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PROBLEM OF IDENTIFICATION OF DETERMINISTIC CHAOS IN THE INTERACTIVE DRIVE SYSTEMS

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Abstract: *Chaos and chaos theory is a field of study in mathematics, computer sciences, electronics, physics and also engineering too. In our article will be explored chaotic behaviour and numerical solutions of the models of drive systems with electric DC motors. These solutions are also bounded like equilibrium, periodic and quasiperiodic solution. There is no precise definition for a chaotic solution because it cannot be represented through standard mathematical functions. However, a chaotic solution is aperiodic solution, which is endowed with some special identifiable characteristics, for example attractors, bifurcations or one-and two dimensional maps.*

Keywords: ***Chaos theory, attractor, assessment, phase portrait, power spectral density.***

1. Introduction

Chaotic behaviour of deterministic system usually means that behaviour is an appearance of absolute random without causing any inherent laws. We can characterize the behaviour of real systems in engineering practice by their trajectories, which remain bounded in the phase space. These time-ordered set of states of a dynamical system are critically dependent on even small changes in initial conditions. That is why the chaotic behaviour of systems is long-term unpredictable. Despite the common belief that chaotic states occurs at a very low frequency, the chaotic behaviour can we mainly observe by the controlled systems. As well as the existence of nonlinearities, the fundamental attributes of everyday reality.

We can describe the behaviour of real systems in engineering practice as a deterministic chaos. Especially, in case of complex interactive systems, those arise in systems of coherent structures. We can observe random elements during system development even though the system as a whole may seem deterministic. It appears that under certain conditions we can see extreme amplification of certain disorders. That is because the systems act as filters, suppressing or amplifying certain disorders.

These partial processes can be initially linear. But they can lead after a certain time to the massive application of pervasive non-linearity (Moon, 1992). Especially in cases, if there is failure to gain over a certain limit, shifting the entire system into a chaotic state. This is a general property of nonlinear dynamical systems described by mathematical equations; sensitivity to small changes in initial conditions, excitation or change some parameters vary depending on the conditions and nature of these disorders. We can observe states of disorder – at first appears extreme increasing vulnerability to certain disorders, then there is a loss of stability of the system changing the quality of its behaviour. The chaotic behaviour of the system increases its complexity (the number of degrees of freedom grows). There is a simple reason why the study of the chaotic states has no research attention so long. It is because the researchers were not able to use linear mathematical models.

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In recent years much attention has been devoted to the study the phenomena of chaos. It is shown that chaos control can be extremely effective and often the only possible solution to many difficult problems. But chaos identification is not simple or straightforward especially in case of complex sets of systems. We are able to identify chaos rapidly in case of simple dynamical systems with typical grading procedures – e.g. Lyapunov exponents, Poincaré maps, conditional and unconditional methods for identification of coherent structures, bifurcation diagrams, etc. All these approaches are often ambiguous and considered as rarely applicable for complex technical systems. Therefore they are unacceptable in engineering practice at the present state of knowledge. For this reason, we assumed alternative approaches and this paper presents some interesting results.

2. Identification of chaotic states of complex dynamical systems

At first, we must remember, that there is no standard definition of chaos in science and technology yet. However, we can specify a range of attributes of chaos. And we use ones which can characterize the notion of chaos and identify states of disorder. There is an important way to evaluate the chaotic states – the assessment of genesis and structure of chaotic attractors.

Some attractors, e.g. point attractor, represent the final state of a system in equilibrium. Others, e.g. cycle attractor, describe the final state of periodically repetitive variation between two or more different states. Chaotic attractor, also known as strange attractor, characterizes the system that never reaches the final state of equilibrium or repeating cycle. We are able to observe the second and third ones, cycle or strange attractors, in phase space – even in response to changes in any critical parameter. Attractors are more than a mathematical abstraction. They represent a very important type, so called in-form. The system state is close to the attractor more probably and therefore more developed, than other possible states. If the trajectory of the dynamical system in the attractor would be in line with the image of the desired future state, the history contains this attractor will be strengthened and increases the likelihood of its implementation. On the contrary, inconsistency of both images will lead to suppression of unwanted history (Kratochvíl & Heriban, 2010).

Very interesting is also the relation between the phase diagrams before and during stabilization of chaotic attractors, and some statistical characteristics of the signals.

And also these changes can predict achieving closeness or reaching chaotic states. This is not about any breakthrough – it is only the ability to correctly interpret the results easily reachable and resulting from realistic simulation experiments.

3. Dynamical system analysis example

We demonstrate the methodology outlined to identify chaotic states of drive system on a model (see Fig. 1).

The drive system model is controlled and the control scheme is as shown in Figure 2. Consider only the basic structure, including the effects of excitation, as shown in Figure 1. We can describe this mathematical model (discretized time-invariant system) in the form of matrix equation (Sprott, 2003)

$$\dot{x} = A(t).x, \;\; x(t) \in R^n \tag{1}$$

where $x(t)$ is the vector of state variables and $A(t)$ the matrix system. Eq. (1) represents an approximate linearization of the system $\dot{x} = f(x), \;\; x(t) \in X \subset R^n$ along a reference trajectory. Due to linearity, we can assume that equation (1) is known and any initial conditions are given by

$$x(t) = \Phi(t, t_0), \;\; \forall(t, t_0) \in R^1 \tag{2}$$

which is called the fundamental matrix of linearized system. Lyapunov coefficients, which indicate the range of sensitivity to initial conditions, are then given by the equation

$$\lambda_i = \underbrace{\lim}_{t \to \infty} \frac{\left\| \Phi(t, t_0) \right\|}{t - t_0} \tag{3}$$

Matrix $A(t)$ obtained by linearization along the trajectory in phase portraits of non-linear system of order n generates n Lyapunov coefficients, which are always real. However, numerical calculations of

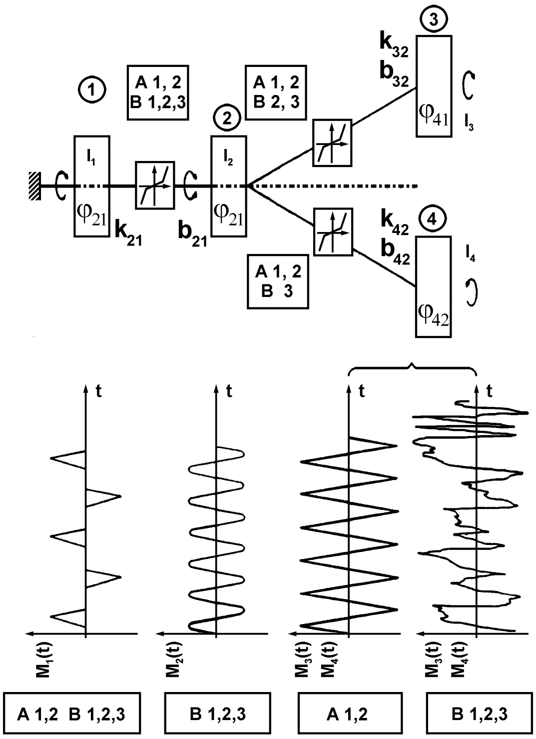

Fig. 1: Drive system model.

their values require different approximations and averaging are often problematic (Moon, 1992). In described case it is doubly true, the solution is given by the extra control method as indicated in Fig. 2.

The analysis of state trajectories but also allows the identification of chaos (Kratochvíl & Heriban, 2010), as we discussed in previous paragraph. In our case, we choose following combinations of tightness in linkage: will assume 0.010 [rad] in the linkage (1-2), and one after another 0.015, 0.030 and 0.050 [rad] in the linkage (2-3). The top row of Fig. 3 shows the phase portraits for third combination and the bottom row second combinations of tightness in second linkage. We will begin with the chaotic state, as shown in the left column, and see how the behaviour of the growing tightness in the linkage (2-3) is gradually changing the system in order to stabilize it. The process is also obvious in the progression of standard stochastic characteristics – the power spectral densities (PSD), see Fig. 4. Left part of Fig. 4 shows a progression of PSD for third linear case without tightness. Middle part of Figure 4 shows results for a combination of tightness 0.010 and 0.015 [rad]. Right part of Figure 4 shows results for a combination of tightness 0.010 and 0.030 [rad]. We can clearly identify differences of linear and nonlinear model of drive. Also, we can describe the suppression of spurious interprocedural cycles for gradual stabilization of the system. At the same time, we demonstrate a correlation between the statistical characteristics of phase portraits in the phase plane.

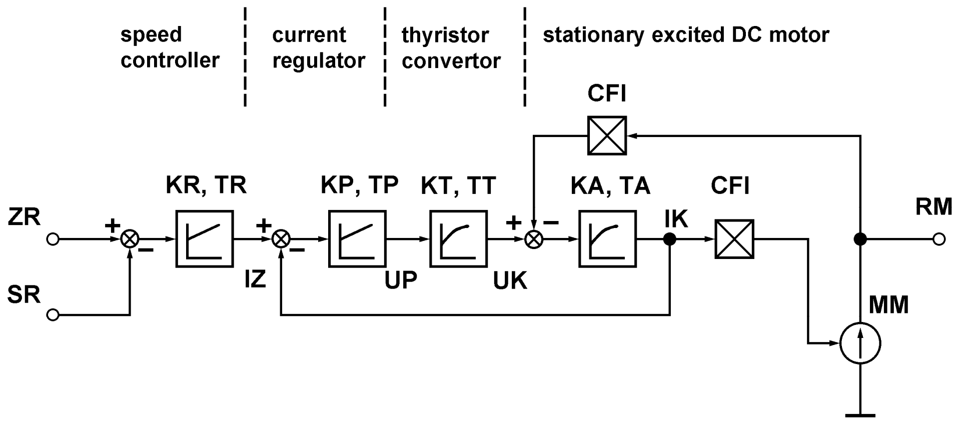

Fig. 2: Control scheme.

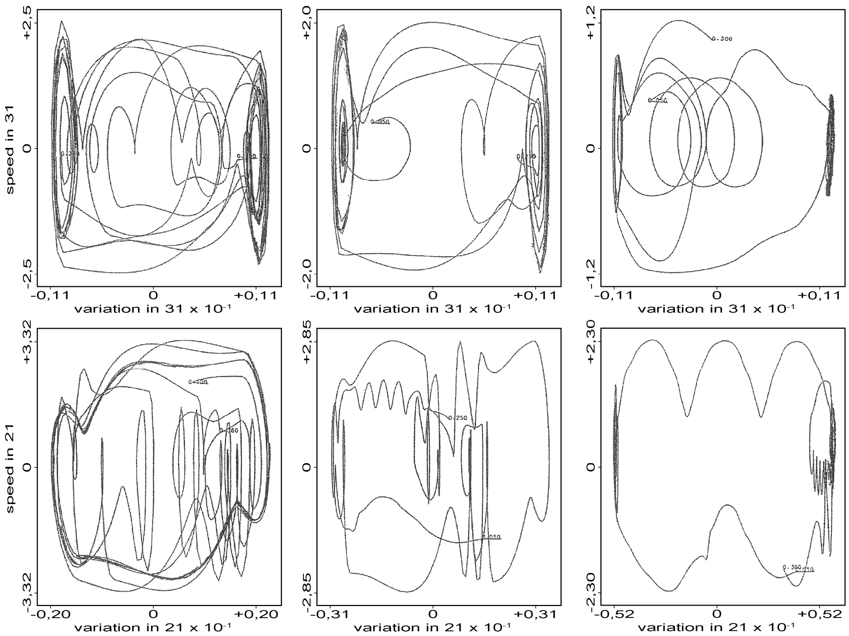

Fig. 3: Phase portraits.

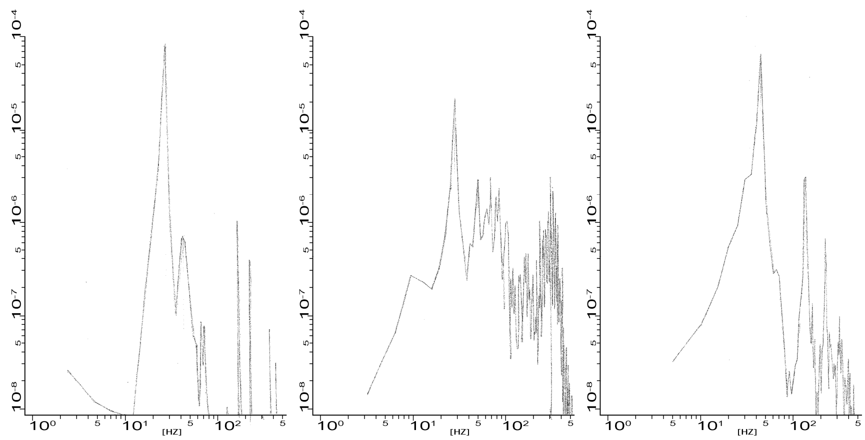

Fig. 4: Power spectral density (PSD).

4. Conclusions

In recent days we analyze different combinations of parameters and models of dynamical systems. And we compare the entire spectrum of the primary statistical characteristics and its correlation. We are able to observe clear links between these waveforms and waveform characteristics of phase portraits in the phase planes. We detect characteristic changes that appear just before the systems reaches chaos states and after the transitions. Also we hope that we will be able to generalize. These results would lead to a significant simplification of the computational analysis of complex dynamical systems.

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