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ANALYSIS OF THE EFFECT OF HYDRODYNAMIC BEARINGS AND DAMPING ON ROTOR STABILITY

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Abstract: *The paper deals with the modeling and evaluation of stability conditions of the balanced rotating system with rigid discs supported on two anisotropic hydrodynamic bearings. The model respects internal and external damping of the shaft considered as one-dimensional continuum. The evaluation of stability is based on eigenvalues in dependence on system rotating speed. Real and imaginary parts of computed eigenvalues are displayed by means of Campbell diagram.*

Keywords: ***Stability, internal damping, external damping, eigenvalues, rotating system.***

1. Introduction

The most of real mechanical systems are continuous and non-homogenous elastic systems. Therefore an approximation, which helps to simplify the description of a real system behavior with enough accuracy, is used. For that reason, it is necessary to specify material and geometric properties of a real system and the impact of each characteristic on its behavior. The damping effects are these properties which are necessary to include into a mathematical model. In case of rotating systems there are two kinds of damping effects. It is so-called external damping effect that is dependent on surroundings of the system and the internal damping effect caused by material properties of the system. Both these damping impacts cause the change of model parameters and the system stability.

The aim of this paper is to present the impact of external and internal damping on the stability of a rotating flexible shaft with rigid discs supported on hydrodynamic bearings.

2. Mathematical model of a rotor

We suppose the rotor which rotates with constant angular velocity ω_0. It is composed of a circular cross-section shaft with continuously distributed mass. The rigid discs are attached to the shaft in given positions. The shaft with discs is supported by two identical hydrodynamic bearings. The layout of test rotor is shown in Fig. 1.

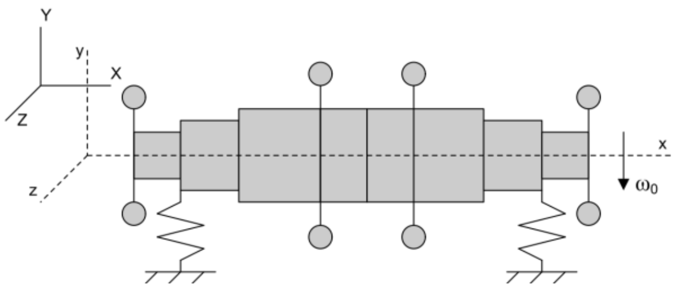

Fig. 1: Scheme of the rotor supported on hydrodynamics bearings.

The mathematical model of the rotor bending vibration is created by the finite element method (FEM), which is based on a shaft division into finite elements (shaft elements). The shaft element *e* of length *l* is defined by two end nodes *i* and *i+1*. The non-deformable cross-section area of each shaft element is considered and it is still perpendicular to the deformed shaft axis after deformation. Therefore the

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motion of the shaft is described by two displacements v, w in Y, Z direction and two angular displacements ϑ, ψ around Y, Z axis. For each shaft finite element, the coordinates are arranged in the vector $\tilde{\mathbf{q}}^{(e)} = \begin{bmatrix} v_i & \psi_i & v_{i+1} & \psi_{i+1} & w_i & \vartheta_i & w_{i+1} & \vartheta_{i+1} \end{bmatrix}^T$. The mass, gyroscopic and stiffness finite element matrices $\mathbf{M}^{(e)}$, $\mathbf{G}^{(e)}$, $\mathbf{K}^{(e)}$ are derived by Lagrange´s equations using an expression for kinetic and potential energy of the shaft element in fixed configuration space *XYZ*. The rigid disc placed in i^{th} node is described by the mass and gyroscopic matrices $\mathbf{M}^{(d)}$, $\mathbf{G}^{(d)}$. All these matrices describing shaft elements and discs are derived according to Slavik et al. (1998).

The anisotropic support located in i^{th} node of the shaft represents the fluid film bearing. Considering the linear relation between forces generated in fluid film, each support can be characterized by the stiffness and damping matrices $\mathbf{K}_{B_i}(\omega_0)$, $\mathbf{B}_{B_i}(\omega_0)$. Their coefficients depend on angular velocity ω_0 according to Muszynska (2005).

2.1. Damping effects

Considering the isotropic external damping the forces are perpendicular to the finite element surface. If the bending vibration is supposed then the external damping impact results from the Rayleigh dissipation function. The external damping in fixed coordinates *XYZ* is expressed by means of external damping matrix

$$\mathbf{B}_E^{(e)} = b_E \begin{bmatrix} \mathbf{S}_1^{-T} \mathbf{I}_\Phi \mathbf{S}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_2^{-T} \mathbf{I}_\Phi \mathbf{S}_2^{-1} \end{bmatrix}, \tag{1}$$

where b_E [kgm^{-1}s^{-1}] is external isotropic damping coefficient per unit length.

In case of internal isotropic damping we assume that internal damping forces are induced by shaft deformation and are of viscous character. Therefore the internal damping forces effect could be expressed in the rotating coordinates *xyz* by means of the internal damping matrix

$$\mathbf{B}_I^{(e)} = b_I EJ \begin{bmatrix} \mathbf{S}_1^{-T} \mathbf{I}_{\Phi''} \mathbf{S}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_2^{-T} \mathbf{I}_{\Phi''} \mathbf{S}_2^{-1} \end{bmatrix}, \tag{2}$$

where b_I[s] is coefficient of viscous internal damping, E [Pa] is Young´s modulus of elasticity and J [m^4] is cross-section area polar moment. Note that $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{I}_\Phi$ and $\mathbf{I}_{\Phi''}$ are constant coefficient matrices of order four derived in Byrtus et al. (2011). But the matrix (3) must be transformed from rotating frame *xyz* to the fixed frame *XYZ* by using a transformation matrix $\mathbf{R}(t)$

$$\mathbf{R}(t) = \begin{bmatrix} \cos(\omega_0 t)\cdot \mathbf{E} & \sin(\omega_0 t)\cdot \mathbf{D} \\ -\sin(\omega_0 t)\cdot \mathbf{D} & \cos(\omega_0 t)\cdot \mathbf{E} \end{bmatrix}, \tag{3}$$

where $\mathbf{D} = \mathrm{diag}(-1 \quad 1 \quad -1 \quad 1)$, $\mathbf{E} = \mathrm{diag}(1 \quad 1 \quad 1 \quad 1)$. Then the relation between a vector of generalized coordinates $\hat{\mathbf{q}}^{(e)}(t)$ in rotating frame *xyz* and vector of generalized coordinates $\mathbf{q}^{(e)}(t)$ in fixed frame *XYZ* is $\hat{\mathbf{q}}^{(e)}(t) = \mathbf{R}^T(t)\mathbf{q}^{(e)}(t)$, the internal damping forces in fixed frame could be expressed as

$$\mathbf{R}(t)\mathbf{B}_I^{(e)}\dot{\hat{\mathbf{q}}}^{(e)}(t) = \mathbf{R}(t)\mathbf{B}_I^{(e)}\mathbf{R}^T(t)\dot{\mathbf{q}}^{(e)}(t) + \mathbf{R}(t)\mathbf{B}_I^{(e)}\dot{\mathbf{R}}^T(t)\mathbf{q}^{(e)}(t), \tag{4}$$

where $\mathbf{R}(t)\mathbf{B}_I^{(e)}\mathbf{R}^T(t) = \mathbf{B}_I^{(e)}(t)$ is so-called dissipation matrix and $\mathbf{R}(t)\mathbf{B}_I^{(e)}\dot{\mathbf{R}}^T(t) = \mathbf{K}_I^{(e)}(t)$ is so-called circulatory matrix. It was demonstrated that for the isotropic shaft element these two matrices are constant in time.

Therefore the mathematical model of the shaft element respecting damping effect in fixed frame *XYZ* could be expressed as

$$\mathbf{M}^{(e)}\ddot{\tilde{\mathbf{q}}}^{(e)}(t) + \left(\mathbf{B}_E^{(e)} + \mathbf{B}_I^{(e)} + \omega_0 \mathbf{G}^{(e)}\right)\dot{\tilde{\mathbf{q}}}^{(e)}(t) + \left(\mathbf{K}^{(e)} + \mathbf{K}_I^{(e)}\right)\tilde{\mathbf{q}}^{(e)}(t) = \mathbf{0}. \tag{5}$$

2.2. Mathematical model of a whole rotating system

Mathematical model of the whole rotating system with bearing supports is derived in the space with the configuration $\mathbf{q}_i = [\ldots \quad v_i \quad w_i \quad \vartheta_i \quad \psi_i \quad \ldots]^T$, therefore matrices describing the shaft element must be transformed from the configuration space defined by vector $\tilde{\mathbf{q}}^{(e)}$ to the new configuration space. Then the equation of the motion could be expressed as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{B}_E + \mathbf{B}_I + \mathbf{B}_B(\omega_0) + \omega_0 \mathbf{G})\dot{\mathbf{q}}(t) + (\mathbf{K} + \mathbf{K}_I + \mathbf{K}_B(\omega_0))\mathbf{q}(t) = \mathbf{0}, \tag{6}$$

where each matrix is composed of transformed block matrices and ω_0 is an angular velocity of the rotation. Matrices $\mathbf{B}_B(\omega_0)$ and $\mathbf{K}_B(\omega_0)$ are global damping and stiffness matrices of the hydrodynamic bearings.

3. Application

The model respecting damping effect was tested using the rotor supported on two hydrodynamics bearings. The shaft is derived into eight shaft elements and equipped with four discs (see Fig. 1). The shaft elements are described by length l [m], material density ρ = 7850 kgm^{-3}, Young´s modulus of elasticity E = 2.10^{11} Pa, outer diameter D [m] and inner diameter d [m]. The rigid discs are characterized by mass m [kg], moment of inertia with respect to lateral axis I_0 [kgm^2] and moment of inertia with respect to axis of symmetry I [kgm^2]. These parameters are written in Tab. 1.

Tab. 1: Parameters of the shaft elements (on the left) and discs (on the right).

element	D [m]	d [m]	l [m]
1	0.43	0	0.9
2	0.53	0	1.12
3	0.735	0	0.52
4	0.735	0	0.995
5	0.735	0	0.925
6	0.735	0	0.52
7	0.53	0	1.12
8	0.43	0	0.75

node	m [kg]	I_0 [kgm^2]	I [kgm^2]
1	670	78.88	39.44
4	549	1035	517.5
6	549	1035	517.5
9	670	78.88	39.44

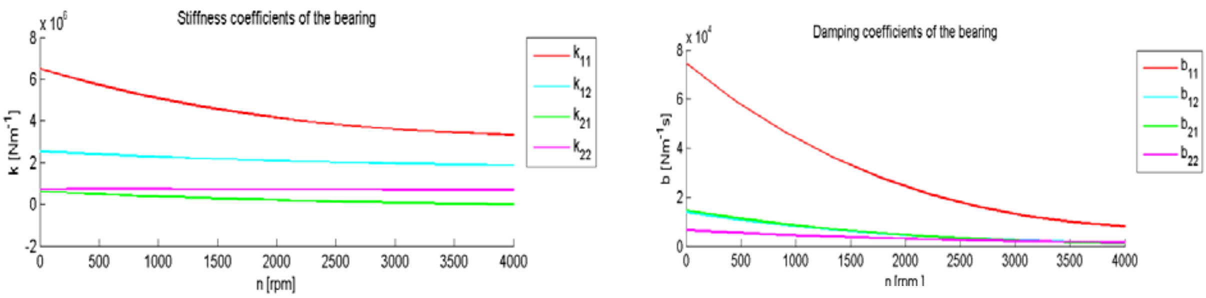

Fig. 2: Stiffness and damping coefficients of hydrodynamic bearings dependent on rotating speeds n.

The bearing stiffness and damping coefficients are approximated by cubic polynomials in dependence of rotating speed $n \in \langle 0{,}4000 \rangle$ rpm. see Fig. 2.

According to Gasch & Pfützner (1980) the internal damping coefficient is set b_I = 0.01 s and the external damping coefficient is set as b_E = 0.1 kgm^{-1}s^{-1}.

The eigenvalues are obtained by solving the eigenvalue problem and the Campbell diagrams expressing the dependence of eigenvalue imaginary parts on the rotor speed n for both systems are shown in Fig. 3. The eight eigenvalues with the smallest positive imaginary parts are written in Tab. 2.

Tab. 2: The eigenvalues with eight smallest imaginary parts corresponding with the system without damping effect and system considering damping effect for n=2000 rpm.

eigenvalue	$b_I = 0$ s. $b_E = 0$ $kgm^{-1}s^{-1}$	$b_I = 0.01$ s. $b_E = 0.1$ $kgm^{-1}s^{-1}$
λ_1	-2.19+3.27i	-2.20+3.27i
λ_2	-0.03+8.02i	-0.03+8.02i
λ_3	-1.32+20.13i	-1.32+20.14i
λ_4	-0.17+71.29i	-0.17+71.29i
λ_5	-0.57+335.77i	-2.92+335.76i
λ_6	-0.44+395.30i	-3.70+395.29i
λ_7	-0.06+627.60i	-11.97+627.51i
λ_8	-0.03+739.28i	-14.18+739.20i

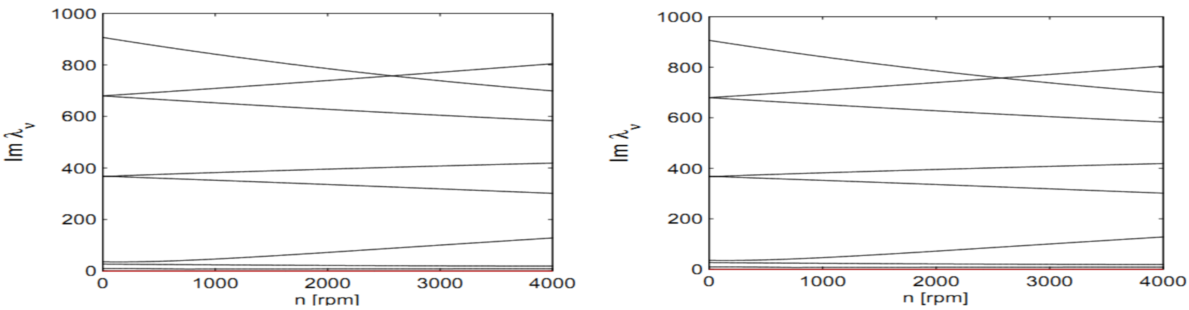

Fig. 3: Campbell diagrams for system without damping effect (on the left) and system with damping effect (on the right).

4. Conclusions

This paper presents the basic dynamic model of the rotating system supported on two hydrodynamic bearings respecting damping effects. The finite element approach was used and the shaft was modeled by means of eight shaft elements and four rigid discs are attached to the shaft in chosen nodes. The hydrodynamic bearings are modeled as supports with stiffness and damping coefficients depending on the rotating speed *n*.

The analysis of the damping effects was performed for rotating speed $n \in \langle 0,4000 \rangle$ rpm. whereas the operating rotating speed is $n = 2000$ rpm. Based on eigenvalues corresponding to both cases of model see Tab. 2. it is obvious that the real parts of eigenvalues of the system with damping are smaller than real parts of eigenvalues of the system without damping. It was shown that external damping causes a stabilizing effect in the analyzed rotating speed range. On the other hand. the internal damping affects the eigenvalues of the system minimally. The conclusions shown in this paper are valid in case of rotor systems supported on hydrodynamic bearings with relatively large values of damping coefficients.

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