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DESIGN AND RELIABILITY ASSESSMENT OF ROOF STRUCTURAL ELEMENTS USING THE NEW DIGITAL GROUND SNOW LOAD MAP OF THE CZECH REPUBLIC

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Abstract: *The paper deals with the problems of design and reliability assessment of roof structural elements exposed to the effects of snow load. A new digital ground snow load map of the Czech Republic is introduced in this paper. This digital map provides to structural designers detailed information about the characteristics of snow load on the ground for arbitrary selected locality in the Czech Republic. The digital map covers the area of the Czech Republic by the net 100 x 100 m. The following data are given to every square 100 x 100 m: (a) characteristic value of snow load on the ground according to European standards; (b) statistical characteristics of annual maximum ground snow load (mean value μ, standard deviation σ, variation coefficient V and skewness α); (c) sorted ground snow load history - so called load duration curves. Possibilities of utilizing the data from digital map are demonstrated in this paper using an example of steel purlin design. The Partial Factors Method as well as Simulation-Based Reliability Method are used for design and assessment in this example.*

Keywords: ***Snow load, probabilistic, reliability assessment, SBRA, steel structures.***

1. Introduction

Transition from national to European standards for reliability assessment of structures (so called Eurocodes) is connected with many problems in the Czech Republic. One of them is an expressive increase in design values of climatic actions, mainly of snow and wind loads. Higher design values of snow loads may affect the economics of roof structural elements design. Lightweight steel and timber roofs are mostly influenced. The assessment of existing structures designed in agreement with national standards could be also complicated due to higher design values of snow loads according to Eurocodes.

Non-negligible economic savings could be achieved by applying some improvements to the reliability assessment procedures proposed in the European standards. The first possibility is improvement (refinement) of European ground snow load maps given in EN 1991-1-3, the second one is the application of more appropriate probabilistic-based design methods if compared to the traditional partial factors method proposed in Eurocodes.

2. Digital ground snow load map

2.1. Conception of the digital map

The new digital ground snow load map covers the area of the Czech Republic by the net 100 x 100 m. Needed snow characteristics were calculated for every square of the net using the Multiple Weighted Linear Regression method, see (Křivý & Stříž, 2010).

The map conception is such as to be user friendly. Snow characteristic of the selected location are obtained either by clicking on a virtual map or directly by entering the GPS coordinates - see www.snehovamapa.cz. The map is applicable not only for the traditional analysis using partial factor method but also for the direct probabilistic assessment of structures. The following data are given to every square 100 x 100 m:

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a) The first data is the characteristic value of snow load on the ground (s_k). The characteristic value is based upon the probability of 0.02 of its time-varying part being exceeded for a reference period of one year, see EN 1990. This is equivalent to a mean return period of 50 years for the time-varying part. The characteristic value (s_k) is applicable for common analysis using partial factor method given in Eurocodes.
b) Statistical characteristics of annual maximum ground snow load (mean value μ, standard deviation σ, variation coefficient V and skewness α) constitute the second group of data provided by the digital map. Arbitrary fractile can be derived from these statistical characteristics (including the characteristic value (s_k) defined in point (a)). The statistical characteristics can be used also for the direct probabilistic analysis according to EN 1900 and JCSS documents (2001). The suitable probabilistic distributions are the three-parametric lognormal distribution or Gumbel distribution.
c) The sorted ground snow load history, so called Load Duration Curves (Marek et al., 1996), is the next characteristic that is given to each section 100 x 100 m. The load duration curves are derived from data being measured during the whole year, *i.e.* including periods when the snow does not occur. The load duration curve is obtained by the ascending sort of the measured data. The most lasting value of ground snow load is $s = 0$. Distribution function and corresponding histogram is very easy to derive from the load duration curve – the distribution function is an inverse function to the load duration curve.

2.2. Characteristic values comparison of snow load on the ground (s_k) for selected localities

The characteristic values of ground snow load (s_k) are mostly applicable in practical design based on partial factor method. Tab. 1 compares the characteristic values (s_k) taken from the current ground snow load map of the Czech Republic given in ČSN EN 1991-1-3 and the characteristic values calculated for new digital map. The comparison is carried out for 28 selected locations in the area of the Czech Republic. GPS coordinates of the localities are given in (Křivý & Stříž, 2010).

Tab. 1: Snow load on the ground according to the ČSN EN 1991-1-3 and new digital map.

Locality	**ČSN EN 1991-1-3** **s_k (kN/m^2)**	**digital map** **s_k (kN/m^2)**	**difference (%)**	**difference (kN/m^2)**
Praha (Prosek)	0.70	0.54	-22.9	-0.16
Pardubice (Pardubičky)	0.70	0.52	-25.7	-0.18
Brno (Žabovřesky)	1.00	0.73	-27.0	-0.27
Ostrava (Hrabůvka)	1.00	0.91	-9.0	-0.09
Havířov (Šumbark)	1.50	0.99	-34.0	-0.51
Valašské Meziříčí	1.50	1.44	-4.0	-0.06
Jeseník	2.00	1.62	-19.0	-0.38
Frýdlant N/O (north)	2.00	1.32	-34.0	-0.68
Frýdlant N/O (south)	2.50	1.42	-43.2	-1.08
Frenštát p/R (Trojanovice)	2.50	2.52	+0.8	+0.02
Vrbno p/P (west)	3.00	2.01	-33.0	-0.99
Moravský Beroun	3.00	2.31	-23.0	-0.69
Hanušovice	4.00	2.36	-41.0	-1.64
Mosty u Jablunkova	4.00	2.45	-38.8	-1.55

Tab. 1 shows significant differences between the characteristic values. Lower values taken from the digital map are more numerous. The comparison results shows that mainly the localities with higher snow load are often classed to higher snow region than necessary. The reasons leading to such differences between the both characteristic values are following:

– The new digital snow map does not work with eight discrete snow regions as defined in the current ground snow load map of the Czech Republic given in ČSN EN 1991-1-3. The net with basic size 100 x 100 m covers the area of the Czech Republic so closely, that we can speak about continuous distribution of the ground snow load. The term “snow region” is irrelevant. The largest differences are at the localities lying closely behind the boundary of snow regions defined in the map given in ČSN EN 1991-1-3.

- It is important to keep in mind that every snow region covers specific range of values, *e.g.* the third snow region covers the range 100 till 150 kg/m^2.
- The local ground characteristics (valleys, solitary hills etc.) are not often taken into account in the printed map in ČSN EN 1991-1-3 because of its resolution limits.
- More sophisticated model for calculating snow characteristics was applied to the digital map by comparing it with the map given in ČSN EN 1991-1-3. The influence of slope gradient, orientation and convexity were not considered when processing the map of snow regions for ČSN EN 1991-1-3. Suitable climatological stations for regression analysis were selected only upon their horizontal distance from investigated grid point.
- Statistical data from the period 1962 – 2009 were used for the new digital map, data from the period 1962 – 2006 were used for the map in ČSN EN 1991-1-3.

3. Design of a roof element

This chapter contains a study dealing with design of steel purlin according to six following procedures:

(a) Partial factors design according to national Czech standards (are not in force anymore) ČSN 73 0035 a ČSN 73 1401.

(b) Partial factors design according to Eurocodes; characteristic values (s_k) taken from the current ground snow load map of the Czech Republic given in ČSN EN 1991-1-3; $\gamma_Q = 1.50$.

(c) Partial factors design according to Eurocodes; characteristic values (s_k) taken from the new digital ground snow load map of the Czech Republic; $\gamma_Q = 1.50$.

(d) Partial factors design according to Eurocodes; characteristic values (s_k) taken from the new digital ground snow load map of the Czech Republic; design value of snow load (s_d) derived according to EN 1990 - Annex C.

(e) Direct probabilistic design according to Eurocodes and JCSS documents; permanent action represented by normal distribution N($\mu = g_k$, $V = 0.02$); snow load on the ground represented by 50-year maxima distribution (Gumbel distribution); $P_d = 7.2 \cdot 10^{-5}$ for safety assessment and $P_d = 6.7 \cdot 10^{-2}$ for serviceability assessment; in accordance with JCSS documents θ_R is considered as lognormal distribution LN0($\mu = 1$, $V = 0.05$) and θ_R as lognormal distribution LN0($\mu = 1$, $V = 0{,}1$); yield strength of steel S235 is according to (Marek et al., 2006) represented by normal distribution N($\mu = 292$ MPa, $V = 0.055$); the probabilistic analysis is carried out using Monte-Carlo method.

(f) Direct probabilistic design according to the SBRA method (Křivý & Marek, 2007); permanent action represented by normal distribution N($\mu = g_k$, $V = 0.02$); snow load on the ground represented by corresponding load duration curves; $P_d = 7.2 \cdot 10^{-5}$ for safety assessment and $P_d = 6.7 \cdot 10^{-2}$ for serviceability assessment (the values of target probability P_d are given for probabilistic analysis based on EN 1990; P_d for analysis based on load duration curves are not available yet); in accordance with JCSS documents θ_R is considered as lognormal distribution LN0($\mu = 1$, $V = 0.05$) and θ_R as lognormal distribution LN0($\mu = 1$, $V = 0.1$); yield strength of steel S235 is according to (Marek et al., 2006) represented by normal distribution N($\mu = 292$ MPa, $V = 0.055$); the probabilistic analysis is carried out using Monte-Carlo method.

The purlin from steel S235 is a part of a flat roof structure. The purlin is designed as a simple supported beam with the length of 5 m. Spacing of purlins is 2.5 m. The purlin is secured against lateral-torsional buckling by roof cladding. Normal topography according to EN 1991-1-3 is considered. Limit vertical deflections are $\delta_{max} = L/200$ a $\delta_2 = L/250$. Only the combination of permanent and snow load is considered in this study. The design is carried out for three selected localities from Tab. 1:

1) Prague (permanent action from roof cladding and self-weight of purlin: $g_k = 35$ kg/m);
2) Havířov (permanent action from roof cladding and self-weight of purlin: $g_k = 45$ kg/m);
3) Hanušovice (permanent action from roof cladding and self-weight of purlin: $g_k = 110$ kg/m).

Results of the study are summarized in Tab. 2. Table contains minimal required section modulus of the purlin W_{min} (safety assessment) and minimal required moment of inertia I_{min} (serviceability assessment).

Tab. 2: Minimal required cress-sectional characteristics.

procedure according to point	W_{min} ($mm^3 \cdot 10^3$)	I_{min} ($mm^4 \cdot 10^6$)	W_{min} ($mm^3 \cdot 10^3$)	I_{min} ($mm^4 \cdot 10^6$)	W_{min} ($mm^3 \cdot 10^3$)	I_{min} ($mm^4 \cdot 10^6$)
	Prague		Havířov		Hanušovice	
(a)	38.54	2.91	53.22	4.07	110.11	8.72
(b)	34.18	2.71	67.92	5.81	179.32	15.50
(c)	29.13	2.32	48.07	3.89	110.51	8.82
(d)	37.53	2.32	62.53	3.89	142.02	8.82
(e)	46.41	3.63	78.60	6.34	179.01	14.31
(f)	21.63	0.70	37.04	1.20	84.20	3.55

4. Conclusions

The values given in Tab. 2 show considerable differences between the results obtained using the six different procedures. It is interesting to compare the results obtained according to the European standards, *i.e.* the results corresponding to points (b) till (e) in Tab. 2. The direct probabilistic design in line with EN 1990 is most conservative. On the contrary, the procedure according to point (c) leads to the lowest required values of cross-sectional characteristics.

Considerably lower values of cross-sectional characteristics are required when applying the direct probabilistic design using SBRA method – see point (f) in Tab. 2. This disproportion is caused by unsuitable selection of target probabilities P_d that are not calibrated for probabilistic analysis with the load duration curves (P_d calibrated for probabilistic analysis applying 50-year maximum distributions according to EN 1990 were used in this study).

The procedure according to point (c) is most suitable for the practical designing of roof structural elements – *i.e.* characteristic value of snow load on the ground is determined from new digital map and partial factor $\gamma_Q = 1.50$ is considered. The procedure according to point (b) is significantly influenced by conservative values taken from the current ground snow load map of the Czech Republic given in ČSN EN 1991-1-3 (see the comparison of characteristic values given in Tab. 1). The procedures according to points (d) and (e) are influenced by the fact that the snow load is derived from the theoretical 50-year maximum distributions. The general problem of both direct probabilistic approaches (e) and (f) is a suitable choice of target probability P_d (mainly the relationship between the definition of random variable quantities and P_d). The study shows that the procedure according to point (e) may lead to unreasonably conservative results. On the contrary, an introducing of target probabilities P_d from EN 1990 to the probabilistic analysis based on load duration curves, see point (f), leads to unreasonably low values of cross-sectional characteristics.

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