovak Republic, e-mail: usarate@savba.sk

functions $S_s \approx S_x, S_y, S_\omega = \int xdA, \int ydA, \int \omega dA$, respectively, (A is the cross-sectional area) together with additional unit sectorial function $w_s = \int S_s \, ds$ along the centre-line *s*.

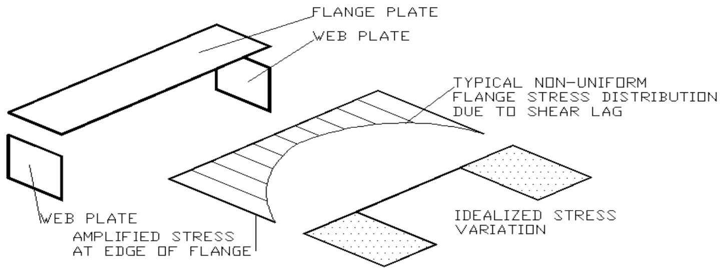

Fig. 1: Shear lag effect.

The equilibrium state of thin-walled box system is given by

$$\delta\pi = \int_0^l \left\{ \int_A (\sigma.\delta\varepsilon + \tau.\delta\gamma)\, dA - p.r.\delta v - \int_s n_s.wds.\delta v' \right\} dz = 0, \tag{7}$$

with shear strain γ and with first terms representing the virtual work of internal forces and further terms specifying the virtual work of external forces on displacements δv.

If in above equation the stress σ is replaced by corresponding functions of displacements υ_s the modified Eq. (7) is given by

$$\delta\pi_S = \int_0^1 [\delta v_S''.E.J_S.v_S'' + \delta v_S'^T.G.K_S.v_S' - \delta v_S'^T.p - \delta v_S'.n.w_S^T]\, \mathrm{d}z = 0, \tag{8}$$

with $E.J_s$ and $G.K_s$ as sectorial and shear rigidities of the cross-section studied and with corresponding load components *p* and *n*. The double integration of Eq. (8) gives

$$\int_0^1 \{\delta v_S [\, E.J_S.v_S^{IV} - G.K_S.v_S^{II} - p - n'.w_S\,]\}\mathrm{d}z = 0, \tag{9}$$

and submits for each $\delta v_S \neq 0$ the system of simultaneous differential equations

$$E.J_S.v_S^{IV} - G.K_S.v_S^{II} = p + n'.w_S \;, \tag{10}$$

for numerical treatment of the problem.

Matrix equation (10) with unknown components of cross-sectional shear deformations v_s is modified into diagonal shape of s differential equations given by

$$E.J_{s,ii}.v_{s,i}^{IV} - G.K_{s,ii}.v_{s,i}^{II} = p_{s,i} + n_{s,i}'.w_{s,i}. \tag{11}$$

Resulting displacements are given by strain components of non-deformable thin-walled cross-section, combined with components of unit shear deformations.

Differential equations (10) and (11) are formally identical with differential equation of the girder with flexural rigidity $E.J_s$ subjected to axial force $G.K_s$. The solution is known with the analogy of components

$$-E.J_{s,ii}.v''_{s,i} = M_{s,i} \;, \quad (12)$$

$$-E.J_{s,ii}.v'''_{s,i} = M'_{s,i} = T_{s,i} \;, \quad (13)$$

with flexural moment M and lateral force T. Corresponding normal and shear stresses are given by

$$\sigma_{s,i} = \frac{M_{s,i}}{J_{s,ii}} . w_{s,i} \tag{14}$$

and

$$\tau_{s,i} = -G . w^{\cdot}_{s,i} . v'_{s,i} . \tag{15}$$

With implementation of the Fourier integral transformation into the Transfer Matrix Method (Tesár, A., 1977, 1988, 2005, 2006 and 2010) is studied the influence of the shear lag in the response of thin-walled box bridges subjected to periodic, non-periodic and moving dynamic loads.

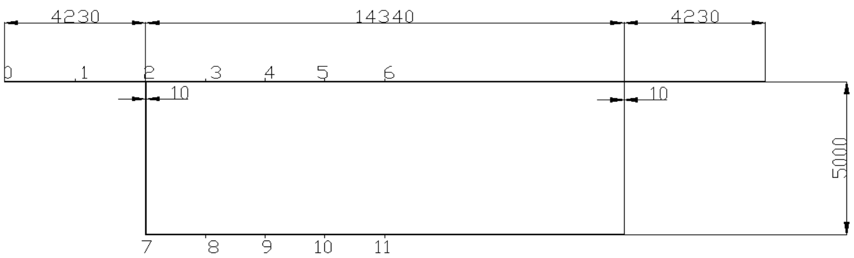

Fig. 2: Thin-walled box bridge with nodal points for assessment of torsion.

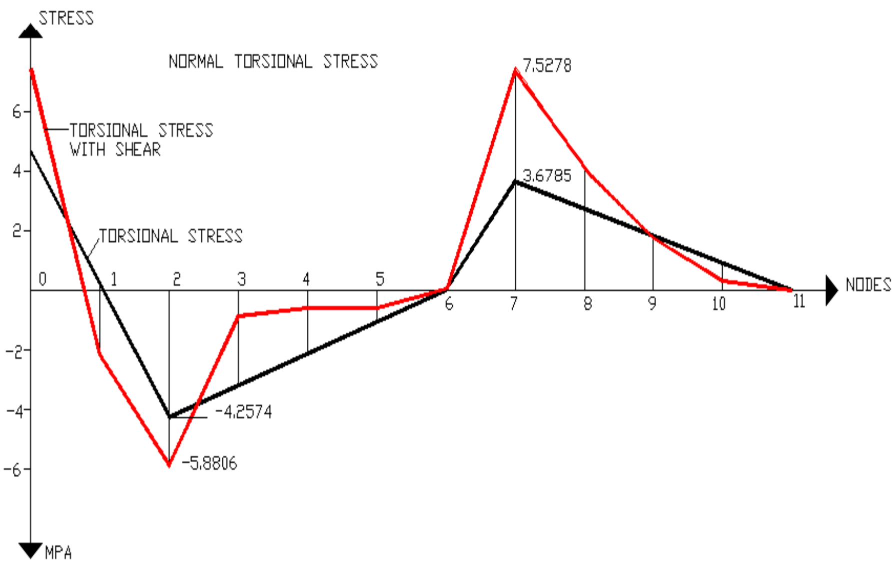

Fig. 3: Comparison of normal torsion stress without and with of shear lag.

3. Application

Cross-sectional geometry of the single span bridge studied is plotted in Fig. 2. The span of the bridge is L = 70 m. Concentrated load P acts in location L/2 = 35 m. Torsion is initiated by two mid-span loads P/2 acting in vertical direction up and down in the flanges of thin-walled cross-section studied. Calculated are the stresses in flexure and torsion due to shear lag. Maximal shear stress and resulting normal stresses in torsion without and with consideration of the shear lag are summed up in Fig. 3. For real situations all stresses obtained are to be multiplied by actual value of the load P.

Such results are to be taken into account in the assessment of ultimate behavior of thin-walled members of slender box bridges.

4. Conclusions

The approach for assessment of the shear lag effect in thin-walled box bridges is suggested. Technical torsion-bending theory of thin-walled structures is adopted for theoretical and numerical treatment of the problem. Such assessment is a part of the bridge design and monitoring in order to specify the safety with respect to collapse and damage due to overloading.

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References

Bornscheuer, F.W. (1952) Systematische Darstellung des Biege- und Verdrehvorganges unter besonderer Berücksichtigung der Wölbkrafttorsion. Der Stahlbau 21, pp. 342-346.

Sedlacek, G. (1967) Systematische Darstellung des Biege- und Verdrehvorganges für prismatische Stäbe unter Berücksichtigung der Profilverformung. Diss. TU Berlin.

Tesár, A. (1977) Vibration of Thinwalled Box Structures. Proceedings of Czechoslovak Academy of Sciences, ACADEMIA, Prague, 87, 1.

Tesar, A. (1988) Transfer Matrix Method. KLUWER Academic Publishers, Dordrecht/Boston/London.

Tesar, A. (2005) Load-bearing control of slender bridges. International Journal for Numerical Methods in Engineering, 62, pp. 924-936.

Tesar, A. (2006) Bionics and fractal configurations in structural engineering. International Journal for Numerical Methods in Engineering, 68, pp. 790-807.

Tesár, A. (2010) Testing of the Old Bridge Crossing Danube in Bratislava. Technical Report, TESAR@PARTNER.