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MODELING OF DYNAMIC FRAGMENTATION: ONE-DIMENSIONAL CASE

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Abstract: *Dynamic fragmentation (disintegration of continuous body into smaller pieces under dynamic loading) is a physical phenomenon observed in many fields of engineering and science. The crack initiation and propagation combined with stress release waves makes the whole situation nontrivial and difficult to describe by analytical means. In this contribution, continuum damage mechanics and finite element method are used for numerical simulation of a ring (or cylindrical shell) subjected to radial loading at a high strain rate (e.g. explosive). The solution is simplified to the one-dimensional case and the results (average fragment size, fragment size distribution and energy dissipation) are compared for different finite element discretizations and for different perturbations of material properties. A comparison is also made between numerical results and analytical models including newly proposed ones.*

Keywords: ***Dynamic fragmentation, finite element method, mesh size dependency, average fragment size, convergence of dissipated energy.***

1. Introduction

A continuous body under rapid dynamic loading is, in contrast to the static case, usually split into many smaller pieces. Numerous studies of this phenomenon (dynamic fragmentation) have been focused on the case of an expanding ring (see Fig. 1), because it is simple to analyze both experimentally and numerically. In this contribution, we present two new analytical models for prediction of average fragment size, and also finite element simulations which are in certain aspects superior to other methods reported in literature.

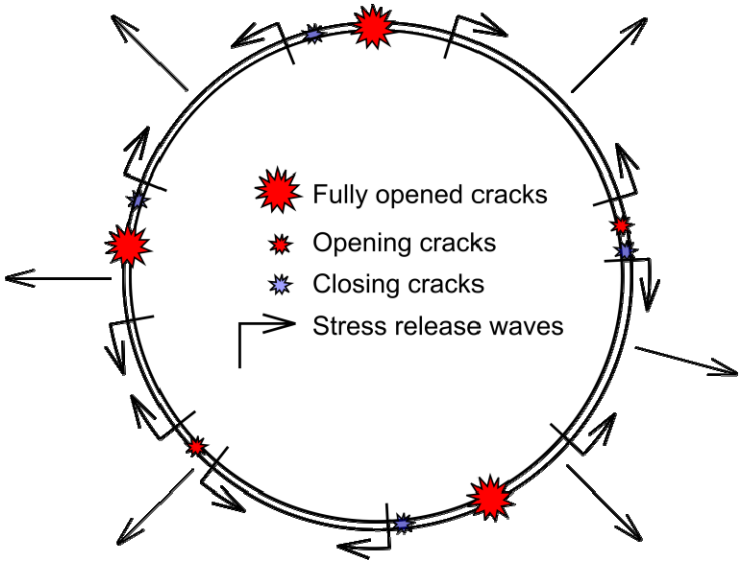

Fig. 1: Schematic representation of an expanding ring according to (Molinari et al., 2007).

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2. Analytical prediction of average fragment size

There exist several simple analytical models for predicting the average fragment size l_{avg} as a function of the strain rate $\dot{\varepsilon}$ and material parameters (e.g. Young's modulus E, bulk density ρ, fracture energy G_f, tensile strength σ_0). One of the most cited approaches is Grady's model (Grady & Olsen, 2003), which assumes that the fragment size is minimizes the sum of kinetic energy density $k = \frac{1}{24}\rho\dot{\varepsilon}l^2$ (Miller et al., 1999; Zhou et al., 2005) and surface energy "volume density" (energy consumed for the fragment creation divided by the fragment volume) $\bar{\gamma} = \frac{G_f}{l}$. This condition yields the estimate

$$l_{avg} = \sqrt[3]{12}\left(\frac{G_f}{\rho\dot{\varepsilon}^2}\right)^{\frac{1}{3}}. \tag{1}$$

The Glenn–Chudnovsky (1986) model (GC) balances the kinetic, surface and strain energy density $w_e = \frac{1}{2}\frac{\sigma_0^2}{E}$ (considered at the moment of fragment creation, therefore computed for tensile strength σ_0) and leads to

$$l_{avg} = 2\sqrt{\frac{\alpha}{3}}\sinh\left(\frac{\phi}{3}\right), \quad \text{where} \quad \phi = \sinh^{-1}\left[\beta\left(\frac{3}{\alpha}\right)^{\frac{3}{2}}\right], \quad \alpha = \frac{12\sigma_0^2}{\rho E\dot{\varepsilon}^2}, \quad \beta = \frac{12G_f}{\rho\dot{\varepsilon}^2}. \tag{2}$$

We propose two new analytical models. The first one is derived from a model presented by Yew & Taylor (1994) and therefore named "modified Yew–Taylor model" (modYT). It assumes, that the time when the peak stress is reached is negligible compared to the time of the whole fragmentation process $t_c = \frac{l}{2c}$, which is equal to half of the fragment length divided by the value of elastic wave speed $c = \sqrt{\frac{E}{\rho}}$. Minimizing the sum of kinetic, surface and strain energy density $w_e = \frac{1}{2}E\varepsilon = \frac{1}{2}E\dot{\varepsilon}t_c = \frac{1}{8}\rho\dot{\varepsilon}^2l^2$ results into

$$l_{avg} = \sqrt[3]{3}\left(\frac{G_f}{\rho\dot{\varepsilon}^2}\right)^{\frac{1}{3}}. \tag{3}$$

The second model is a combination of modYT and GC model, and therefore named YTGC. The assumptions are: the time of fragment creation is again $t_c = \frac{l}{2c}$, but it is small such that the stress distribution along the fragment is not affected and is equal to σ_0. Balancing of strain energy density increment $\Delta w_e = \sigma_0\dot{\varepsilon}t_c = \sigma_0\dot{\varepsilon}\frac{1}{2c}l$ and surface energy density (as well as minimization of their sum) results into

$$l_{avg} = \sqrt{\frac{2G_fc}{\sigma_0\dot{\varepsilon}}}. \tag{4}$$

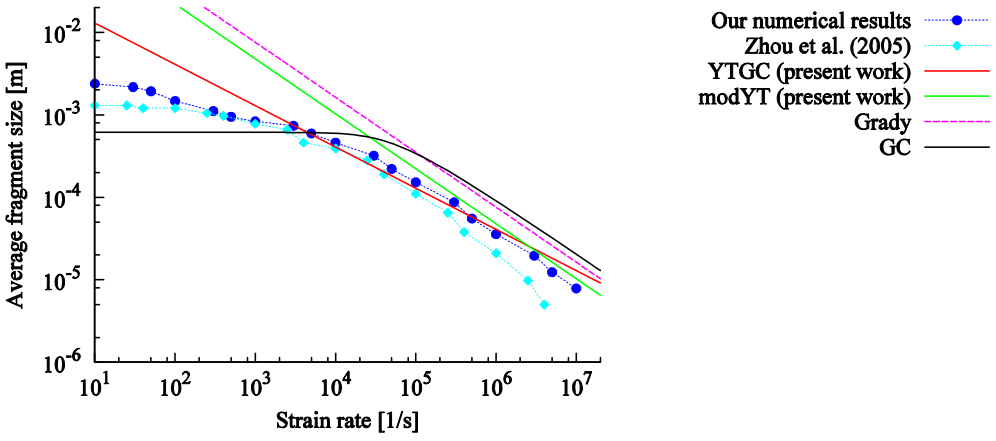

Fig. 2: Comparison of different analytical models and numerical results.

For comparison of analytical models, the following material properties are considered: E = 275 GPa, ρ = 2750 kg/m^3, G_f = 100 J/m^2, σ_0 = 300 MPa. The results are summarized in Fig. 2. Clearly, for the present set of material parameters the newly proposed models modYT and YTGC predict the average fragment size better than previously existing models (Grady and Glenn–Chudnovsky)

3. Numerical solution

The expanding ring is simulated as a one-dimensional bar (approximately representing a certain small part of the ring). The numerical solution is based on the finite element method (FEM) with linear approximation of the displacement field. Cracking is modeled in the smeared manner using 1D damage mechanics with linear softening. The initial condition prescribes a constant strain rate (and therefore linear velocity along the bar length). To "macroscopically retain" the constant strain rate, the velocity of boundary nodes is prescribed by a constant value for the whole simulation, see Fig. 3.

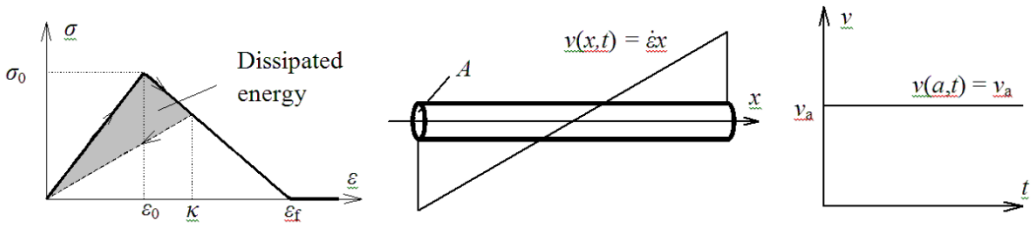

Fig. 3: Considered material model and initial and boundary conditions.

For an ideal rod with all material parameters constant along its length, a uniform solution exists but is instable (similar to the static case). To eliminate such solution, it is sufficient to slightly perturb the material parameters. The simplest approach is to define the material parameter value independently in each element (to consider the parameter values as uncorrelated). Physically more realistic is to assume a certain spatial correlation of material parameters. We assume a simple exponential covariance function $C(x_1, x_2) = \sigma^2 e^{|x_1 - x_2|/r}$, where σ is the standard deviation and r is the correlation length.

4. Results

Convergence of the results with increasing number of elements for different uncorrelated perturbations is shown in Fig. 4. Corves marked by A correspond to 1% perturbation of the cross-section area, curves marked by σ_0 to 1% perturbation of the tensile strength and curves marked by l_e to 40% perturbation of element length. Results from an ideal rod exhibit non-monotonic convergence (given by the instability of results, which is triggered by round-off error in if the number of elements is sufficiently high). Fig. 5 shows the convergence for different strain rates. Finally, convergence results considering spatial correlation are shown in Fig. 6. Dissipated energy is higher for a higher correlation length.

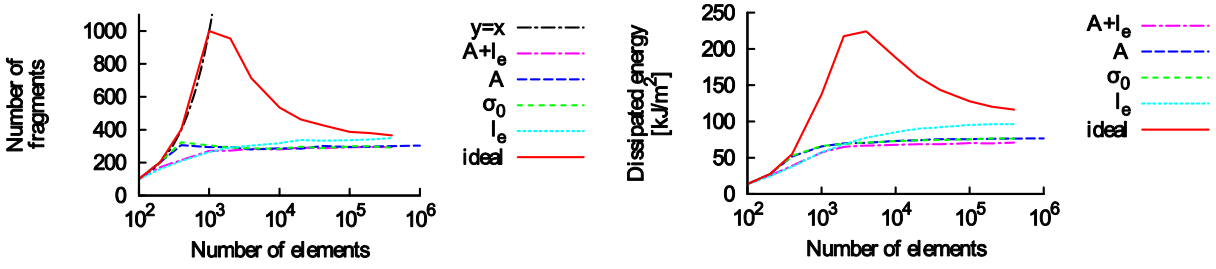

Fig. 4: Number of fragments (left) and dissipated energy (right) for $\dot{\varepsilon} = 10^5\ s^{-1}$ and for various uncorrelated perturbations.

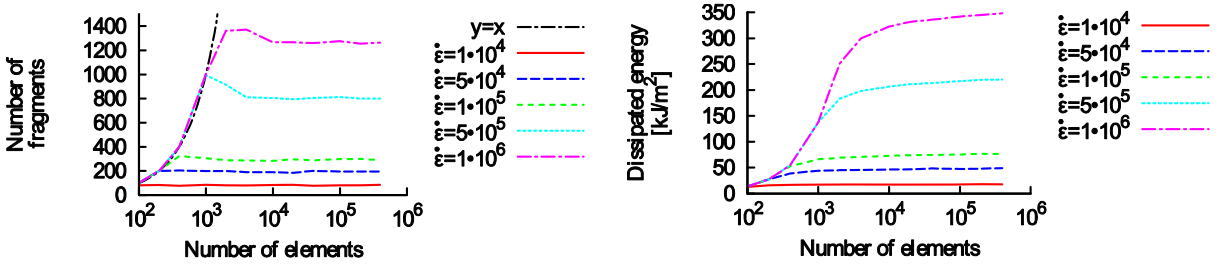

Fig. 5: Number of fragments (left) and dissipated energy (right) for uncorrelated 1% perturbation of tensile strength σ_0 and for various strain rates $\dot{\varepsilon}$.

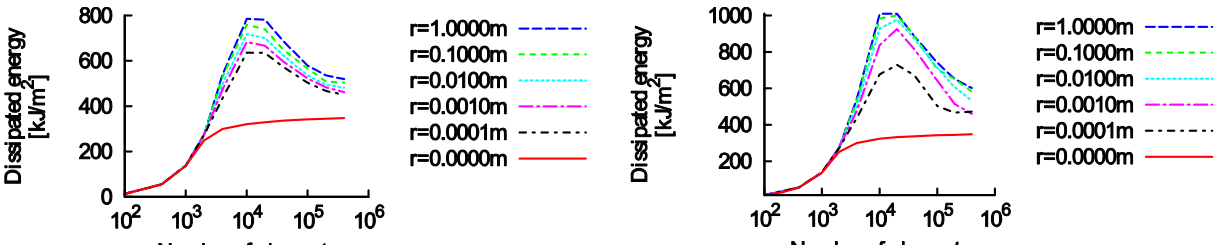

Fig. 6: Dissipated energy for $\dot{\varepsilon} = 10^6\ s^{-1}$, various correlation length and 1% perturbation of cross–section area A (left) and tensile strength σ_0 (right).

5. Conclusions

The present article deals with dynamic fragmentation of materials under high strain rates, with focus on the case of an expanding ring. The main results can be summarized as follows:

- New analytical models for prediction of average fragment size are proposed. In comparison with existing models from the literature they agree better with numerical results.
- Combination of the finite element method and damage mechanics leads to monotonic convergence of dissipated energy. This is in contrast to the method of characteristics combined with cohesive elements using 1% perturbation of strength presented by Molinari et al. (2007), which gives non-monotonic convergence of dissipated energy.
- For a bar with a spatial correlation of material properties corresponding to a finite correlation length, the results are almost always between two extremes – the ideal bar (equivalent to an infinite correlation length) and uncorrelated parameters (equivalent to zero correlation length).

Future work on this topic will address in more detail the influence of spatial correlation (including a larger standard deviation), validation of newly proposed analytical models for different material parameters, extension of the analysis to more dimensions, etc.

Acknowledgement

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