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MODAL ANALYSIS OF THE IMPERFECT BLADED DISK WITH FRICTION ELEMENTS

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Abstract: *This paper is concerned with mathematical modeling of the imperfect bladed disk with friction elements, which are used to suppress blade vibration. These kinds of systems are mainly used in steam turbines where our interests are aimed. The friction elements are placed between shrouds of selected central symmetrical mounted blades (imperfection). A slipping is considered on the both contact surfaces. The model of the bladed disk is based on decomposition into a disk subsystem and a blading subsystem. The finite element method is used for modeling of the both subsystems. All influences of steady-state rotation are respected as centrifugal forces, gyroscopic effects, centrifugal stiffening of blades and dynamic softening. The model is used for a modal analysis of non-rotating and rotating imperfect bladed disk. Selected natural modes of non-rotating system will be shown. Dependency of eigenfrequencies on the angular velocity is presented in Campbell diagram. The methodology of simulation and post-processing are implemented in an in-house software.*

Keywords: ***Bladed disk, modal analysis, FEM, friction element.***

1. Introduction

Requirements on higher efficiency and wide operation range of steam turbines lead to thinner blade profile, which has better properties in term of computation of fluid dynamics (CFD) but dynamic properties get worse. The purpose of friction elements is to decrease potential high amplitudes of blade vibration, which may occur due to resonances or high applied forces of steam flow fluctuation. The aim of this paper is to present methodology for bladed disk vibration analysis of damped blades applicable for test imperfect bladed disk used in Institute of Thermomechanics AS CR.

The presented method is based on discretization of 3D disk (Šašek & Hajžman, 2006) and 1D blades (Kellner & Zeman, 2006) by FEM. A harmonic balance method will be further used for modeling of bladed disk vibration with friction elements, which are placed between blade shrouds (see Fig. 1 and Fig. 2). The comparison between flexible and rigid shroud is performed in Zeman et al. (2009) entitles the shroud modeled as a rigid body fixed with blade in the end of blade's profile.

2. Model of imperfect bladed disk

The imperfect bladed disk is consists of a disk, which is fixed on inner radius to rigid shaft, and two kinds of blades. Blades of the first type are mounted to disk with rigid joint. A rigid shroud is placed in the end of the second type of blades. Friction elements are placed between shrouds (see Fig. 2). Each type of blades is separated into two sets (2x25 blades of the first type, 2x5 blades of the second type), which create bladed disk with two perpendicular axis of symmetry. There are 2x4 friction elements in the both sets of the second type of blade.

The stiffness coupling matrix between friction elements and blade shrouds is derived from deformation of adjacent shroud contact areas caused by relative movements of blades and friction elements and by centrifugal forces acting on friction elements.

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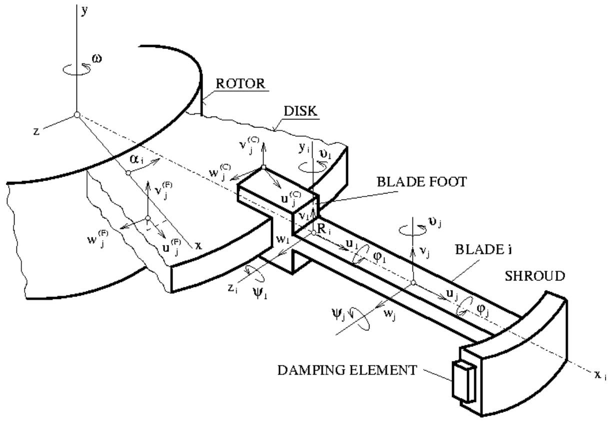

Fig. 1: The second type of blades with friction elements.

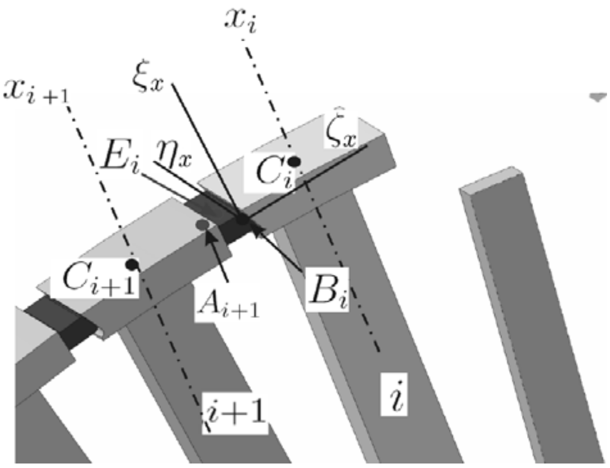

Fig. 2: Detail of shrouds with friction elements.

This model respects contact stiffnesses between blades and damping elements E_i on both contact areas, defined by contact stiffness matrices

$$\mathbf{K}_C^{(X)} = diag\begin{pmatrix}0 & 0 & k_{\varsigma_x} & k_{\xi_x\xi_x} & k_{\eta_x\eta_x} & 0\end{pmatrix}, X = A_{i+1}, B_i, \tag{1}$$

The matrices express the constraint between the circumferential displacement and two rotations by means of contact stiffness k_ς in normal direction to concrete contact area $\xi_x\eta_x$ and two flexural stiffnesses $k_{\xi_x\xi_x}$, $k_{\eta_x\eta_x}$, whereas $\mathbf{K}_C^{(B_i)} = \mathbf{K}_C^{(B)}$, $\mathbf{K}_C^{(A_{i+1})} = \mathbf{K}_C^{(A)}$.

These matrices are expressed in local coordinate systems $\xi_x, \eta_x, \varsigma_x$ placed in central contact point B_i of the radial contact area of blade shroud i, respective in central contact point A_{i+1} of the blade shroud $i+1$ on the slope area. The point E_i is the center of gravity of the friction element. The coupling (deformation) energy between two adjacent blades i and $i+1$ is expressed as

$$\begin{aligned} E_C^{i,i+1} &= \frac{1}{2}\left(\mathbf{q}_{B,i} - \mathbf{T}_{B,E}\mathbf{q}_{E,i}\right)^T \mathbf{K}_C^{(B)}\left(\mathbf{q}_{B,i} - \mathbf{T}_{B,E}\mathbf{q}_{E,i}\right) + \\ &+ \frac{1}{2}\left(\mathbf{T}_{A,E}\mathbf{q}_{E,i} - \mathbf{q}_{A,i+1}\right)^T \mathbf{K}_C^{(A)}\left(\mathbf{T}_{A,E}\mathbf{q}_{E,i} - \mathbf{q}_{A,i+1}\right), \end{aligned} \tag{2}$$

where $\mathbf{q}_{B,i}, \mathbf{q}_{A,i+1}$ are vectors of blade shroud displacements of points B_i, A_{i+1} and $\mathbf{q}_{E,i}$ is vector of gravity center E_i displacements of friction elements, where i is blade index. The transformation between vectors $\mathbf{q}_{B,i}, \mathbf{q}_{A,i+1}$ in local coordinate systems $\xi_x\eta_x\varsigma_x$, $x = B_i, A_{i+1}$ and vectors $\mathbf{q}_{C,i}, \mathbf{q}_{C,i+1}$ of global coordinates of end nodal points of the adjacent blades is

$$\mathbf{q}_{B,i} = \mathbf{T}_B\mathbf{q}_{C,i}, \quad \mathbf{q}_{A,i+1} = \mathbf{T}_A\mathbf{q}_{C,i+1}. \tag{3}$$

Matrices $\mathbf{T}_{B,E}$ and $\mathbf{T}_{A,E}$ transform vector $\mathbf{q}_{E,i}$ into displacements of contact points B_i and A_{i+1} in the local coordinate systems $\xi_x\eta_x\varsigma_x$, $x = B_i, A_{i+1}$.

The disk is modeled analogously as in paper (Šašek & Hajžman, 2006) by means of 3D continuum FE discretization. The mathematical model of the bladed disk rotating with angular velocity ω ad interim with smooth friction elements was derived in configuration space $\mathbf{q} = [\mathbf{q}_D^T \quad \mathbf{q}_D^T]^T$, where $\mathbf{q}_D$ is vector of generalized coordinates of the disk and $\mathbf{q}_R$ is vector of generalized coordinates of all blades and friction elements. The connection of the blades and the disk is realized by rigid joint which is described in Kellner, J. & Zeman, V. (2010).

The conservative mathematical model of the centrally clamped bladed disk rotating with constant angular velocity ω around y axis has the form

$$\mathbf{M}\ddot{\mathbf{q}} + \omega\mathbf{G}\dot{\mathbf{q}} + \left(\mathbf{K}_s + \mathbf{K}_C + \omega^2\mathbf{K}_\omega - \omega^2\mathbf{K}_d\right)\mathbf{q} = \omega^2\mathbf{f}_2, \tag{4}$$

where $\mathbf{M}$ is mass matrix, $\omega\mathbf{G}$ is skew-symmetric matrix of gyroscopic effects, $\mathbf{K}_s$ is static stiffness matrix, $\mathbf{K}_\omega$ is matrix of blade centrifugal stiffening and $\mathbf{K}_d$ is matrix of dynamic softening of 1D and 3D continuum in centrifugal field. The contact stiffness matrix $\mathbf{K}_C$ between friction elements and shroud contact areas are calculated from identity $\frac{\partial E_C}{\partial \mathbf{q}} = \mathbf{K}_C\mathbf{q}$, where $E_C = \sum_i E_C^{i,i+1}$.

3. Modal analysis of bladed disk

Correctness of the modeling methodology of the imperfect bladed disk is advised by modal analysis of the test bladed disk (Půst & Pešek, 2009). First eight natural modes of non-rotating bladed disk are rigid ($f_{1,2\ldots8} = 0$ Hz). These natural modes are characterized by oscillation of the friction elements in y direction because of smoothness of the friction elements. The next four mode shapes are shown on Fig. 3 to Fig. 6. We can separate the modes into 2 pairs. The first pair, where blades oscillate in the opposite way, is consists of the 9th and 11th natural mode. The 10th and 12th natural mode, where blades oscillate in the same direction, create second pair. All pairs are characterized by 1 nodal diameter (ND), where the disk and the blades don't oscillate. The nodal diameters are perpendicular in corresponding mode shapes of one pair. Different values of pair eigenfrequencies are given by imperfect blades.

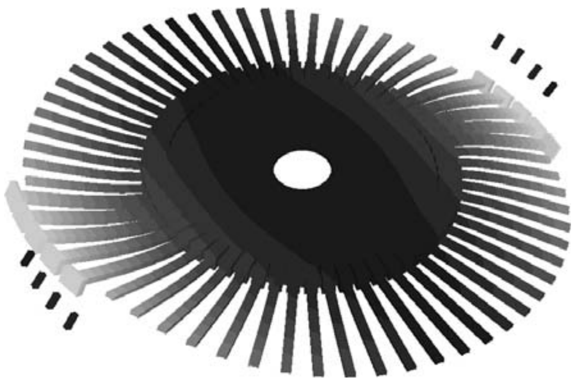

Fig. 3: The 9th natural mode ($f_9 = 89.23$ Hz, 1 ND).

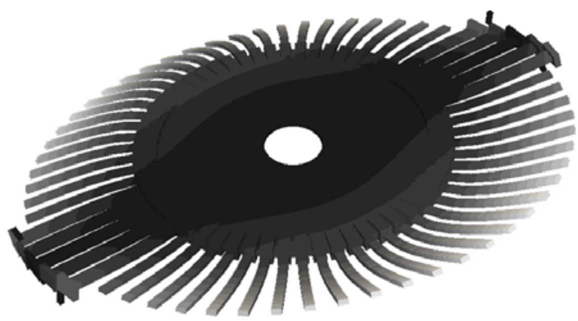

Fig. 4: The 11th natural mode ($f_{11} = 110.41$ Hz, 1 ND).

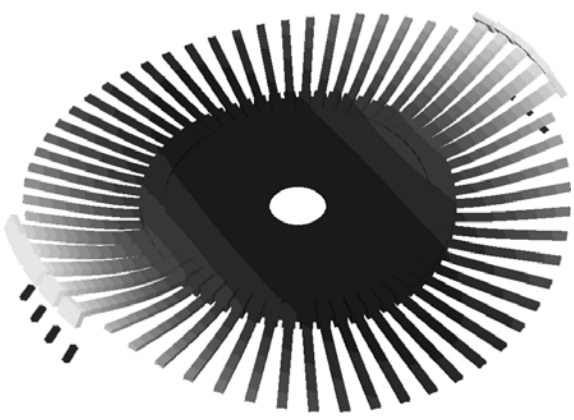

Fig. 5: The 10th natural mode ($f_{10} = 93.73$ Hz, 1 ND).

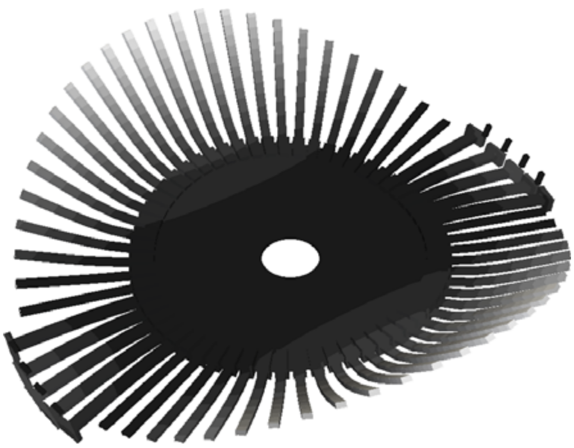

Fig. 6: The 12th natural mode ($f_{12} = 120.38$ Hz, 1 ND).

The dependency of eigenfrequencies is shown in the Campbell diagram (see Fig. 7) diagram for 9th to 16th eigenfrequencies and for angular velocity $\omega = \langle 0, 3000 \rangle$ RPM. The blade centrifugal stiffening effect represented by matrix $\mathbf{K}_{\omega}$ influences the eigenfrequencies mostly. The other effects of rotation affects less the eigenfrequencies.

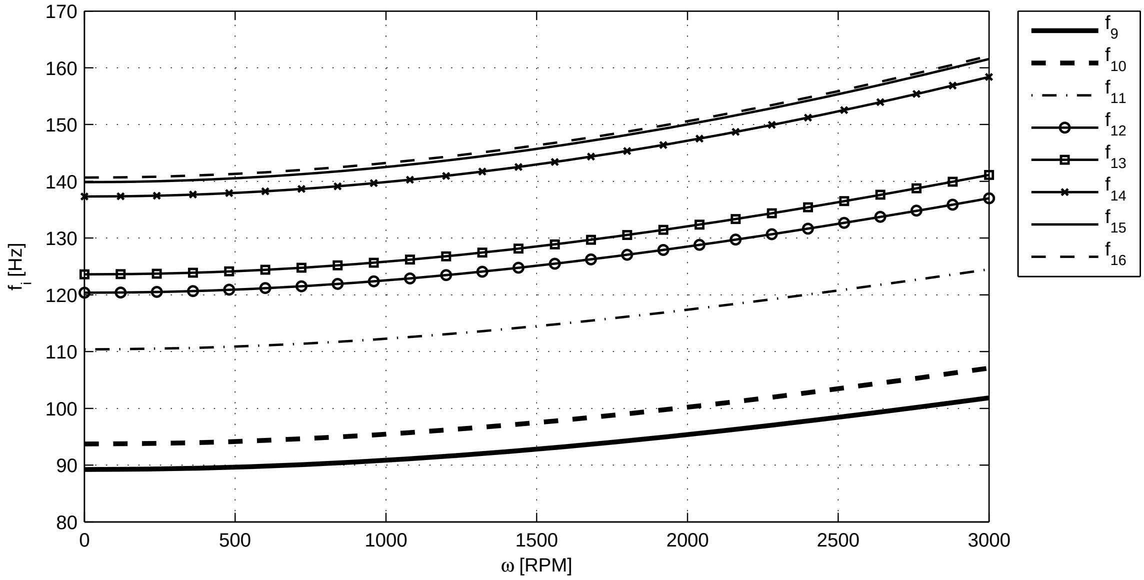

Fig. 7: Campbell diagram for 9th to 16th eigenfrequencies and for angular velocity $\omega = \langle 0, 3000 \rangle$ *RPM.*

4. Conclusions

The presented model of the imperfect bladed disk with the smooth friction elements is the first step for bladed disk vibration analysis of damped blades. The correctness of the methodology is checked on the modal analysis of non-rotating and rotating testing bladed disk, where all influences of steady-state rotation are considered.

The developed methodology and mathematical model of the imperfect bladed disk will be further used for modeling of the test bladed disk vibration respecting friction forces in contact surfaces between blade shrouds and friction elements using harmonic (balance) linearization method.

Acknowledgement

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