

# IMPELLER BLADE DESIGN BASED ON THE DIFFERENTIAL GEOMETRY

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Abstract: A new method of blade design based on the previously determined meridional velocity field is introduced in this contribution. The aim of calculation is the spatial shape of the blade, based on the field of  $\beta$  angles across the blade. This field, obtained through the system of equations, originates from differential geometry principles. The blade angles prescribed on the leading and the trailing edges serve as the boundary conditions.

Keywords: Impeller blade, meridional section, curvilinear coordinate system, differential geometry.

## 1. Introduction

The commonly used way of pump designing consists of more or less interconnected steps. The results of calculations come from various numerical methods and are interpolated with simple or weighted polynomials or splines. Generally, sophisticated flow simulation program is not used until the basic design is finished. The centrifugal pumps are widely used in various applications and there are also intentions to enter new features (Choi et al., 2006). Large efforts were also made to streamline the process of pump design and to maximize the pump efficiency using CFD (Anagnostopoulos, 2009). Another field of improvements was defining the appropriate designing methods for the different ranges of pump usage (Kyparissis et al., 2009) or seeking for the designs with the best characteristics (Zhou et al., 2003).

The improved design approach, part of which is presented below, is based on the differential geometry in the curvilinear coordinate system. Its main aim is the interconnection of pump components based on the assumptions of flow in the early stages of the design. It would allow prevention of errors made in the initial stages of design, which consequently means time savings. Below presented paper deals with one part of the pump design, the blade design. The blade shape is closely connected with previously performed meridional cut design associated with determination of the meridional velocity.

### 2. Meridional velocity

The cornerstone of succesfull blade design is properly designed meridional section. The applied method uses meridional section defined as Bezier surface. Data computed in the intersections of the Bezier surface curves serve for the characterization of flow.

Consider curvilinear coordinate system  $u, v \in \langle 0, 1 \rangle$ ;  $w = \varphi \in \langle 0, 2\pi \rangle$  (Brdička et al., 2005).  $\mathbf{g}_i$  denotes the vector tangent to the appropriate coordinate curve, see Fig. 1. The coordinate curves are not generally orthogonal. The calculations presented below require mutually orthogonal vectors  $\mathbf{g}_i$ . Such orthogonalization can be performed by following equation, (Sloupenský, 2010):

$$\mathbf{c}(t) = \left(\mathbf{g}_1 - \frac{\mathbf{g}_1 \cdot \mathbf{g}_2}{\mathbf{g}_2 \cdot \mathbf{g}_2} \mathbf{g}_2\right) \frac{du}{dt}$$
(1)

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Initial assumption and boundary conditions:

$$\mathbf{c} \cdot \mathbf{g}_2 = \mathbf{0} \tag{2}$$

$$v = v_0; u = 0 \tag{3}$$



Fig. 1: Curvilinear coordinate system.

The interior of the meridional section is determined by iterative method of progressive selection according to required flow in the specific area of the impeller. It is possible to obtain required values of meridional velocity in different parts of the impeller through this procedure. The flow assumptions of rotational, quasi-potential and potential flow and adjustment of the flow within the curvilinear coordinate system are described in (Sloupenský & Pochylý, 2010). Equations which determine the system of coordinate curves for potential, rotational and quasi-potential flow are presented in this reference.

#### 3. Spatial form of the blade

The blade proper performance is ensured by its spatial shape. This is based on the  $\beta$  angles of the velocity triangles in each of the points of the blade. It is necessary to know some initial conditions to define those angles. The  $\beta$  angles entered in six points on the blade serve for this purpose. Three of them are specified on the leading edge and three of them on the trailing edge, see Fig. 2.



Fig. 2: Location of the entered points.

To calculate angles across the blade the polynomial (4) is used. Changes in the longitunidal direction v are linear and in the traverse u quadratic. Experiments were also carried out with quadratic definition in the longitudinal direction. However, the results were poor.

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$$\beta(u,v) = A_1 + A_2 u + A_3 v + A_4 uv + A_5 u^2 + A_6 u^2 v$$
(4)

There are six unknown variables in the previous equation. The prescribed values of  $\beta$  angles shall serve as the boundary conditions. It is recommended to take into account the total length of the blade during the procedure. Excessive length could cause substantial friction losses.



(5)

Fig. 3: Example of calculated values.

Lets say, surface u = const. It is possible to design the blade shape based on this assumption and with known  $\beta(u, v = const.) = \beta(v)$ , see Fig. 3.



Fig. 4: Calculation of the blade from the trailing edge.

$$\tan \beta = \frac{ds}{yd\varphi} = -\frac{|\mathbf{g}_2|du}{yd\varphi} \tag{6}$$

$$\frac{d\varphi}{du} = -\frac{|\mathbf{g}_2|}{y} \cot \beta \tag{7}$$

Boundary conditions:

$$u = u_i \tag{8}$$

$$\varphi = f(\zeta) \tag{9}$$

$$y = r_2 \tag{10}$$

Where  $\zeta$  is prescribed trailing edge angle.



Fig. 5: Example of the blade curvature on the shroud (black line) and on the hub (red line).

#### 4. Conclusions

The computational procedure introduced in this paper offers the possibility to control the blade spatial shape. The blade spatial shape is controlled by the values of the blade angles prescribed on the leading and the trailing edges and by the trailing edge angle. The blade shape is also interconnected with previously designed meridional section. The initial values mostly serve only for the preliminary blade rendering. The following procedure is the step by step adjustment of the prescribed values until the desired shape is obtained. It is possible to transfer the final blade shape into one of the flow simulation programs.

#### Acknowledgement

Support of following grant projects is gratefully acknowledged:

FR-TI1/418 Research of impeller pumps

The specific research for the year 2011 supported by Faculty of Mechanical Engineering, Brno University of Technology.

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