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DYNAMIC ANALYSIS OF COLLAPSE OF A HIGH BUILDING

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Abstract: *The differential equation of collapse of a high building is derived taking into account many influences. Computer simulation of the collapse of the WTC building is presented using two independent programs for some variations of parameters. The results of both, differential equation and computer simulation, are compared.*

Keywords: ***Dynamics, structural mechanics, collapse.***

1. Introduction

This article deals with collapse of a high building. Its aim is not to investigate a cause of the collapse but to examine the falling process itself. The article studies theory of collapse of a high building. Process of the falling is investigated from the point of view of basic laws of mechanics. Differential equation of a high building collapse is derived and all major influences of the falling process are included. Several parameters which influence the falling process are introduced. The conditions which are to be considered to let the building collapse completely are set and the falling speed is examined.

2. Derivation of a differential equation of collapse of a high building

Let's assume that columns in the location between the coordinates x' and x_0 lose stability and a top part of the building above x' starts to fall and hits the still undamaged lower part of the building under the location x_0 with velocity v_0.

We will introduce equation of dynamical equilibrium for the location x:

$$G - F_N - F_m - F_c - F_a = 0 \tag{1}$$

Where G is weight of a part of the building above the location *x*, for which equilibrium equation is formulated

F_N is resistance put up by the columns against the collapse

F_m is resistance originated by hitting of a falling part of the building into a motionless mass

F_c is a viscous damping

F_a is an inertial force of a falling mass

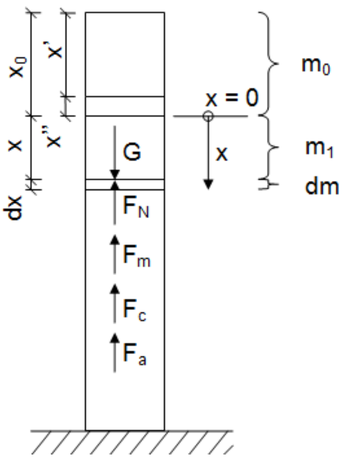

Fig. 1: Scheme of a high building with acting forces.

Derivation of individual parts of the equilibrium equation (1)

a) The weight of the building above the location *x*:

$$G = mg\beta\ , \tag{2}$$

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where m is the mass of the building above the location x, g is acceleration of gravity, β is portion of the total mass above the location x which pushes to a lower part of the building. The mass which falls outside of the building is subtracted.

b) The columns resistance:

$$F_N = mgs\kappa \tag{3}$$

where s is a rate of the ultimate force of columns to the current force in columns in the moment of the collapse, κ is the factor of the ultimate force of columns which represents average column resistance during its deformation related to an ultimate force.

The assessment of the column pressing was done using the method of controlled deformation in order to obtain this factor and receive its operational chart (see picture below). Factor κ is then the rate of the ultimate force to its median value.

c) The resistance of a motionless mass:

It is inertial force of a still mass *dm* accelerated in a time *dt* to the speed v ($a = dv/dt$). We can use the term $a = v/dt$ for acceleration in the equation due to acceleration starting from zero up to the speed *v*.

The force F_m can be then expressed in this way:

$$F_m = dm \cdot a = dm \cdot \frac{v}{dt} \tag{4}$$

When considering that $v = dx/dt$, the equation (4) can be rewritten as follows:

$$F_m = dm \cdot \frac{v^2}{dx} = \mu v^2, \tag{5}$$

where $\mu = dm/dx$ is a line density of the building.

d) The viscous damping:

$$F_c = C \cdot v = m\alpha v, \tag{6}$$

where C is a factor of the viscous damping. We are considering Rayleigh damping here which is depending on mass quantity $C = m\alpha$ only.

e) The inertial force of a falling mass:

$$F_a = \beta m \cdot a = \beta m \frac{dv}{dt} = \beta m v \frac{dv}{dx} \tag{7}$$

Again, only the inertial force of a mass which does not fall outside of the building is considered here.

3. Differential equation of the building collapse

By substituting relations which we derived above into the equation (1) we get:

$$mg\beta - mgs\kappa - \mu v^2 - m\alpha v - \beta m v \frac{dv}{dx} = 0 \tag{8}$$

We will divide the equation with speed v and mass m and adjust:

$$\frac{b}{v} - \frac{v}{x + x_0} - \alpha - \frac{\beta dv}{dx} = 0, \tag{9}$$

where $b = g(\beta - s\kappa)$. We used relation $\mu(x + x_0) = m$ when adjusting the equation.

Analytical solution of the differential equation was found only when influence of damping was omitted:

$$v(x)=\sqrt{\frac{2b(x+x_0)}{2+\beta}+(x+x_0)^{\frac{-2}{\beta}}\cdot C_1} \tag{10}$$

To specify the constant C_1 the magnitude of the speed v_0 is needed. This is the speed of the mass m_0 above x_0 falling into the undamaged part of the building. We will start from the same differential equation where we will modify factor b into $b_0=(\beta-s_0\kappa)g$, whereas $s_0<1$:

$$\frac{b_0}{v(x)}-\frac{v(x)}{x+x'}-\frac{\beta\, dv(x)}{dx}=0 \tag{11}$$

The solution will thus have a similar form. We will use boundary conditions $v(0)=0$ for finding the magnitude of the integration constant. Then we are looking for $v(x'')=v_0(x)$. After that we can return to searching the integration constant C_1:

$$v_0(x)=\sqrt{\frac{2b_0(x+x_0)}{2+\beta}+(x+x_0)^{\frac{-2}{\beta}}\cdot C_1} \tag{12}$$

$$C_1=\frac{v_0^{\,2}(x)(2+\beta)-2b_0(x+x_0)}{(x+x_0)^{\frac{-2}{\beta}}(2+\beta)} \tag{13}$$

We will establish C_1 in (10):

$$v(x)=\sqrt{\frac{2b(x+x_0)}{2+\beta}+(x+x_0)^{\frac{-2}{\beta}}\cdot\frac{v_0^{\,2}(x)(2+\beta)-2b_0(x+x_0)}{(x+x_0)^{\frac{-2}{\beta}}(2+\beta)}} \tag{14}$$

We have now the solution of the equation (14). Therefore we can find out when the collapse of the building will stop so that the speed will be zero $v(x)=0$. There was not found any solution of this equation in the closed form so we are going to return to treat the equation (9) with numerical method. The Euler implicit method of solving a differential equation was found to be the most suitable method. Its principle is:

$$v_{i+1}=v_i+h\cdot f\left(x_i,v(x_i)\right) \tag{15}$$

We will get this in our equation:

$$v(x_{i+1})=v(x_i)+\frac{h}{\beta}\left(\frac{b}{v(x_{i+1})}-\frac{v(x_{i+1})}{x_{i+1}+x_0}-\alpha\right) \tag{16}$$

After deduction of the speed $v(x_{i+1})$ we get this relation:

$$v(x_{i+1})=\frac{1}{2(h+\beta x_{i+1}+\beta x_0)}*$$
$$*\left\{\begin{matrix}-\alpha h x_{i+1}+\beta v(x_i)x_{i+1}-\alpha h x_0+\beta v(x_i)x_0+\\+\sqrt{-4(h+\beta x_{i+1}+\beta x_0)(-bhx_{i+1}-bhx_0)+(\alpha h x_{i+1}-\beta v(x_i)x_{i+1}+\alpha h x_0-\beta v(x_i)x_0)^2}\end{matrix}\right\} \tag{17}$$

We will compute the speed $v(0)$ in a similar way.

Discussion of the magnitude of damping, the safety factor s and the ultimate force ratio κ

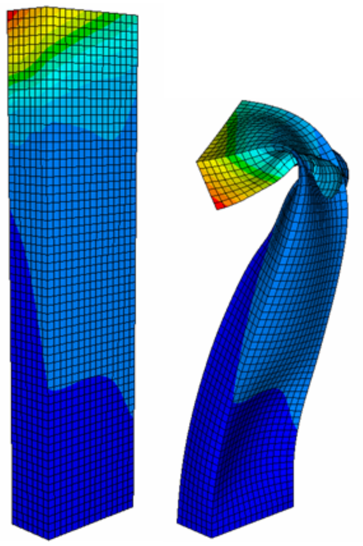
Before we start solving equation in the numerical way we are going to clarify the magnitude of the damping α.

$$\alpha=\frac{C}{m}=\frac{2m\omega_n\xi}{m}=2\omega_n\xi \tag{18}$$

For the ratio of damping, we will consider the value 10-30% and the limiting value 0 that yields the damping ratios $\alpha=0{,}147, \alpha=3{,}18, \alpha=0$. We will consider value 0 in order to solve the collapse without the effect of damping.

Much less uncertainty is about the value of the parameter s. We are assuming it to be around 2.5 – 3. e.

The coefficient of the ultimate strength in columns κ was computed from simulation of pressing of the columns by the method of controlled deformation.



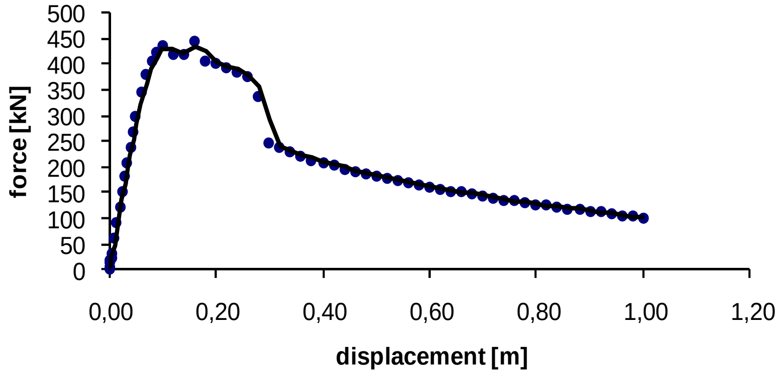


Fig. 2: Response diagram and the deformed shape of a box column.

It is clear from the graph that the value of the κ coefficient will be around 0.25.

Results – times and extends of the fall for various parameters

s[-]	alpha	The theory without falling away of a mass x[m]	The theory with falling away of a mass x[m]	RFEM x[m]	FyDiK x[m]	The theory without falling away of a mass t[s]	The theory with falling away of a mass t[s]	RFEM t[s]	FyDiK t[s]
2.0	0.147	330	330	331.0	278.7	15.2	25.8	15.7	19.0
2.0	3.18	330	330	18.2	63.2	83.1	357.1	7.4	31.5
3.0	0	330	330	259.8	287.1	20.5	36.1	17.9	20.4
3.0	0.147	330	330	324.5	304.2	24.2	103.0	28.6	33.1
3.0	0.5	330	330	79.6	72.3	36.5	330.5	12.1	28.1
3.0	1.06	330	330	64.6	66.5	60.2	707.3	15.9	37.4

4. Conclusions

It was possible to solve a differential equation of the building collapse in closed form only when the damping was omitted. Real damping is indispensable considering massive destruction of all components of a building construction. Therefore this solution represents limit of a speed and an extent of the collapse. General form of the differential equation was solved only in the numerical way. Two independent computer programs were used for the simulation, named RFEM (Němec et al., 2010) and FyDik. Despite the difference in the approaches both computer programs gave comparatively similar solutions. The difference between the solution of the differential equation and that of the computer simulation is greater. The computer simulation gives more reliable solution as there is no need to keep continuity of all values used in differential equation.

Reference

Němec, I. at al. (2010) Finite Elements Analysis of Structures. Aachen: Shaker Verlag.