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SELECTED APPROACHES TO MATHEMATIC MODELING OF RAIN-WIND INDUCED VIBRATIONS

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Abstract: *Large amplitude vibrations of cylindrical structural elements are observed when under the simultaneous action of wind and rain. Due to the action of wind and rain a translating water rivulet forms on the skew cable surface changing significantly the aerodynamic properties of the cable. General mathematic description of the problem is in the form of nonlinear equations, thus this problem is frequently simplified to 1 or 2 degree of freedom system. Another possibility of avoiding complex mathematic description is numerical simulation of the problem using CFD computer program. The paper describes the fundamental principles of modeling rain-wind flow interacting with an obstacle.*

Keywords: ***Computational fluid dynamics, fluid-solid interaction, multiphase flow, rain-wind induced vibrations.***

1. Introduction

There are many real-world situations where engineering solutions use in advance mathematical modeling of multiphase flow, e.g. mixers and vessels analysis, spraying of fluid in fire mitigation or in agriculture. Wind-rain conditions also appear to be crucial in analysis of many everyday requirements.

There have been observed large amplitude vibrations of cables in some of the constructed cable suspended bridges in the last decades. These vibrations occur at simultaneous wind and rain conditions (RWIV) and were for the first time described on a French bridge (Wianecki, 1979). A detailed investigation of this phenomenon was performed in Japan where vibrations neither of vortex shedding nor galloping type were observed on cable-stayed bridge Meikonishi (Hikami & Shiraishi, 1988).

2. Types of Water Rivulet Vibration

Under the conditions of rain accompanied by low velocity wind, a water rivulet forms on the inclined cable. This rivulet dominantly changes aerodynamic properties of the cable. Its movement on the cable surface is influenced by wind action, inertia action of the cable and gravity force. Generally the formation and movement of the rivulet(s) can be divided into three types: two rivulets (one on the upper side, the other on the lower side of the cable) symmetrically vibrating (in phase vibration); two rivulets (one on the upper side, the other on the lower side of the cable) anti-symmetrically vibrating (anti-phase vibration); one rivulet (on the lower side of the cable), vibrating dominantly in the wake.

The opinions on the formation and principle of vibration differ. In (Matsumoto et al., 1992) is stated that the vibration is determined by the formation of the upper rivulet and the subsequent flow change in the cylinder wake (deviation of the resultant force out of the cylinder's gravity center axis). On the contrary, (Bosdogianni & Oliver, 1996) declare that predominantly, the position of the rivulet is of essence, not its vibrating movement. (Ruscheweyh, 1999) completely disagrees with this opinion and asserts that it is exactly the cyclic movement of the water rivulet which is crucial for the development of the phenomenon.

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3. Models of rain-wind induced vibration

A cylindrical body is symmetric (with respect to the flow) and thus no galloping type vibration arises under common conditions. This fact significantly changes if a small body is attached to the cylindrical surface, altering the surrounding flow to non-symmetry. Similar conditions can be obtained by placing a simulated solid rivulet that is used in experimental analysis of RWIV. The mechanism of loosing stability under this type of vibration is known and can be understood using quasi-stationary models and analyses. An important role is played here by the so called den Hartog criterion, expressing the non-stability condition as

$$c_D + \frac{dc_L}{d\alpha} + 2\beta / K < 0\,, \tag{1}$$

where $2\beta = c/m\omega$ and $K = \rho d L_R U / 2m\omega$. Detailed information can be obtained for example in (Van der Burgh & Hartono, 2004).

General mathematic description of cylinder vibration with attached moving water rivulet leads to nonlinear equations (Van der Burgh & Hartono, 2004). This problem is therefore often simplified to one degree of freedom (Ruscheweyh, 1999 and Wang & Xu, 2003) or two degrees of freedom problem (Yamaguchi, 1990). Another possibility how to avoid complex mathematic description is numerical simulation of the problem using computation fluid dynamics code, for example ANSYS CFX or Fluent (ANSYS Revision 12.0, 2009).

3.1. Computational fluid dynamics model

Due to complexity of the wind, rain and cylinder interaction problem, it is usually necessary to apply many simplifications. Often, reduction of the dimension into 2D problem is utilized. Many input data (e.g. upper rivulet position, water mass vibrating together with the cylinder, aerodynamic properties of the system, etc.) must be obtained from scientific literature, or by wind tunnel experiments.

Mathematic theory of fluid flow in rain-wind-obstacle interaction problem is based on the behavior of incompressible Newtonian fluid. The governing equations of the fluid motion are Navier-Stokes equations

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{S}\,, \tag{2}$$

where ρ is the fluid density, **v** is the velocity vector, p is the pressure, μ is the kinematic viscosity and **S** if the body forces vector.

Aerodynamic properties of cylindrical body are changed by the formation of water film on the cylinder's surface. In this case for Eulerian-Eulerian inter-fluid transfer is used the homogeneous model. The free surface then uses the full buoyancy model described as

$$\frac{dp}{dz} = -\left(\rho - \rho_{ref}\right)g\,, \tag{3}$$

where ρ_{ref} is the reference density, g is the gravity acceleration vector and z is the coordinate in the direction of the gravity vector.

Interaction of the fluids on their contact can be described, for example, according to Brackbill, where normal force (dependent upon curvature effect) and tangent force (dependent on variation in the surface tension coefficient) appear.

An illustrative example of the water film formation on the cylinder of D = 0.154 m with uniformly distributed water source $m = 1.389\cdot10^{-3}$ kg·m^{-2}·s^{-1} around the surface of the cylinder is depicted in Fig. 1. Wind velocity with constant value of U = 9.5 m·s^{-1} and SAS SST (Scale Adaptive Simulation Shear Stress Transport) turbulent model were used in the simulation. Turbulence intensity in the inlet was 1 %; the outlet was defined using the relative pressure value of 0 Pa averaged over the whole outlet. Wall contact angle of the water was expected to be approx. 70°. In the discretization process authors use high resolution advection scheme and second order Euler backward scheme for time discretization.

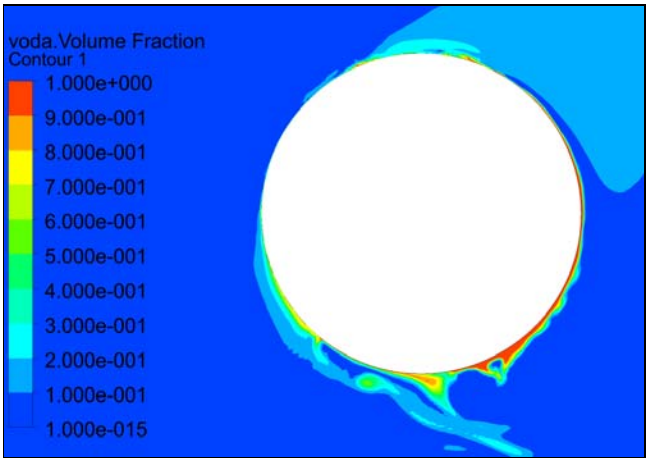

Fig. 1: Formation of lower rivulet in a cylinder wake (CFD analysis at Re = 2.5 · 10^5).

3.2. One and two degree of freedom models

One degree of freedom models usually presume that the rivulet vibrates with the same frequency as the cylinder, their amplitude ratio is constant and depends only on wind speed, and rivulet's mass can be neglected. Although the obtained equation of motion (4) can be easily solved, it is a problem to evaluate the actuating force F, see formula (5), as described in (Wilde & Witkowski, 2003).

$$\ddot{y} + 2\xi_S \omega \dot{y} + \omega^2 y = -\frac{F}{m} \tag{4}$$

$$F = \frac{\rho U_{rel}^2}{2} d\left[c_L(\phi_e)\cos(\phi^*) + c_D(\phi_e)\sin(\phi^*)\right], \tag{5}$$

where $\phi_e = \phi^* - \theta - \theta_i$. The meaning of the individual quantities is identical with the following engineering model. Relative wind velocity U_{rel} and angle of rivulet position θ are depicted in Fig. 2.

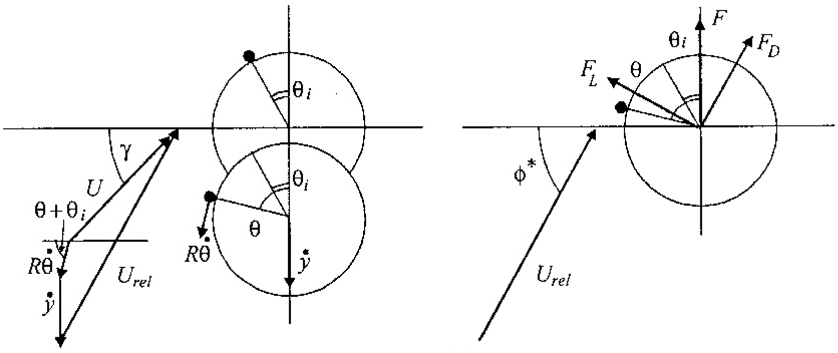

Fig. 2: Action of quasi-static wind force on cylinder (Wilde & Witkowski, 2003).

3.3. Engineering models

A simple mathematic model suitable for engineering application was derived by (Ruscheweyh, 1999). Forces action upon the cylinder are qualified by the following relations

$$\Delta F_x = \Delta c_D \frac{\rho}{2} \bar{V}^2 d L_R \sin(\omega_0 t + \phi_x), \tag{6}$$

$$\Delta F_y = \Delta c_L \frac{\rho}{2} \bar{V}^2 d L_R \sin(\omega_0 t + \phi_y), \tag{7}$$

where ρ is air density, d is cylinder diameter, L_R is rivulet length, ω_0 is natural frequency of cylinder vibrations, $\bar{V}$ is wind mean velocity, Δc_D and Δc_L are first harmonic components of coefficient of drag and lift and ϕ_x, ϕ_y are phase shifts between translations and forces acting upon the cylinder. Quantities Δc_D, Δc_L, ϕ_x and ϕ_y can be obtained from model test in wind tunnel or using the computational fluid dynamics.

An example of velocity field around a plane stationary cylinder model with diameter $D = 0.154$ m and an attached virtual stationary rivulet at position $\alpha = -30°$ calculated by authors of the paper is presented in Fig. 3. Corresponding values of Δc_D, Δc_L can be calculated from simulations carried out with different wind angle of attack.

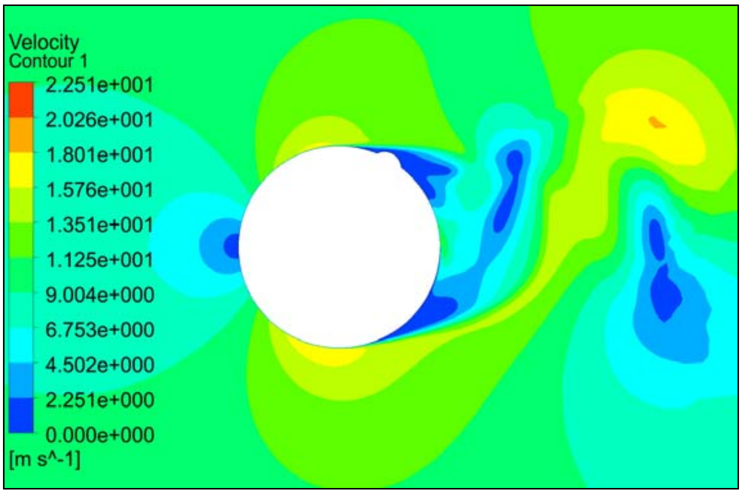

Fig. 3: Flow over a cylinder with permanently attached rivulet in the wake.

4. Conclusions

The paper provides an introduction into the area of vibration of cylinder-like structural elements under the conditions of simultaneous wind and rain. Fundamental characteristics and types of RWIV are presented. There is introduced an overview of mathematic models of RWIV, from simple one or two degree of freedom systems to nonlinear model. The question of advantageous use of numerical CFD simulation for acquiring input data into simple models is discussed.

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