17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

SIMULATION OF VIBROIMPACT ROTOR-SEALING RING SYSTEM

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Abstract: *Mathematical model describing the vibratory impact motion of rotor and floating sealing ring is suggested. Hertz's theory is used. The model allows studying the stationary and nonstationary oscillations of rotor and ring.*

Keywords: ***Rotor, shaft, floating seal, impact.***

1. Introduction

In the new liquid-propellant rocket engines a fuel feed is fulfilled by turbine pumps of the latest generation (Patent of Russian Federation). Main working part of these machines is a rotor with floating seals which is the rings with comparatively high stiffness and established with small radial clearance (0.05…0.1 mm) (Childs, 1993). A short-term drift of rotor vibration above acceptable limits often causes the inflammation of turbine pumps due to the contacts between rotor and seals (Gurov & Shestakov, 2000). Vibration problem becomes more sharply in case of the flexible rotor. Lateral (bending) oscillations of rotor rotating with angular velocity close to the natural frequency are attended by considerable amplitudes. Obviously, the breakdown risk may be come to zero by means of theoretical research for vibratory impact regimes in the rotor-sealing ring system on the basis of correct mathematical model.

Choice of the impact model for solving of assigned task is made in agreement with the following. Local deformations occur in the rotor-sealing ring system at the impacts because an impact velocity and a hardness of system bodies correspond to the low-velocity impact not causing the bodies interpenetration. It is possible to neglect an effect of elastic oscillations and to consider a character of contact interactions at the impact same as in a static condition because a period of the slowest natural oscillations for the colliding bodies or a time of elastic waves passing in the bodies is much less than a impact duration. Such assumptions underlie static impact theory from which Hertz and other authors proceeded (Babitsky & Krupenin, 2001; Hunt & Grossley, 1975). The discrete models of impact describing deformations partially are its foundation. In these models it is supposed that bodies motion during impact is described by the differential equations of motion of a solid (by the equations «force - acceleration» $P_{\text{y}} = m\ddot{x}$) which rather easily are solved by the known methods. Forces P_{y} acting during contacting take into account the viscid and elastic properties of real bodies and are modeled by a set of springs and dampers.

2. Mathematical model of rotor-sealing ring system

Considering constructive and dynamic features, the motion equations of researched mechanical system (Fig.1) should be written down taking into account the mass and inertia moment of rotor m and J, torsion torque M_0, rotor imbalance $a = O_1G$, equivalent stiffness of the shaft and supports $k = k_{\text{оп}}k_{\text{в}}/(k_{\text{оп}} + k_{\text{в}})$, hydrodynamic and impact forces as components P_n and P_τ initiated by the floating sealing ring $m_{\text{к}}$ which is pressed to the frame $\Pi_{\text{к}}$ of turbine pump by the pressure difference $\Delta p = p_1 – p_2$.

Due to small length of ring (in comparison with length of rotor) along the axis z, it is possible to suppose the rotor motion in the ring is flat. Thus let x, y are coordinates for an axis of rotor O_1, $x_{\text{к}}$, $y_{\text{к}}$

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are coordinates for an axis of ring O_2, x_G, y_G are coordinates for the line penetrating through the centre of rotor gravity G, θ is polar angle of centers line (angle between relative displacement $e=O_1O_2$ and axis x).

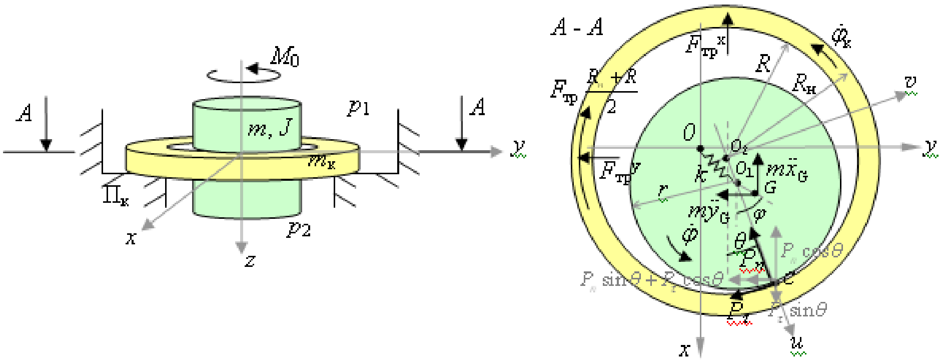

Fig. 1: Dynamical model for rotor with floating sealing ring.

Considering the model, it is possible to notice that the point O_1 doesn't have a mass but has the inertia moment J, the forces $P_\tau \sin\theta$, $P_n \cos\theta$, $m\ddot{x}_G$ and kx act in the direction x, the forces $P_n \sin\theta$, $P_\tau \cos\theta$, $m\ddot{y}_G$ and ky act in the direction y. Besides around the point O_1 the moments M_0, $m\ddot{x}_G \cdot a\sin\varphi$, $m\ddot{y}_G \cdot a\cos\varphi$ and $P_\tau r$ operate. Thus, the motion equations for a rotor are:

$$\begin{cases} 0 = -m\ddot{x}_G - kx - P_n\cos\theta + P_\tau \sin\theta \\ 0 = -m\ddot{y}_G - ky - P_n\sin\theta - P_\tau \cos\theta \\ J\ddot{\varphi} = M_0 + ma\left(\ddot{x}_G\sin\varphi - \ddot{y}_G\cos\varphi\right) - P_\tau r \end{cases}, \tag{1}$$

where φ is an angular displacement of rotor gravity center or a rotation angle of the rotor. Coordinates x, y are connected to coordinates x_G, y_G by the expressions $x_G = x + a\cos\varphi$ and $y_G = y + a\sin\varphi$ so: $\ddot{x}_G = \ddot{x} - a\dot{\varphi}^2\cos\varphi - a\ddot{\varphi}\sin\varphi$ and $\ddot{y}_G = \ddot{y} - a\dot{\varphi}^2\sin\varphi + a\ddot{\varphi}\cos\varphi$. Substituting these expressions in (1), it is easy to transform the received motion equations to homogeneous and standard form for the oscillations theory:

$$\begin{cases} m\ddot{x} + b\dot{x} + kx = ma\left(\dot{\varphi}^2\cos\varphi + \ddot{\varphi}\sin\varphi\right) - P_n\cos\theta + P_\tau\sin\theta \\ m\ddot{y} + b\dot{y} + ky = ma\left(\dot{\varphi}^2\sin\varphi - \ddot{\varphi}\cos\varphi\right) - P_n\sin\theta - P_\tau\cos\theta \\ \left(J + ma^2\right)\ddot{\varphi} + ma\left(\ddot{y}\cos\varphi - \ddot{x}\sin\varphi\right) = M_0 - P_\tau r \end{cases} \tag{2}$$

The inevitable friction obstructing real rotor motion as a linear viscous force with proportionality coefficient b is brought here too.

In turbo-pumps the torque moment M_0 depends on pressure of gases in front of the turbine and angular velocity $\dot{\varphi}$. This dependence is usually set as the joint characteristic of the turbine (the driving torque M_1) and the pump (the resisting moment M_2 induced by working load) (Grobov, 1959):

$$M_0 = M_1 - M_2 = M_n\left(2 - \dot{\varphi}/\omega_n\right) - M_n\left(\dot{\varphi}/\omega_n\right)^2 \tag{3}$$

where M_n and ω_n is nominal torque moment and nominal angular speed accordingly.

Definitions for external forces in the equations system (2) are:

$$P_n = \begin{cases} k_h e, \text{ при } e < \delta \\ k_h e + P_y, \text{ при } e > \delta \end{cases}, \quad P_\tau = \begin{cases} (\dot{\theta} - 0.5\dot{\varphi})d_h e, \text{ при } e < \delta \\ (\dot{\theta} - 0.5\dot{\varphi})d_h e + fP_y, \text{ при } e > \delta \end{cases} \tag{4}$$

where $\delta=R-r$ is value of radial clearance, $k_h e$ is hydrodynamic normal force, $(\dot{\theta} - 0.5\dot{\varphi})d_h e$ is hydrodynamic tangential force, P_y is impact force between rotor and ring, f is coefficient of sliding friction.

Here the model of "short seal» (Simonovsky, 1986) is used, that is allowable proceeding from the real sizes of floating sealing rings. Accordingly the hydrodynamic stiffness and damping of ring are:

$$k_{\text{h}} = \frac{\pi L R \eta}{2\delta(1+\eta)^2}\Delta p\ ,\ d_{\text{h}} = \frac{\pi \mu_{\text{c}} k_z L^3 R}{12\delta^3} \tag{5}$$

where $\eta = 75\delta/L$, L is length of sealing surface of ring, μ_{c} is fluid viscosity, $k_z = 0.005\text{Re}$ is turbulence coefficient dependent on the Reynolds' number.

It is important to note also that in the turbo-pumps a pressure difference at the seals of rotor wheels depends on a pressure of turbine stage which is proportional to a square of angular velocity of rotor $\Delta p = \Delta p_{\text{n}}\, \omega^2/\omega_{\text{n}}^2$, where Δp_{n} is the pressure difference appropriate to nominal angular velocity ω_{n}. In this connection, hydrodynamic stiffness of sealing rings changes on dependences of kind $k_{\text{h}} = k_{\text{h}}^{\text{n}}\, \omega^2/\omega_{\text{n}}^2$, where k_{h}^{n} is value for ω_{n}. The hydrodynamic damping of sealing rings is proportional to number $\text{Re} = 2\rho\delta w/\mu_{\text{c}}$, where ρ is fluid density, $w = \sqrt{2\Delta p\eta/\rho}$ is fluid velocity in the rings clearance. From here, hydrodynamic damping is proportional to rotor velocity $d_{\text{h}} = d_{\text{h}}^{\text{n}}\, \omega/\omega_{\text{n}}$.

Known Hertz's formula is used for definition of the impact force:

$$P_{\text{y}} = K e_{\max}^{3/2} = K^{2/5}\left(\frac{5}{4}MV^2\right)^{3/5} \tag{6}$$

where $K = 4/(3\eta)\sqrt{Rr(R-r)^{-1}}$ is coefficient dependent on materials $\eta = (1-\mu_{\text{р}}^2)E_{\text{р}}^{-1} + (1-\mu_{\text{к}}^2)E_{\text{к}}^{-1}$ and curvature of rotor and ring surfaces at the contact point, $\mu_{\text{р}}$, $E_{\text{р}}$, $\mu_{\text{к}}$ and $E_{\text{к}}$ are Poisson's ratio and elastic modules for materials of rotor and ring, $e_{\max}$ is the maximal approach of their centers, $M = mm_{\text{к}}/(m+m_{\text{к}})$ is equivalent mass, $V=\dot{e}$ is approach velocity for the centers of rotor and ring before impact.

The motion equations of ring as a solid also can be written down proceeding from d'Alembert principle and taking into account that to the ring are applied same external forces P_n and P_τ but other sign. Then (see Fig.1) the point O_2 has mass $m_{\text{к}}$ and inertia moment $J_{\text{к}}$, the forces $P_n\cos\theta$, $P_\tau\sin\theta$ and $F_{\text{тр}}^{\text{x}}$ act in the direction x, the forces $P_n\sin\theta$, $P_\tau\cos\theta$ and $F_{\text{тр}}^{\text{y}}$ act in the direction y. Besides around ring axis the moments $P_\tau R$ and $F_{\text{тр}}(R_{\text{н}}+R)/2$ operate. Thus, the required motion equations of sealing ring are:

$$\begin{cases} m_{\text{к}}\ddot{x}_{\text{к}} = P_n\cos\theta - P_\tau\sin\theta - F_{\text{тр}}^{\text{x}} \\ m_{\text{к}}\ddot{y}_{\text{к}} = P_n\sin\theta + P_\tau\cos\theta - F_{\text{тр}}^{\text{y}} \\ J_{\text{к}}\ddot{\varphi}_{\text{к}} = P_\tau R - F_{\text{тр}}(R_{\text{н}}+R)/2 \end{cases} \tag{7}$$

where $F_{\text{тр}}^{\text{x}} = F_{\text{тр}}\dot{x}_{\text{к}}\big/\sqrt{\dot{x}_{\text{к}}^2+\dot{y}_{\text{к}}^2}$, $F_{\text{тр}}^{\text{y}} = F_{\text{тр}}\dot{y}_{\text{к}}\big/\sqrt{\dot{x}_{\text{к}}^2+\dot{y}_{\text{к}}^2}$, $F_{\text{тр}} = fN$, $N=F_{\text{пр}}+\Delta p S_{\text{тр}}+m_{\text{к}}g$, $S_{\text{тр}}=\pi(R_{\text{н}}^2-R^2)$ is friction surface of ring about the turbo-pump frame, g is gravitational acceleration, $F_{\text{пр}}$ is force of an axial spring (it is widely used in turbine pumps of the latest generation).

Hydrodynamic and impact forces connect the rotor and ring in closed system. For bunch of equations (2) and (7) it is necessary to express relative displacement e and polar angle θ through coordinates of the rotor centre x, y and coordinates of the ring centre $x_{\text{к}}$, $y_{\text{к}}$.

Returning to Fig. 1, it is possible to find $x-x_{\text{к}}=e\cos\theta$ and $y-y_{\text{к}}=e\sin\theta$ consequently:

$$e = \sqrt{(x-x_{\text{к}})^2+(y-y_{\text{к}})^2},\quad \cos\theta = \frac{x-x_{\text{к}}}{e},\quad \sin\theta = \frac{y-y_{\text{к}}}{e} \tag{8}$$

Using the appropriate derivative expressions $\dot{x}-\dot{x}_{\text{к}} = \dot{e}\cos\theta - e\dot{\theta}\sin\theta$ and $\dot{y}-\dot{y}_{\text{к}} = \dot{e}\sin\theta + e\dot{\theta}\cos\theta$, it is possible to obtain from first of them:

$$\dot{e} = \frac{\dot{x}-\dot{x}_{\text{к}}+e\dot{\theta}\sin\theta}{\cos\theta},\ \dot{\theta} = \frac{\dot{e}\cos\theta-(\dot{x}-\dot{x}_{\text{к}})}{e\sin\theta}.$$

Further substituting the expression for $\dot{e}$ initially and then for $\dot{\theta}$ in second of them and also taking into account (8), it is possible to obtain to required dependences:

$$\dot{e} = \frac{(x - x_к)(\dot{x} - \dot{x}_к) + (y - y_к)(\dot{y} - \dot{y}_к)}{e}, \quad \dot{\theta} = \frac{(\dot{y} - \dot{y}_к)(x - x_к) - (\dot{x} - \dot{x}_к)(y - y_к)}{e^2} \tag{9}$$

The received set of equations (2) and (7) together with conditions (4) and dependences (3), (5), (6), (8), (9) is mathematical model and describes the impact dynamics of considered system.

3. Conclusions

Integration of these equations is possible by the only numerical methods. Calculation of transient impact oscillations of rotor-sealing ring system is made by the Runge-Kutta method. Results are presented in the Fig. 2.

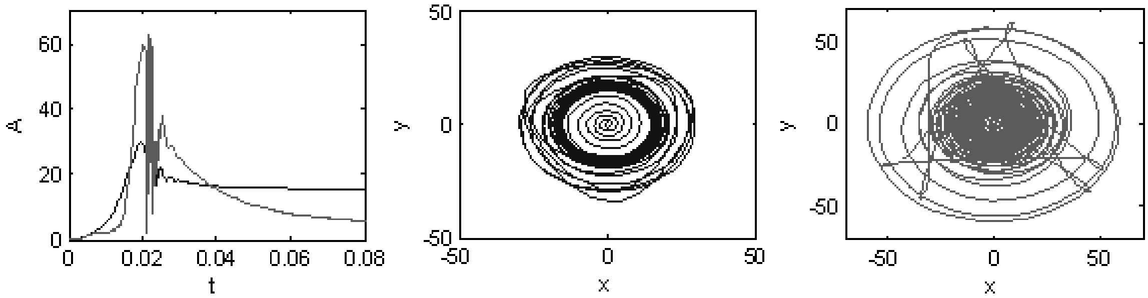

Fig. 2: Amplitudes ($A = \sqrt{x^2 + y^2}$) and trajectories (x, y) of transient impact oscillations of flexible rotor and floating sealing ring.

It is start up of rotor (from 0 to ω_n). In the beginning the motion is non-impact. But further there are eight impacts occurring at the passage of critical speed. It is visible from characteristic two jumps and six loops in the ring trajectory. Impact regime in system after the resonance is replaced by the non-impact regime again.

The developed model allows studying the stationary and nonstationary oscillations of rotor and ring at the impacts between them for specified parameters (Banakh & Nikiforov, 2007, 2008).

Acknowledgement

The financial support provided by the Russian Foundation for Basic Research and RFBR Grant No 10-08-00500-a award scheme is gratefully acknowledged.

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