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CHANGE OF MODAL PROPERTIES DUE TO DAMAGE IN BEAM BUCKLING PROBLEM

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Abstract: *The paper is focused on analysis of changes of eigenfrequencies due to local damage in beam buckling problem. The aim is determination of suitable initial parameters for consequent inverse analysis. Relative changes of the first five eigenfrequencies when local damage in individual parts of the beam occurs were studied together with influence of axial force size to those changes. Even with the knowledge of significant simplification, such a task can serves as a prefiguration for damage identification of pre-stressed structures.*

Keywords: ***Eigenfrequencies, damage, beam buckling problem.***

1. Introduction

In the field of damage identification using structural health monitoring data (SHM, ambient vibration monitoring) the analysis of change of modal properties due to damage is important (Wenzel & Pichler, 2005; Lehký & Novák, 2009). The task of damage identification using vibration data is based on the premise that damaged structure has smaller stiffness in some parts – and this difference will affect vibration (modal properties). The comparison of vibration of virgin (undamaged) structure and damaged structure can be then used for the detection of damaged parts (localization of damage). Integral part of research in this field is the study of modal properties and analysis of its applicability for damage identification, see e.g. Frantík et al. (2007) for numerical analyses and Lehký & Novák (2009) for experimental analysis.

In the paper, beam buckling problem from the damage point of view is studied. Such a task can serve as a prefiguration for pre-stressed bridges. Limiting factor for damage identification using changes of modal properties are their relatively small values compared to the size of the damage. The aim of the study is to find out if changes in vibration are bigger with higher pre-stressing of a structure and if there is a change in dominancy of eigenfrequencies changes in individual damaged parts.

2. Computational model

Computational model was created using physical discretization method implemented in application FyDiK (Frantík, 2009). The beam is divided into a specific number of elements with the same length. Every element has an inner normal spring. The elements are connected together by hinges with rotational springs, see Fig. 1. Both types of springs are considered to be linear, but large deflections are taken into account in the mathematical representation. For the sake of simplicity the mass of the beam is concentrated in hinges. Model is then formulated as a nonlinear dynamical system.

In our study the model of a wooden beam with 20 elements is used. The beam has length $l = 80$ cm, mass $m = 84$ g, normal stiffness $EA = 1.26 \cdot 10^6$ N, bending stiffness $EI = 2.625$ Nm^2. According to analytical solution (Brepta a kol., 1994):

$$f_j = \frac{\lambda_j^{\,2}}{2\pi}\sqrt{\frac{EI}{ml^3}}, \quad \lambda_j = j\pi,\ j = 1,2,3,..., \tag{1}$$

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the unloaded beam should have following bending eigenfrequencies $f_1 = 12.27$ Hz, $f_2 = 49.09$ Hz, $f_3 = 110.45$ Hz, $f_4 = 196.35$ Hz, $f_5 = 306.80$ Hz. For the comparison, model (with 20 elements) gives $f_1 = 12.25$ Hz, $f_2 = 48.69$ Hz, $f_3 = 108.41$ Hz, $f_4 = 189.97$ Hz, $f_5 = 291.35$ Hz.

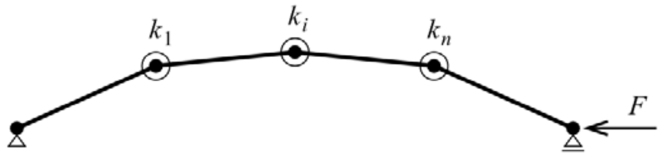


Fig. 1: Discrete model of a beam with rotational springs.

3. Parametric study

Changes of modal properties were studied on simply supported beam without imperfection and pre-loaded by axial force. The aim of this study was to find out which eigenfrequencies are affected by change of stiffness in individual location on the beam. This kind of information is important for damage identification using SHM data. Let's mention that evaluation of eigenfrequencies (especially the higher ones) from experiment is not an easy task. For study purposes, first five eigenfrequencies were taken into account. Ratio between the first one and the fifth one is approximately 1:25. As mentioned in chapter 2, bending of the beam is realized by 19 rotational springs with stiffness $k_1, k_2, \ldots, k_{19}$ corresponding to bending stiffness EI_1 to EI_{19}. A damage of beam in individual location is modeled by decrease of bending stiffness down to one tenth of its nominal value. Such damages were applied step by step for all parts (only one part was damaged at each time).

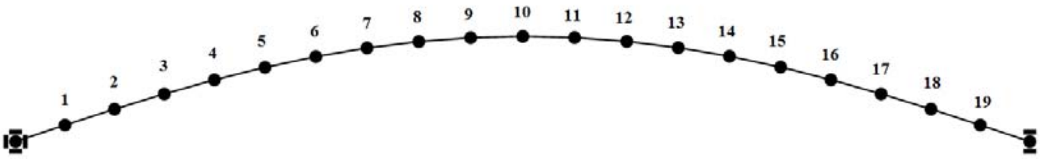


Fig. 2: Scheme of computational model of simply supported beam.

3.1. Unloaded beam

First study was performed on a beam without axial force. Figure 3a shows the relative changes of the first eigenfrequency f_1 as a function of the level of damage and position of damage. It can be seen that changes of eigenfrequency correspond in some sense to the first mode shape of the beam – position of extreme value corresponds to position of amplitude of mode shape. The same results were also obtained for higher eigenfrequencies. A distinctive change of eigenfrequencies (more than 5 %) can be detected only for higher levels of damage (more than 50 % of nominal stiffness). The damage in nodes of mode shape will not influence corresponding eigenfrequency. It is obvious that for efficient damage identification a higher number of eigenfrequencies must be taken into account because the absence of frequency change in certain part of the structure (node of mode shape) is then compensated by the change of another frequency. That can be seen in Fig.3b which shows which eigenfrequency has changed the most due to damage in certain part of the beam. In the middle part of the beam (parts 7-13) the first eigenfrequency is dominant. But in outer parts its influence decreases and higher eigenfrequencies become more important.

3.2. Pre-loaded beam

The main aim of the study is to analyze an influence of axial (pre-stressing) force on eigenfrequency change of the beam due to damage. With increasing axial force, eigenfrequencies are decreasing for undamaged as well as damaged beam. Fig.4a shows such decrease for one particular case where middle part of the beam was damage to one half of the nominal stiffness. Critical force of that case is 36.65 N, for undamaged beam it is 40.40 N. In case of damaged beam, decrease of all eigenfrequencies is faster compared to undamaged beam (dashed line vs. solid line in Fig.4a). In the figure, there are no lines of the second and the fourth eigenfrequencies of damaged beam. The reason is that damage is located in the node of both corresponding mode shapes and therefore those eigenfrequencies are not affected by damage.

a) b)

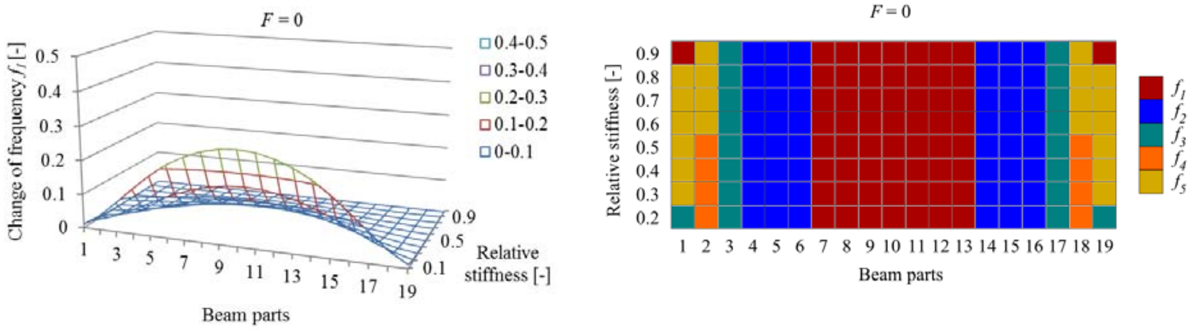


Fig. 3: a) Relative changes of the first eigenfrequency for different levels of stiffness in individual damaged parts of unloaded beam; b) division of unloaded beam to regions with maximum relative changes of each eigenfrequency.

Fig. 4b shows relationship between axial force and relative changes of eigenfrequencies. It is obvious that with increasing force (pre-stressing) a relative changes of eigenfrequencies are being increased too. Especially change of the first eigenfrequency is very strong. Higher eigenfrequencies are less affected. Again, the second and the fourth eigenfrequencies are not depicted in the figure since their values are not influenced by damage of the middle part of the beam. More detailed picture of dependence of axial force on eigenfrequencies changes caused by damage of individual parts of the beam to 50% of its nominal value gives Fig. 5. Fig. 5a shows relative changes of the first eigenfrequency which increases hand by hand with axial force. Fig.5b shows relative changes of the second eigenfrequency. Changes increase again together with axial force but not so quickly as for the first eigenfrequency. One can also see zero change of the second eigenfrequency if the middle part (No. 10) of the beam is damaged – that for any size of axial force.

In Fig. 6a there are relative changes of the first eigenfrequency for different levels of damage and axial force $F = 30$ N. Increase of eigenfrequency changes is distinctive compared to unloaded beam (see Fig.3a). It is valid for lower levels of damage too. In the figure, one can see that critical forces for cases where damage of parts close to middle of the beam is relatively large (stiffness is reduced to 20% of its nominal size) is lower than axial force $F = 30$ N. Fig.6b shows regions of beam loaded by axial force $F = 20$ N where each eigenfrequency is dominant (relative change of that eigenfrequency outweigh the others). In comparison with unloaded beam a region where the first eigenfrequency is dominant is wider for pre-loaded beam.

a) b)

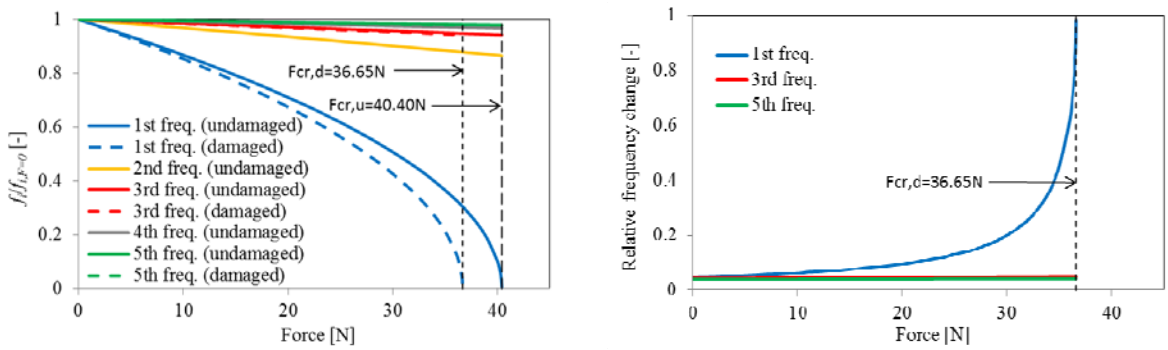
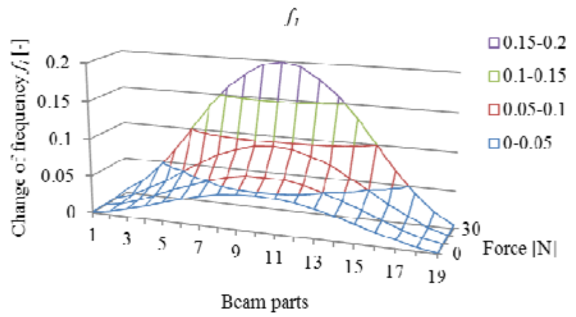


Fig. 4: a) Changes of eigenfrequencies with increase of axial force for undamaged and damaged beam; b) relative changes of eigenfrequenies due to damage (in both graphs the damage is represented by decrease of stiffness of middle part No. 10 to 50% of nominal stiffness).

a) b)



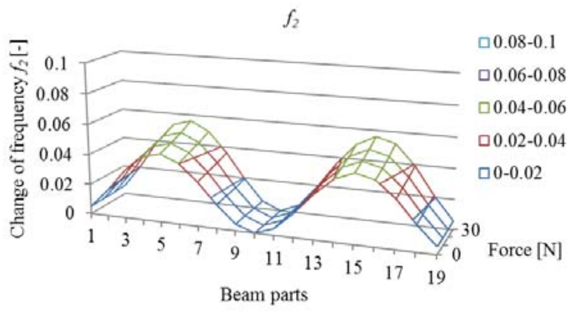
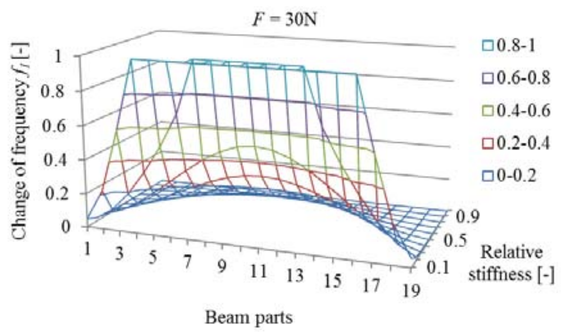


Fig. 5: Relative changes of (a) first and (b) second eigenfrequency caused by damage to 50% of nominal stiffness of individual damaged parts and their dependence on increase of axial force.

a) b)



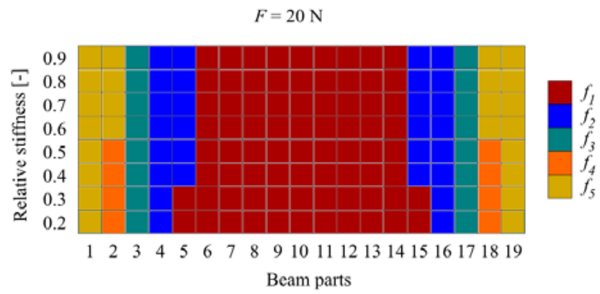


Fig. 6 a) Relative changes of the first eigenfrequency for different levels of stiffness in individual damaged parts of beam pre-loaded by axial force 30 N; b) division of pre-loaded beam (20 N) to regions with maximum relative changes of each eigenfrequency.

4. Conclusions

A dominancy of changes of particular eigenfrequencies when damage in individual parts of the buckled beam occurs was studied together with an influence of axial force size to those changes. Even with the knowledge of significant simplification, such a task can serves as a prefiguration for damage identification of pre-stressed structures. Results show thanks to pre-stressing there are more significant changes of modal properties due to damage. It will be positive for damage identification using SHM data. Spreading region of dominancy of the first eigenfrequency due to increasing axial force is also positive since the relative changes of frequency are bigger in comparison with higher ones.

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