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# MECHANICS OF FLAT TEXTILE FABRICS – THEORY

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**Abstract:** *The presented work is focused, from wide textile problems, on research of mechanical properties of flat textile fabrics, the geometrically and also physically non-linear directionally oriented formations. The method of continuum mechanics is used, where textile flat fabric is substitute by continuous medium with the same mechanical properties.*

**Keywords:** ***Continuum mechanics, conjugated pair, biaxial loading of fabric.***

## 1. Introduction

Identification of mechanical properties of uniaxially or biaxially loaded flat textiles is physical problem, which leads in general case to seven unknowns' task.

Fabric is so unique formation, that it is necessary to describe its mechanical properties (contrary to solids) for each concrete state of stress and transformation. Constitutive dependence in plane composition is possible to express by three equations.

Substitution of fabric with expressive structure by plain continuum with the same mechanical properties allows to use continuum mechanics equations and to define basic mechanical properties of textile fabric. Dependence between Euler and Lagrange coordinates of points (previously described by Chandrasekharaiah & Lokenath Debnath, 1994; Okrouhlik, 1995; Striz, 2003) is possible to determine from measured movements of observed points on the flat textile (Fig. 1):

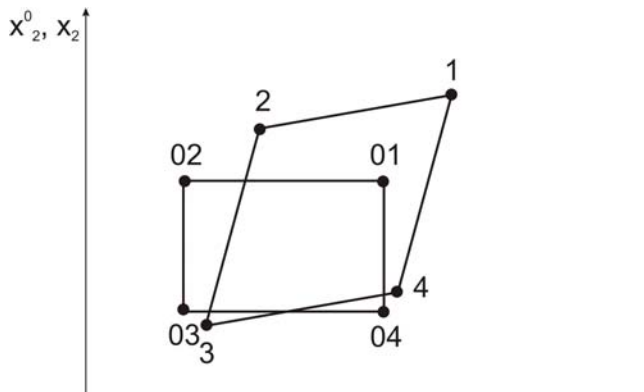


*Fig. 1: Show of original and deformed sample and way of its coordinates reading.*

$$x_i^p = x_i^{0p} + w_i^p, \quad i = 1, 2, \tag{1}$$

where $p$ is a number of point, circle indicates Lagrange coordinates. If we replace derivations in equation (1) by differences of ending points of diagonals of selected generally irregular element (Fig. 1), we can define material deformational rate $F$ by solution of equation systems:

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$$\Delta x_{ir} = \Delta x^0_{ir} + w_{i;j}\, \Delta x^0_{jr}, \quad i, j, r = 1,2, \tag{2}$$

where $\Delta x_{i1} = x_i^1 - x_i^3, \quad \Delta x_{i2} = x_i^2 - x_i^4, \Delta x^0_{i1} = x_i^{01} - x_i^{03}, \quad \Delta x^0_{i2} = x_i^{02} - x_i^{04}.$

Differential of movement wi;j, i, j = 1, 2 in a plain of element is set like a constant and labeling (;), i.e. derivation according to Lagrange coordinate, is not used and $w_{i;j} = v_{ij}$ is implemented. From difference equation (2) the algebraic equation system is obtained:

$$\begin{aligned}
&(1+v)\left(x_1^{01} - x_1^{03}\right) + v_{12}\left(x_2^{01} - x_2^{03}\right) = x_1^1 - x_1^3,\\
&(1+v)\left(x_1^{02} - x_1^{04}\right) + v_{12}\left(x_2^{02} - x_2^{04}\right) = x_1^2 - x_1^4,\\
&v_{21}\left(x_1^{01} - x_1^{03}\right) + (1+v_{22})\left(x_2^{01} - x_2^{03}\right) = x_2^1 - x_2^3,\\
&v_{21}\left(x_1^{02} - x_1^{04}\right) + (1+v_{22})\left(x_2^{02} - x_2^{04}\right) = x_2^2 - x_2^4.
\end{aligned} \tag{3}$$

Material deformational rate $F = \dfrac{\Delta x_i^r}{\Delta x_j^{0r}}$ is given by equation system (3) solution. The result is:

$$F = \begin{pmatrix} 1+v_{11} & v_{12} \\ v_{21} & 1+v_{22} \end{pmatrix}, \quad F^T = \begin{pmatrix} 1+v_{11} & v_{21} \\ v_{12} & 1+v_{22} \end{pmatrix}.$$

Let's extend tensor F with non-dimensional movement $w_{3;3} = v_{33}$. The result is

$$F = \begin{pmatrix} 1+v_{11} & v_{12} & 0 \\ v_{21} & 1+v_{22} & 0 \\ 0 & 0 & 1+v_{33} \end{pmatrix} \tag{4}$$

and jacobian

$$J = (1+v_{33})\,J_0 = (1+v_{33})\left[(1+v_{11})(1+v_{22}) - v_{12}\, v_{21}\right]. \tag{5}$$

Quantity $v_{33}$ is determined by fabric thickness change:

$$h = h_0\,(1+v_{33}), \tag{6}$$

where $h_0$ is original measured thickness of fabric. Elongation tensor $U$ and tensor of rotation $R$ is defined by material deformational rate $F$. It is valid:

$$U^2 = F^T.F, \quad F = R.U. \tag{7}$$

Elongation tensor $U$ is possible to define e.g. by projector method defined e.g. by Striz (2001). Let's label tensors $U$, $R$ with components:

$$U = \begin{pmatrix} u_{11} & u_{12} & 0 \\ u_{12} & u_{22} & 0 \\ 0 & 0 & 1+v_{33} \end{pmatrix}, \quad R = \begin{pmatrix} r_{11} & r_{12} & 0 \\ -r_{12} & r_{11} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{8), (9}$$

The clamping of measured fabric sample has to allow movements in both directions of loading. In case of angular rotation of main axes of fabric sample anisotropy the clamping has to allow sample's slope.

On a basement of these assumptions the values of $v_{ij}$ defined on an element of sample is possible to spread on a whole sample. Coordinates of peaks, lengths of deformed sides and goniometric functions of bevel angle are defined from sample shape. Tensor of real specific forces Σ [N/m] is determined with help of forces $Q_1$, $Q_2$ (operate in axes of loading) from equations of mechanics. Let's mark

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

## 2. Conjugated Pairs

The continuum mechanics introduces so-called conjugated pairs, i.e. connection among different types of definitions of stress tensor and strain tensor. Their scalar product on chosen interval of loading has to satisfy condition of equability of mechanical work or power. Just perfect sample loading and its homogenous shape can fulfill this.

Conjugated pairs differ in parameter „*n*“. The best known are conjugated pairs with parameters 2, 1, 0, -1, -2. The big number of them can exist, but special importance has Biott’s pair ($n$ = 1), which very often gives on so-called “conventional” tension, related to original size of the sample.

Biott’s tension and deformation can be expressed like:

$$S(1) = \frac{J_0}{2\,h_0}\left[F^{-1}\,\Sigma\,R + R^T\,\Sigma\,F^{-1T}\right], \tag{11}$$

$$\varepsilon_{ij}(1) = U - I, \tag{12}$$

where $I$ is unit tensor. Other stress tensors (except Cauchy’s) can be expressed like:

$$S(n) = \frac{1}{2}\left[S(1)U^{1-n} + U^{1-n}\,S(1)\right] \tag{13}$$

and for strain tensors is valid

$$\varepsilon_{ij}(n) = \frac{1}{n}\left(U^n - I\right). \tag{14}$$

All conjugated pairs have to fulfill condition of mechanical work (power) equality. The following equation is for textile fabric plane state of stress

$$I(n) = \int_0^{x_{\max}} \left[\sigma_{11}\frac{\mathrm{d}\varepsilon_{11}}{\mathrm{d}x} + \sigma_{22}\frac{\mathrm{d}\varepsilon_{22}}{\mathrm{d}x} + 2\,\sigma_{12}\frac{\mathrm{d}\varepsilon_{12}}{\mathrm{d}x}\right]\mathrm{d}x, \tag{15}$$

where $x = \dfrac{Q_1(x)}{Q_1\max}$. Permanency of equation (15), for given load and for all conjugated pairs depends on accuracy of reading of fabric sample peaks’ coordinates (fig. 1) and on determination of material deformational gradient $F$ (4).

## 3. Nonlinear Task

Six mechanical modules determine monoclinic anisotropy and can be expressed by this tensor:

$$\overline{E}_{ij} = \begin{pmatrix} \overline{E}_{11} & \overline{E}_{12} & \overline{E}_{14} \\ \overline{E}_{12} & \overline{E}_{22} & \overline{E}_{24} \\ \overline{E}_{14} & \overline{E}_{24} & \overline{E}_{4} \end{pmatrix}. \tag{16}$$

Strip over the modules indicates plane state of stress according to $\overline{E}_{ij} = \dfrac{E_{ij}}{\left(1-\nu_{ij}^2\right)}$, where $E_{ij}$ is modulus of triaxial state of stress, $\nu_{ij}$ is Poisson’s ratio.

Equation system for modules determination $\overline{E}_{ij}$:

$$\begin{aligned}
&\overline{E}_{11}\,\varepsilon_{11} + \overline{E}_{12}\,\varepsilon_{22} + 2\,\overline{E}_{14}\,\varepsilon_{12} - J_0\,\sigma_{11} = 0,\\
&\overline{E}_{12}\,\varepsilon_{11} + \overline{E}_{22}\,\varepsilon_{22} + 2\,\overline{E}_{24}\,\varepsilon_{12} - J_0\,\sigma_{22} = 0,\\
&\overline{E}_{14}\,\varepsilon_{11} + \overline{E}_{24}\,\varepsilon_{22} + 2\,\overline{E}_{4}\,\varepsilon_{12} - J_0\,\sigma_{12} = 0,\\
&\tan 2\omega\left(\overline{E}_{11} - \overline{E}_{22}\right) - 2\left(\overline{E}_{14} + \overline{E}_{24}\right) = 0,\\
&\tan 4\omega\left(\overline{E}_{11} + \overline{E}_{22} - 2\,\overline{E}_{12} - 4\,\overline{E}_{4}\right) - 4\left(\overline{E}_{14} - \overline{E}_{24}\right) = 0,\\
&\left(\overline{E}_{11}\,\overline{E}_{22} - \overline{E}_{12}^{2}\right)\left(\varepsilon_{11} - \varepsilon_{22}\right)^2 - \left[\overline{E}_{4}\left(\left(\varepsilon_{11} - \varepsilon_{22}\right)^2 + 4\,\varepsilon_{12}^2\right) -\right.\\
&\left.- 2\,J_0\,\sigma_{12}\,\varepsilon_{12}\right]\left(\overline{E}_{11} + \overline{E}_{22} + 2\,\overline{E}_{12}\right) = 0,
\end{aligned} \tag{17}$$

where $\tan 2\omega = \dfrac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}}$. Angle ω is between main anisotropy axis and load axis.

Shear stress is zero on the main axis. This enables to define angle ω. The E'14 and $\overline{\text{E}}$'24 are on this axes zero. Then it is valid $\overline{E}'_{14} + \overline{E}'_{24} = 0,\quad \overline{E}'_{14} - \overline{E}'_{24} = 0$ and from these conditions fourth and fifth equation follows (17). The following equation is used for isotropic materials:

$$G = \frac{\tau_i}{\gamma_i}, \tag{18}$$

Where the intensity of shear stress of plane state of stress $\tau_i = \dfrac{1}{\sqrt{6}}\sqrt{\left(\sigma_{11} - \sigma_{22}\right)^2 + \sigma_{11}^2 + \sigma_{22}^2 + 6\,\sigma_{12}^2}$

and the slope intensity $\gamma_i = \sqrt{\dfrac{2}{3}}\sqrt{\left(\varepsilon_{11} - \varepsilon_{22}\right)^2 + \left(\varepsilon_{22} - \varepsilon_{33}\right)^2 + \left(\varepsilon_{33} - \varepsilon_{11}\right)^2 + 6\,\varepsilon_{12}^2}$ are invariant. $G = \dfrac{E}{2\left(1+\nu\right)}$ is shear modules. Let's change modulus *G* for $\overline{K}_4$ in equation (18) and we get equation

$$\overline{K}_4 = \frac{1}{2}\left[\frac{1}{4}\left(\overline{E}_{11} + \overline{E}_{22} - 2\,\overline{E}_{12}\right) + \overline{E}_4\right] = \frac{J_0\,\tau_i}{\gamma_i}, \tag{19}$$

where just $\varepsilon_{33}$ is unknown. Deformation $\varepsilon_{33}$ is possible to express in dependence on conjugated pair

$$n\,\varepsilon_{33} = \left(1 + v_{33}\right)^n - 1. \tag{20}$$

Transverse proportion of fabric $h = h_0\left(1 + v_{33}\right)$ is in equation (20) dependent on choice of conjugated pair. If the method of fabric attenuation measurement (under defined loading) will be realized, then exponent „*n*“ and choice of conjugated pair can be done.

## 4. Conclusion

Determination of explicit conjugated pair depends on perfect biaxial loading apparatus, equipped with stepper, with possibility of fabric free welts movement during loading increment. This equipment is prepared. Development of methodic for fabric attenuation measurement (for given loading) is further condition. Only measurement of real change of fabric transverse proportion allows choosing suitable conjugated pair. These are conditions for fabric mechanical properties determination.


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