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INFLUENCE OF THE SPHEROID PROLONGATION ON THE DRAG FORCE

N. Lukerchenko[*], I. Keita[*], Y. Kvurt[**], J. Miles[*]

Abstract: *The drag force acting on a spheroid moving perpendicularly to its axis of rotation in water was studied experimentally. Along the spheroid axis, which is normal to its axis of rotation, a round narrow hole was bored. The spheroid moved along a thin vertical thread stretched in water. A video system recorded the spheroid motion and the spheroid velocity was determined from the record. The drag force coefficient was calculated from the balance of forces acting on the spheroid. Two oblate, two prolate spheroids and one sphere with ratio of the axes 0.67; 0.81; 1.33; 2 and 1 (sphere), respectively, with approximately the same volumes, were used. The friction coefficient between the thread and spheroid was determined from the comparison of the experimental and calculated motions of the sphere, for which the drag force coefficient is known. The dependence of the drag force coefficient of the spheroid on the ratio of its semi-axes was obtained.*

Keywords: ***Spheroid, drag force coefficient.***

1. Introduction

Spherical particle is usually used for modelling of solid particle movement in fluid (e.g., Nino & Garcia, 1994; Kholpanov & Ibyatov, 2005; Lukerchenko et al., 2006, 2009a,b). However, the influence of the particle shape on its motion in fluid can be significant. For example, in the case of particle saltation in a channel with rough bed, the elongated shape of the particles leads to a significant increase of the angular velocity (Nino & Garcia, 1998), which strongly affect numerical models. A spheroid is the simplest shape, which can be compared with a sphere, and it is suitable to study the influence of the particle elongation.

Let the semi-axis of a spheroid that corresponds to the axis of rotation (axis of symmetry) is a_0 and the semi-axis that is normal to the axis of rotation, i.e. the equatorial radius, is b_0 (Fig. 1). If the spheroid moves slowly in fluid without rotation (i.e for Stokes conditions, Re<<1), its drag force can be calculated using the drag force components parallel to the axis of rotation $F_{||}$ and perpendicularly to this axis $F_{\perp}$ (Happel, 1973). The correlations for the drag force coefficients $C_{||}$ and $C_{\perp}$ that are dimensionless analogues of the drag force components $F_{||}$ and $F_{\perp}$ were derived for Stokes flow theoretically (e.g., Happel, 1973; Clift et al., 2005). Unfortunately, there are not available relationship for the drag force coefficients $C_{||}$ and $C_{\perp}$ for different ratios of the spheroid semi-axes $E=a_0 / b_0$ when Reynolds number Re>>1. Experimental results for axisymmetric motion ($F_{\perp} = 0$) outside the Stokes range appear to be limited to oblate spheroids ($E < 1$) for which is the preferred orientation (Clift et al., 2005). List et al. (1973) studied experimentally the drag force acting on the oblate spheroids with axis ratios $0.5 < E < 0.79$ over a range of Reynolds numbers from 40 000 to 400 000 at various inclination angles of the fluid flow.

The present paper deals with an experimental evaluation of the drag force coefficient $C_{\perp}$ for the semi-axes ratio E ranging from 0.67 to 2 (Tab. 1) and $Re \sim 10\,000$.

[*] Nikolay Lukerchenko, PhD., Ibrahima Keita, PhD. and Ing. Jiri Miles: Institute of Hydrodynamics of Academy of Sciences of the Czech Republic, v.v.i.; Pod Patankou 30/5, 166 12, Prague 6; CZ, e-mails: lukerchenko@ih.cas.cz, keita@ih.cas.cz, miles@ih.cas.cz

[**] Dr. Yury Kvurt, CSc.: Institute of Problems of Chenical Physics of Russian Academy of Sciences; Chernogolovka, Moscow reg., 142432 Russia, e-kvurt@icp.ac.ru

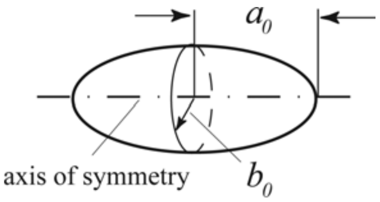

Fig. 1: Test spheroid.

Tab. 1: Parameters of spheroids.

No.	a_0, mm	b_0, mm	$E=a_0 / b_0$	
1	12.6	18.9	0.67	Oblate spheroids
2	14.6	18.0	0.81	
3	16.4	16.4	1.0	Sphere
4	20.0	15.0	1.33	Prolate spheroids
5	26.2	13.1	2.0	

2. Experimental procedure

The experiments were carried out in a rectangular glass vessel 0.780 m long, 0.580 m wide, and 0.980 m high. The water depth was about 0.900 m.

The spheroid masses were measured using the electronic balance SARTORIUS BA2100S; the maximum absolute error was 10^{-5} kg.

The five Plexiglas models of spheroid were used. Along one of the equatorial diameters, i.e. normally to the axis of rotation, a round narrow hole of diameter 1 mm was bored. The spheroid moved along a thin thread of diameter 0.1 mm that was passed through the hole and was stretched in water vertically.

The measurement of spheroids sedimentation start just under the water surface in order to avoid air entrainment end the effect of the water surface on its motion.

The spheroid movement in water was recorded using the digital video camera NanoSenze MKIII+ with frequency up to 1000 frames per second. The spheroid terminal fall velocity was determined by evaluation of the video record. The drag force coefficient was calculated using the balance of forces acting on the spheroid.

3. Mathematical model

The following forces act on the spheroid of mass m moving steady in water along the vertical thread: gravity force $F_g = mg=\rho_s\, \Omega g$, Archimedean force $F_A = \rho\Omega g$, friction force $F_f = k_f\, l$ and drag force $F_d = C_\perp\, S\, \rho V^2/2$, where ρ_s is the spheroid density, ρ is the water density, g is the gravitational acceleration. $\Omega = 4/3\ \pi a_0 b_0^2$ is the spheroid volume, k_f is the friction coefficient, $l = 2b_0$ is the length of the hole in spheroid, V is the fall velocity, and $S = \pi\ a_0\ b_0$ is the spheroid cross-section area perpendicular to direction of movement. The force F_f due to the mutual friction between the spheroid and the thread is supposed to be proportional to the length of the hole l.

The balance of the forces gives:

$$m\, g = \rho\, \Omega\, g + 2k_f\, b_0 + C_\perp\, \pi\, a_0\, b_0 \rho V^2 / 2. \tag{1}$$

For the each spheroid, Eq. (1) includes two unknowns: the friction coefficient k_f and the drag force coefficient $C_\perp$. However, for the sphere, the drag force coefficient is known and the Eq. (1) can be used for the definition of the friction coefficient k_f.

4. Results

The spheroid volume can be specified in two ways: (I) using the formula for the spheroid volume $\Omega_0 = 4/3\ \pi a_0\, b_0^2$ and, (II) using the values of the Archimedean force $\Omega_A = F_A / \rho g$. The Archimedean force was measured the following way. A beaker filled with water was put on the balance, which was zeroed. The spheroid was submerged into the water in the beaker without touching the beaker wall and bed. The balance shows the Archimedean force. For the used balance the accuracy of the volume measurement for this method is 10^{-8} m^3. The calculated and measured data in Tab. 2 shows that the spheroid production error (shape inaccurancy) was less than 5%.

For the drag force coefficient of the sphere the following correlation was used (Nino&Garcia, 1994):

$$C_{\perp} = \frac{24}{Re}\left(1+0.15\left(Re\right)^{\frac{1}{2}}+0.017Re\right)-\frac{0.208}{1+10^{4}Re^{-0.5}}, \tag{2}$$

where $Re = 2rV/v$ is the Reynolds number, $r=a_0=b_0$ is the sphere radius, v is the water kinematical viscosity. The value of the drag force coefficient for the sphere of measured size is $C_{\perp} = C_{d0} = 0.445$. The fall velocity is 0.35 m/s. The value of the friction coefficient from the Eq. (1) is $k_f = 0.353$ N/m.

Tab. 2: The spheroid production accuracy.

No.	$E=a_0/b_0$	$\Omega_0 \cdot 10^{-5}$ m^3	$\Omega_A \cdot 10^{-5}$ m^3	$\Omega_0 - \Omega_A)/\Omega_A$ %
1	0.67	1.885	1.889	0.2
2	0.81	1.970	1.883	4.6
3	1.0	1.831	1.831	0.0
4	1.33	1.885	1.850	1.9
5	2.0	1.883	1.871	0.6

The values of the drag force coefficient $C_{\perp}$ for the oblate and prolate spheroids were calculated using Eq. (1) and they are presented in Tab. 3. The equivalent Reynolds number is $Re_e = 2r_e V/v$, where the equivalent radius $r_e = (a_0 b_0)^{0.5}$ is the radius of the sphere with the same midlength section area.

Tab. 3: The drag force coefficients of the spheroids.

No.	$E=a_0/b_0$	V, m/s	Re_e	$C_{\perp}$
1	0.67	0.42	13 000	0.34
2	0.81	0.38	12 400	0.39
3	1.0	0.35	11 400	0.45
4	1.33	0.31	10 600	0.55
5	2.0	0.26	9 800	0.72

The dependence of the drag force coefficient $C_{\perp}$ on the ratio of the spheroid semi-axes $E = a_0/b_0$ is depicted in Fig. 2. This dependence matches well with the correlation:

$$C_{\perp} = C_{d0}E^{0.7}, \tag{3}$$

where $C_{d0} = 0.445$ is the drag force coefficient of the sphere in turbulent regime.

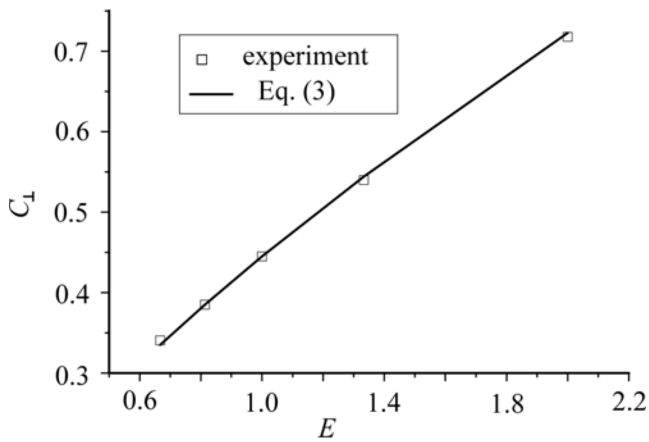

Fig. 2: The dependence of the spheroid drag force coefficient $C_{\perp}$ on the ratio of the spheroid semi-axes E.

5. Conclusions

The dependence of the drag force coefficient of the spheroids moving perpendicularly to the axis of rotation on the spheroid semi-axes ratio were determined for the spheroid semi-axes ratio ranging from 0.67 to 2. The values of the Reynolds number are in the range from 9 800 to 13 000. The dependence corresponds well with the experimental data and it can be used in the numerical models of the particle movement in fluid. The experimental method can be used for investigation of the drag force coefficients of spheroids as well as the bodies with more complex shape.

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